Complex Analysis II (M7270) Course Information

Prerequisits. Linear Algebra I and II, Mathematical Analysis III and IV, Complex Analysis I.

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Course credits: 4

Timetable: Wed, Fri 12:00 – 13:50, M3. Lectures start Fri, Sep 20.

Overview. Complex Analysis is one of the classical and possibly most beautiful areas of mathematics, which has numerious applications in Geometry, Analysis, Number Theory, Applied Mathematics and Physics. Complex Analysis studies functions of (one or several) complex variables satisfying the complex differentiability condition (holomorphic functions). It turns out that the latter simple condition forces unbelievably strong properties of a holomorphic function, such as its infinite differentiability and analyticity (local representation as a sum of a power series). The special feature of Complex Analysis is that very strong results, e.g. the Riemann Mapping Theorem ("every proper simply connected domain in complex plane is conformally equivalent to the unit disc"), appear in an extremely elegant way from a sequence of simple geometric/analytic statements, each of which can be exposed to the students "in one line". This illuminates the effectiveness of Complex Analysis in application to other fields, where Complex Analysis gives a simple geometric picture for numerous difficult and important phenomena.

The course is dedicated to studying in detail advanced aspects of Complex Analysis which fall out of the Complex Analysis I course. At a glance, it is structured as follows.

We first study advanced topics from Complex Analysis in one variable: the theory of conformal mappings, Schwarz reflection principle, the argument principle and its applications, multi-valued analytic functions and Riemann surfaces, Riemann mapping theorem, Runge approximation theorem, harmonic functions of two variables.

Next, we study selected topics from Complex Analysis in Several Variables: holomorphic functions of several variables, Hartogs theorem, analytic continuation, envelopes of holomorphy, holomorphic mappings, basics of the d-bar theory.

Syllabus:

- Brief overview of the Complex Analysis I course
- Riemann sphere. Linear-fractional mappings.
- Conformal mappings of domains. The Schwarz reflection principle.
- Schwarz Lemma. Automorphisms of model domains in the Riemann sphere.

- Analytic continuation. Multi-valued analytic functions. Isolated singularities. Riemann surfaces.
- The argument principle, Rouche's Theorem, and applications. Openness principle, Hurwitz's theorem.
- Families of holomorphic functions. Compactness principle. Riemman Mapping Theorem.
- Additional topics: the modular function, Small Picard Theorem, Runge Theorem, Weierstrass Factorization Theorem.
- Harmonic functions in two variables. Dirichle Problem.
- The geometry of complex space. Holomorphic functions in several variables. Cauchy formula. Power series expansion. Hartogs separate analyticity theorem.
- The phenomenon of forced analytic continuation. Holomorphic convexity. Analytic discs. Envelopes of holomorphy.
- Holomorphic mappings. Schwarz Lemma. Automorphism groups of domains in complex space.
- Basics of the d-bar theory.

Literature.

[1] Gamelin, Theodore W. Complex analysis. (English) Undergraduate Texts in Mathematics. Springer-Verlag, New York, 2001.

[2] Shabat, B. V. Introduction to complex analysis. Part II. Functions of several variables. (English) Translated from the third (1985) Russian edition by J. S. Joel. Translations of Mathematical Monographs, 110. American Mathematical Society, Providence, RI, 1992.

[3] Chabat, B. Introduction à l'analyse complexe. Tome 1. (French) Fonctions d'une variable. Translated from the Russian by Djilali Embarek. Traduit du Russe: Mathématiques. "Mir", Moscow, 1990.