**Ústav fyzikální elektroniky**
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# Fyzikální praktikum 4

# Planck's radiation law and measurement of temperature

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## 1 Planck's law

The emission of radiation from heated bodies (incandescence) is manifested with a continuous optical spectrum. Besides real heated bodies, a model of a so-called black body, absorbing all the light at all wavelengths, plays a fundamental role, since its spectral dependence of intensity is directly described by Planck's distribution. This distribution expressed particularly either as energy density $\rho$ (energy of radiation present in unit volume), radiance $L$ (radiation power emitted from unit projected perpendicular surface into unit solid angle) or irradiance $M$ (radiation power emitted from unit surface). Since for isotropic radiation and ideal diffuse surface

$$L = \frac{1}{4\pi}\rho c, \qquad M = \pi L,$$

the expressions of Planck's law have the forms

$$\begin{aligned}
\rho_\mathrm{b}(\lambda) &= \frac{8\pi hc}{\lambda^5}\frac{1}{\mathrm{e}^{\frac{hc}{k_\mathrm{b}T\lambda}}-1} \qquad (\mathrm{J m^{-3}\,m^{-1}}),\\
L_\mathrm{b}(\lambda) &= \frac{2hc^2}{\lambda^5}\frac{1}{\mathrm{e}^{\frac{hc}{k_\mathrm{b}T\lambda}}-1} \qquad (\mathrm{W m^{-2}\,sr^{-1}\,m^{-1}}),\\
M_\mathrm{b}(\lambda) &= \frac{2\pi hc^2}{\lambda^5}\frac{1}{\mathrm{e}^{\frac{hc}{k_\mathrm{b}T\lambda}}-1} \qquad (\mathrm{W m^{-2}\,m^{-1}}).
\end{aligned}$$

where $T$ is thermodynamic temperature, $\lambda$ is wavelength, $h$ is Planck's constant ($6.625 \times 10^{-34}$ J s), $c$ vacuum speed of light ($2.998 \times 10^8$ m s$^{-1}$) and $k_\mathrm{b}$ Boltzmann constant ($1.38 \times 10^{-23}$ J K$^{-1}$). Here, the energy is expressed per unit wavelength interval (last m$^{-1}$ or nm$^{-1}$ in units) in a form convenient for the measurements in the visible spectral region. From the formulas usually derived for energy per frequency interval (Hz$^{-1}$) they are obtained as

$$L(\lambda) = L(\nu)\left|\frac{\mathrm{d}\nu}{\mathrm{d}\lambda}\right|.$$

The differentiation of Planck's law with respect to the wavelength and the integration of Planck's law over all wavelengths give two important laws:

1. Wien displacement law of intensity maximum

$$\lambda_\mathrm{max}T = \mathrm{konst} = 2.88 \times 10^{-3}\,\mathrm{m\,K},$$

2. Stefan–Boltzmann law of total radiation intensity

$$M_\mathrm{b} = \sigma T^4, \qquad \sigma = 5.67 \times 10^{-8}\,\mathrm{W m^{-2} K^{-4}}.$$

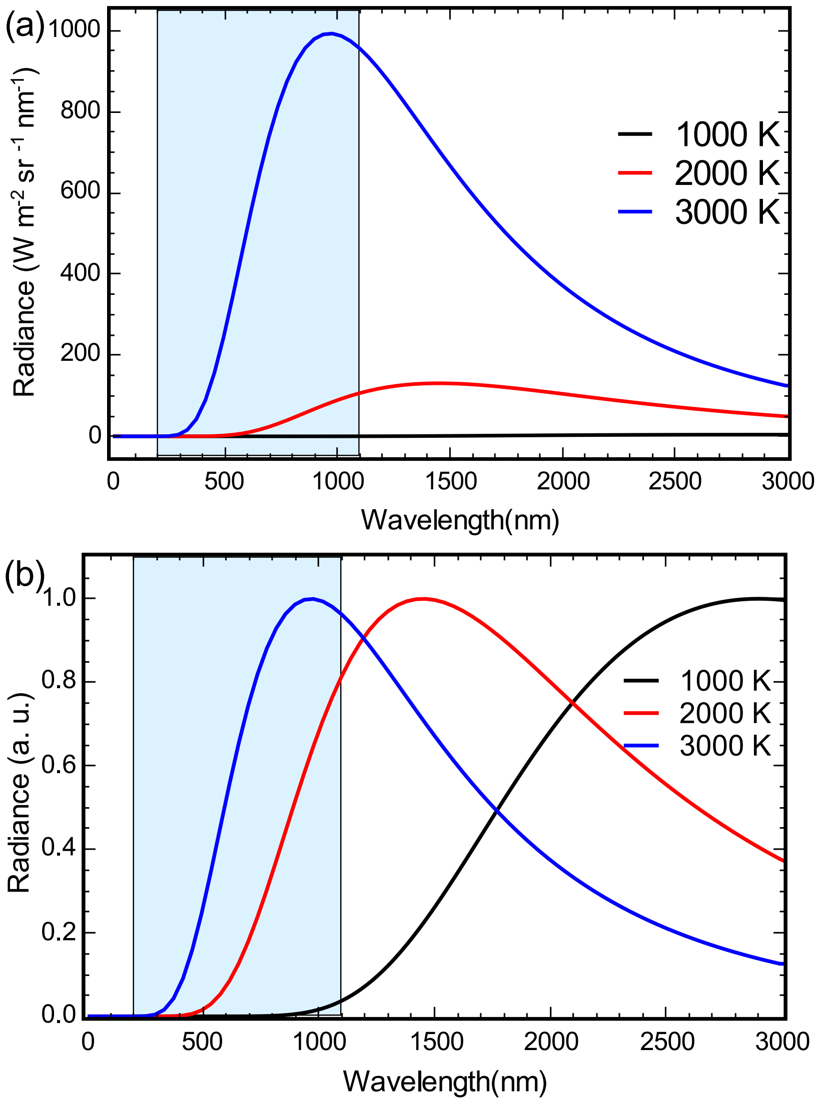


Figure 1: Planck's law plotted (a) as radiance $L(\lambda)$ in absolute values and (b) normalized to unit intensity maximum for three temperatures accessible with common incandescent sources. Region of wavelengths accessible with spectrometer is demarked in the plot.

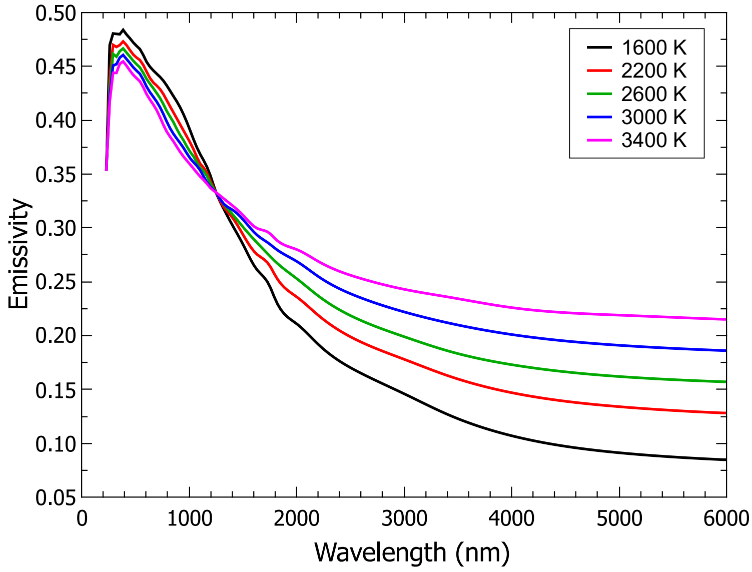


Figure 2: Emissivity of tungsten as a function of wavelength and temperature. The data were taken from [2].

Note, that Stefan–Boltzmann law is obtained by integration over the wavelengths and also over the relevant solid angle and therefore $\sigma$ of this value is the only constant in expression for irradiance.

The spectral radiance $L_\mathrm{b}(\lambda)$ for three temperatures accessible with common incandescent sources is shown in figure 1 (a), the normalized distributions with unit intensity maxima are shown in figure 1 (b). Obviously, only at temperatures of $\approx 3000\,\mathrm{K}$ and higher can the distribution maximum be registered with a spectrometer range $200 - 1100\,\mathrm{nm}$. Using Wien displacement law, the intensity maximum lies at 2898 nm for 1000 K, at 1440 nm for 2000 K, at 966 nm for 3000 K and at 781 nm for 3687 K (melting point of pure tungsten [1]).

Without an absolute intensity measurement, the registration of intensity maximum simplifies principally the correct determination of temperature from the measured spectra. Anyway, the relative spectrometer sensitivity must be known.

The deviation of emission of radiation of real bodies from Planck's law is described by an emissivity $\varepsilon$

$$L(\lambda;T) = \varepsilon(\lambda;T)L_\mathrm{b}(\lambda;T),$$

where $L(\lambda, T)$ is the radiance of a real-body surface with thermodynamic temperature $T$ at wavelength $\lambda$ and $L_\mathrm{b}(\lambda, T)$ is the radiance of black body with the same temperature and at the same wavelength. Emissivity of black body is always a unit, otherwise $\varepsilon < 1$.

With a constant emissivity with respect to the wavelength, the radiator is called a grey body. Otherwise it is called a selective radiator. E.g., in the case of tungsten the emissivity in visible light weakly depends on the wavelength, decreasing towards the longer wavelengths (see figure 2). Therefore, the tungsten filament lamp is a slightly more efficient light source than the black body at the same temperature, although emitting only fraction of the total blackbody radiation intensity.

Without absolute intensity measurement, the constant emissivity $< 1$ of tungsten cannot be determined. The only important influence on the measured spectra may therefore arise from the spectral dependence of the emissivity. In order to resolve, whether the tungsten may be handled as grey body (no emissivity correction is needed) or selective radiator (the measurements must be compared with $L_\mathrm{W}(\lambda) = \varepsilon_\mathrm{W}(\lambda)L_\mathrm{b}(\lambda)$), Planck's distribution with and without emissivity correction was calculated for 3000 K and plotted normalized in figure 3 (a). The spectral emissivity

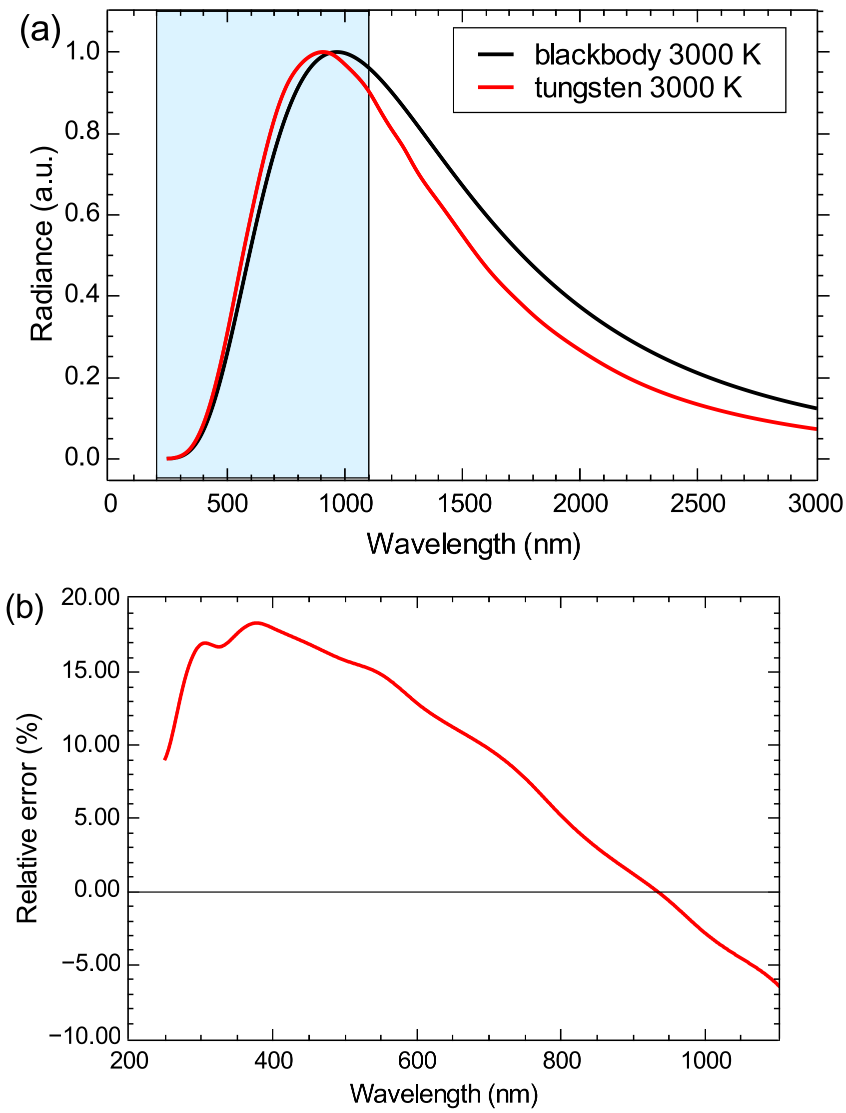


Figure 3: Comparison of black body ($L_{\mathrm{b}}(\lambda)$) and tungsten ($\varepsilon(\lambda)L_{\mathrm{b}}(\lambda)$) for temperature 3000 K. The curves were normalized according to their maxima (a). A relative difference between the normalized distributions (b).

of tungsten for temperature 3000 K was taken in these calculations (see again figure 2). A relative difference between the curves

$$\frac{L^{\mathrm{n}}_{\mathrm{W}}(\lambda) - L^{\mathrm{n}}_{\mathrm{b}}(\lambda)}{L^{\mathrm{n}}_{\mathrm{W}}(\lambda)}$$

is shown in figure 3 (b) with superscript n denoting normalizated distribution with unit intensity maximum. The relative differences are generally lower than 20 %.

## 2 Experimental set-up

Tungsten-filament halogen lamps belong to the the most available and applicable incandescent light sources operated at high temperatures. The halogen lamp is preferred to standard tungsten lamp, since the halogen cycle ensures the transport of tungsten atoms from the neighbourhood of the glass walls back to the filament and thus enables the operation of the lamp at a higher filament temperature. In our experiment it is probably better to avoid types such as ECO or with a UV-

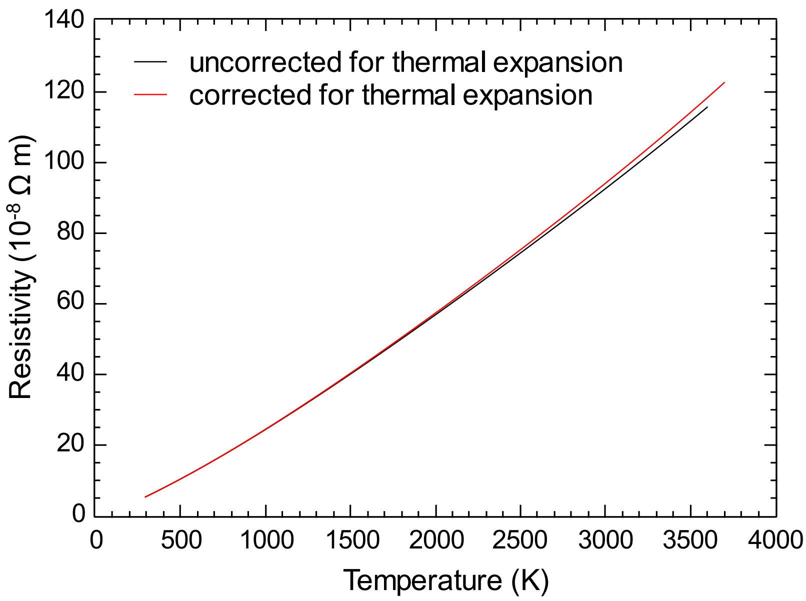


Figure 4: Temperature dependence of tungsten resistivity. The uncorrected curve, showing in principle the dependence of resistance on the temperature, differs from the true resistivity dependence due to thermal expansion.

stop filter. The infrared coating on glass bulb of the halogen lamp reflects IR radiation back to the filament, reducing the necessary power input. On the other end of visible spectrum, the UV-stop filter doped quartz glass absorbs mostly UV-C and UV-B radiation, but also substantially reduces UV-A radiation [3]. Although absorption from both coatings in the visible spectrum is probably negligible, it is better to use a bulb without such modifications. There are also several types of lamps with the same voltage and wattage, differing in output luminous fluxes and lifetimes. In this case, the lamp with the highest luminous flux and the lowest lifetime is expected to have the highest filament temperature at nominal voltage. Osram HLX 64642 24 V/150 W halogen lamp for slide and overhead projectors with no reflector nor luminaire can be taken as a convenient source for investigation.

The large advantage of the tungsten lamp is that the temperature of the filament may be deduced independently from the temperature dependence of tungsten resistance. The temperature dependence of tungsten resistivity is approximated in range 100 – 3600 K with formula [4]

$$\begin{aligned} \varrho(T_*) &= -0.968 + 19.274\,T_* + 7.826\,{T_*}^2 - 1.8517\,{T_*}^3 + 0.2079\,{T_*}^4 \\ [T_*] &= 1000\,\mathrm{K}, \quad [\varrho] = 10^{-8}\Omega\,\mathrm{m}. \end{aligned}$$

The estimated uncertainty of the data used for polynomial expression was about ±3% and the dependence was corrected for thermal expansion. Since in our case the measured quantity is the resistance, in which thermal expansion is included, the uncorrected resistivity dependence should be used instead. However, in the case of tungsten the difference is about 2 % (see figure 4). Dividing the resistivity dependence with the room temperature value

$$\varrho_{293} = 5.28 \times 10^{-8}\ \Omega\,\mathrm{m}\ (293\,\mathrm{K}),$$

an inverse function to $\varrho(T)/\varrho_{293}$ may be fitted with a simple parabolic formula

$$T = 154.6 + 186.4(\varrho/\varrho_{293}) - 1.314(\varrho/\varrho_{293})^2\ \mathrm{K},$$

from which the temperature is evaluated when the ratio of measured resistances is substituted instead of $\varrho/\varrho_{293}$. Note, that due to the large temperature range the formula is only approximate and does not provide precise values especially at low temperatures.

The optical spectra in 200 − 1100 nm wavelength range can be measured with small USB grating spectrometer Avaspec 3648 with 300 lines/mm grating and linear CCD array with 3648 pixels. The fixed slit width is 10 $\mu$m. The light from the studied sources can be collected with an optical fibre. The spectrometer is also equipped with filters providing the removal of higher spectral orders.

The instrument must be intensity-calibrated with a deuterium and tungsten halogen lamp with tabulated true spectrum.

## Problems

1. Measure the optical emission spectra of heated tungsten filament. Use supplied calibration data to correct the spectra for a non-uniform spectral sensitivity of the spectrometer.
2. Estimate the blackbody temperature by fitting the corrected spectra with Planck's law

$$f(\lambda) = \frac{A}{\lambda^5} \frac{1}{\mathrm{e}^{\frac{hc}{k_\mathrm{b} T \lambda}} - 1},$$

   $T$ is thermodynamic temperature, $A$ is scalling coefficient. Use e.g. QtiPlot [5] equipped with Marquardt-Levenberg algorithm of the least squares method.
3. Compare the obtained blackbody temperature with the temperature derived from the measurement of filament resistance.
4. Study the effect of spectral dependence of emissivity on the resulting blackbody temperature.

# References

[1] Lide D R 2005 CRC Handbook of Chemistry and Physics, Boca Raton: CRC Press.

[2] Flesch P 2006 Light and Light Sources, High-Intensity Discharge Lamps. Berlin: Springer.

[3] Halogen lamps – professional and compact technical knowledge. [On-line] `http://www.osram.com`.

[4] White G K and Minges M L 1997 *Int. J. Thermophys.* **18** 1269.

[5] QtiPlot. [On-line] `http://www.qtiplot.ro`.