5.3 The electrons inside a system of two coaxial magnetic mirrors can be described by the so-called *loss-cone distribution function*

$$f(\mathbf{v}) = \frac{n_0}{\pi^{3/2}\alpha_\perp^2 \alpha_\parallel}\left(\frac{v_\perp}{\alpha_\perp}\right)^2 exp\left[-\left(\frac{v_\perp}{\alpha_\perp}\right)^2 - \left(\frac{v_\parallel}{\alpha_\parallel}\right)^2\right]$$

where $v_\parallel$ and $v_\perp$ denote the magnitudes of the electron velocities in the directions parallel and perpendicular to the magnetic bottle axis, respectively, and where $\alpha_\parallel^2 = 2kT_\parallel/m$ and $\alpha_\perp^2 = 2kT_\perp/m$.
(a) Verify that the number density of the electrons in the magnetic bottle is given by n_0.

(b) Justify the applicability of the loss-cone distribution function to a magnetic mirror bottle by analyzing its dependence on $v_\parallel$ and $v_\perp$. Sketch, in a three-dimensionsal perspective view, the surface for $f(\mathbf{v})$ as a function of $v_\parallel$ and $v_\perp$.

6.3 For the loss-cone distribution function of problem 5.3 (in Chapter 5), show that

$$\tfrac{1}{2}m < v_\parallel^2 >= \tfrac{1}{4}m\alpha_\parallel^2$$

$$\tfrac{1}{2}m < v_\perp^2 >= m\alpha_\perp^2$$

6.7 Consider (5.6.4), which is the solution of the Boltzmann equation with the relaxation model for the collision term, in the absence of external forces and spatial gradients, and when $f_{\alpha 0}$ and the relaxation time τ are time-independent. Show that, according to this simplified equation, we have

$$G_\alpha(t) = G_{\alpha 0} + [G_\alpha(0) - G_{\alpha 0}] \exp{(-t/\tau)}$$

where

$$G_\alpha(t) = \int_v f_\alpha \, \chi \, d^3v = n_\alpha < \chi >_\alpha$$

$$G_{\alpha 0} = \int_v f_{\alpha 0} \, \chi \, d^3v = n_\alpha < \chi >_{\alpha 0}$$

Thus, according to the relaxation model for the collision term, every average value $< \chi >_\alpha$ approaches equilibrium with the same relaxation time.

$$f_\alpha(\mathbf{v},t) = f_{\alpha 0} + [f_\alpha(\mathbf{v},0) - f_{\alpha 0}]\, e^{-t/\tau} \qquad (6.4)$$