$$\underbrace{E_k \cdot \ldots \cdot E_2 \cdot E_1}_{B} \cdot A = I_n \qquad B \cdot A = I_n \Rightarrow B = A^{-1}$$

Dk $\Leftarrow$:

$$A = E_k \cdot \ldots \cdot E_1$$

Zřejmé součin reg. je reg.

$$\Longrightarrow (A^{-1})^{-1} \cdot A^{-1} = I_n$$

$$(A^{-1})^{-1} = E_k \cdot \ldots \cdot E_1 \quad \checkmark$$

$\Rightarrow A \sim B$

$$A = \underbrace{E_k \cdot \ldots \cdot E_1}_{= P} \cdot B$$

$\Leftarrow$ $A = P \cdot B$, P reg

$$P = E_k \cdot \ldots \cdot E_1 \Rightarrow A \sim B$$

$$\dim[r_1(A), \ldots, r_m(A)] = h(A)$$

$$\dim[r_1(B), \ldots, r_m(B)] = h(B)$$

$$\underset{m\times n}{B} = \underset{m\times m}{P}\,\underset{m\times n}{A}$$

$$(A \mid B) \sim \ldots\ldots \sim (\underline{\underline{I_n}} \mid \underset{?}{\widetilde{A^{-1}B}})$$

$$A^{-1}A = I_n$$

$$A^{-1}B = \checkmark$$

$$A\mathbb{x} = \mathbb{b} \quad / A^{-1}$$

$$\underbrace{(A^{-1}A)}_{=I_n} \cdot \mathbb{x} = A^{-1}\mathbb{b}$$

$$\mathbb{x} = I_n \cdot \mathbb{x} = A^{-1}\mathbb{b} \quad \checkmark$$

$$(id_V)_{\alpha,\beta} = \left((v_1)_\alpha, \ldots, (v_n)_\alpha \right)$$

$$P_{\beta,\alpha} = \left((u_1)_\beta, \ldots, (u_n)_\beta \right)$$

$$(f(x))_\alpha = (f)_{\alpha,\beta} \, (x)_\beta \qquad f = id_V$$

$$(x)_\alpha = \underset{P_{\alpha,\beta}}{\underset{\|}{(id)_{\alpha,\beta}}} \cdot (x)_\beta \qquad (i) \Rightarrow (iii)$$

(I) $\Rightarrow$ (II) viz před.

$\| \; o_j(P_{\alpha,\beta})$

(II) $\Rightarrow$ (III) $\quad v_j = \alpha\,(v_j)_\alpha = o_j(\alpha \cdot P_{\alpha,\beta})$

$$(v_1, \dots, v_n) = \alpha \cdot P_{\alpha,\beta}$$

(III) $\Rightarrow$ (I) $\quad \alpha \cdot P = \beta \Rightarrow P \overset{?}{=} P_{\alpha,\beta}$

$$P_{\alpha,\beta} = ((v_1)_\alpha, \dots, (v_n)_\alpha) \qquad \underset{\|}{(v_j)_\alpha}$$

$$v_j = o_j(\beta) = o_j(\alpha \cdot P) = \alpha \cdot o_j(P) \quad \checkmark$$

$$P_{\alpha,\alpha} = I_n \quad \checkmark \qquad \alpha = (v_1, \dots, v_n)$$

$$\parallel$$

$$((v_1)_\alpha, \dots, (v_n)_\alpha) \qquad (v_1)_\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$v_1 = 1 \cdot v_1 + 0 \cdots + 0$$

$$(id)_{\alpha,\beta} \cdot (id)_{\beta,\gamma} = (id)_{\alpha,\gamma}$$

$$P_{\alpha,\beta} \cdot P_{\beta,\alpha} = P_{\alpha,\alpha} = I_n \Rightarrow P_{\alpha,\beta} = (P_{\beta,\alpha})^{-1}$$

$$\gamma = \alpha \cdot P^{-1} \Rightarrow \gamma \cdot \underset{\parallel P_{\gamma,\alpha}}{P} = \alpha$$

lis 15-10:48

$$P = \varepsilon \cdot P$$

$$\underbrace{(o_1(P), \ldots, o_n(P))}_{\alpha} = \varepsilon \cdot P_{\varepsilon,\alpha}$$

$$I_n = \varepsilon = P^{-1} \cdot P = \underbrace{(o_1(P^{-1}), \ldots, o_n(P^{-1}))}_{\beta} \cdot P$$

$$\varepsilon = \beta \cdot P_{\beta,\varepsilon}$$

$$\alpha \cdot P_{\alpha,\beta} = \beta \qquad / \alpha^{-1}$$

$$P_{\alpha,\beta} = \alpha^{-1}\beta$$

$$(\alpha \mid \beta) \sim (I_n \mid \alpha^{-1}\beta)$$

$$P_{\beta_2\alpha_2} \cdot (\varphi)_{\alpha_2,\alpha_1} \cdot P_{\alpha_1,\beta_1} =$$

$$(id)_{V_2 \beta_2,\alpha_2} \cdot (\varphi)_{\alpha_2,\alpha_1} \cdot (id)_{V_1 \alpha_1,\beta_1} =$$

$$(id_{V_2} \circ \varphi \circ id_{V_1})_{\beta_2,\beta_1} = (\varphi)_{\beta_2,\beta_1}$$

lis 15-11:06

(i) $\Rightarrow$ (ii) $A = (\varphi)_{\alpha_2, \alpha_1}$

$B = (\varphi)_{\beta_2, \beta_1}$

$B = P_{\beta_2, \alpha_2} \cdot A \cdot P_{\alpha_1, \beta_1}$

$P = P_{\beta_2, \alpha_2}$, $Q = P_{\alpha_1, \beta_1}$

$B = P \cdot A \cdot Q$

(ii) $\Rightarrow$ (iii) $h(A) = h(P \cdot A) = h(P \cdot A Q)$

$= h(B)$

$(iii) \Rightarrow (ii)$ $\quad h(A) = h(B) = k$ RST

$$A \sim \cdots \overset{RST}{\sim} \bar{A} \qquad B \sim \cdots \sim \bar{B}$$

$$\bar{A} = {}^{k}\Big(\ \ \Big) \qquad \bar{B} = {}^{k}\Big(\ \ \Big)$$

schodiskový

$$\bar{\bar{A}} = \begin{pmatrix} I_k & 0 \\ 0 & 0 \end{pmatrix} = \bar{\bar{B}}$$

$$\bar{\bar{A}} = P_1 A \cdot Q_1$$

$$\parallel$$

$$\bar{\bar{B}} = P_2 B \cdot Q_2$$

$$B = (P_2^{-1} P_1) \cdot A (Q_1 Q_2^{-1})$$

(ii) $\Rightarrow$ (i) $\quad B = PAQ$

α_1 báze V, α_2 báze U

$A \longleftrightarrow \varphi : V \to U$, $\quad A = (\varphi)_{\alpha_2,\alpha_1}$

$(\varphi)_{\beta_2,\beta_1} = P_{\beta_2,\alpha_2} \cdot (\varphi)_{\alpha_2,\alpha_1} \, P_{\alpha_1,\beta_1}$?

$Q = P_{\alpha_1,\beta_1} \qquad \alpha_1 \cdot P_{\alpha_1,\beta_1} = \beta_1 \qquad \alpha_1 Q = \beta_1$

$P = P_{\beta_2,\alpha_2} \qquad \beta_2 \cdot P_{\beta_2,\alpha_2} = \alpha_2 \qquad \beta_2 = \alpha_2 \cdot P^{-1}$

$A \qquad r_1(A), \dots, r_m(A)$

$\varphi(x) = Ax \qquad r_{j_1}(A), \dots, r_{j_n}(A)$

báze $\mathrm{Im}\,\varphi = \mathrm{Im}\,A$

$B = (r_{j_1}(A), \dots, r_{j_n}(A))$

$a_j := r_j(A) = \boxed{B \cdot c_j} \qquad \boxed{A = BC}$

$m = n = 10^4$ řádků $\qquad 10^8$ paměťových míst

$n = 100 \qquad B \quad m \times n = 10^6$ paměti

$C \quad n \times m = 10^6$