GB471
Stabilita a dynamika přírodních systémů

Josef Zeman

2020

4 Nelineární systémy

Deterministický chaos

Fluidní tok: Navier-Stokesovy rovnice

Edward N. Lorenz 1963

Deterministic Nonperiodic Flow[1]

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ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

1. Introduction

Certain hydrodynamical systems exhibit steady-state flow patterns, while others oscillate in a regular periodic fashion. Still others vary in an irregular, seemingly haphazard manner, and, even when observed for long periods of time, do not appear to repeat their previous history.

These modes of behavior may all be observed in the familiar rotating-basin experiments, described by Fultz, *et al.* (1959) and Hide (1958). In these experiments, a cylindrical vessel containing water is rotated about its axis, and is heated near its rim and cooled near its center in a steady symmetrical fashion. Under certain conditions the resulting flow is as symmetric and steady as the heating which gives rise to it. Under different conditions Thus there are occasions when more than the statistics of irregular flow are of very real concern.

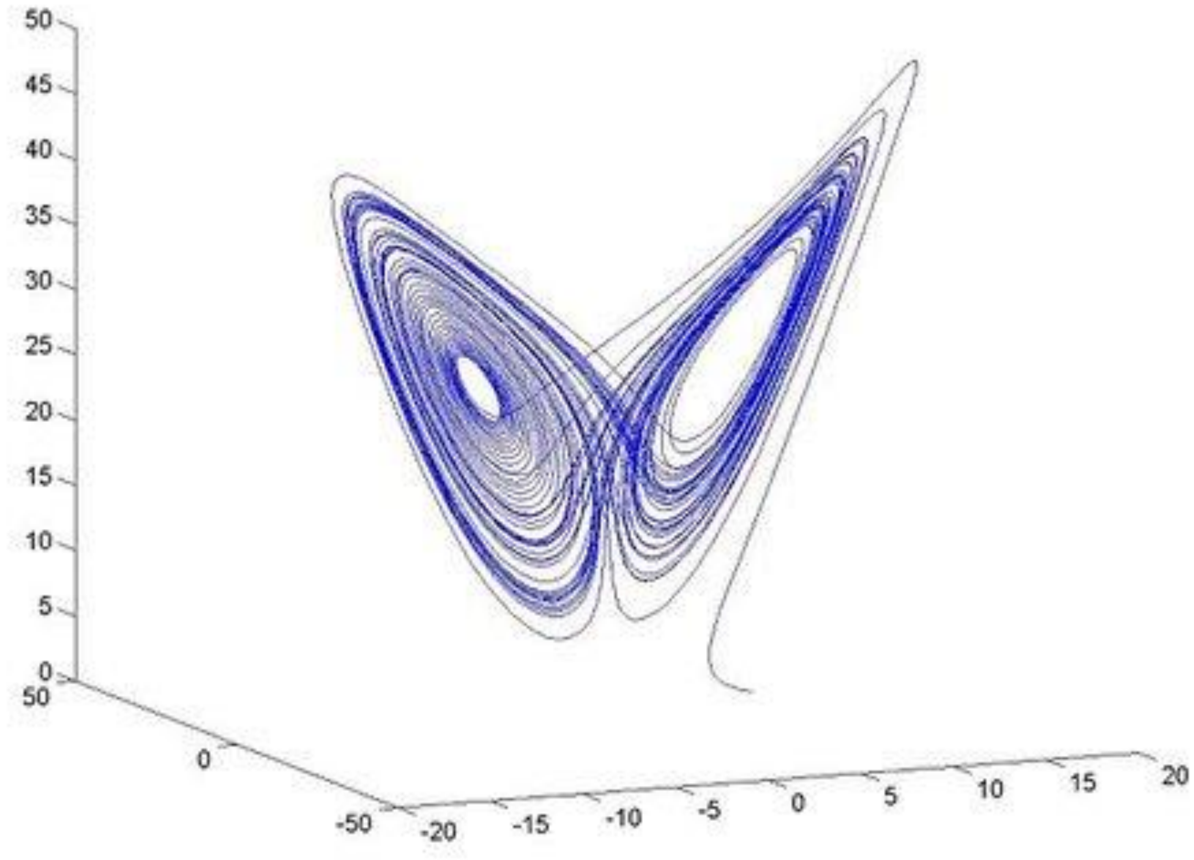
In this study we shall work with systems of deterministic equations which are idealizations of hydrodynamical systems. We shall be interested principally in nonperiodic solutions, i.e., solutions which never repeat their past history exactly, and where all approximate repetitions are of finite duration. Thus we shall be involved with the ultimate behavior of the solutions, as opposed to the transient behavior associated with arbitrary initial conditions.

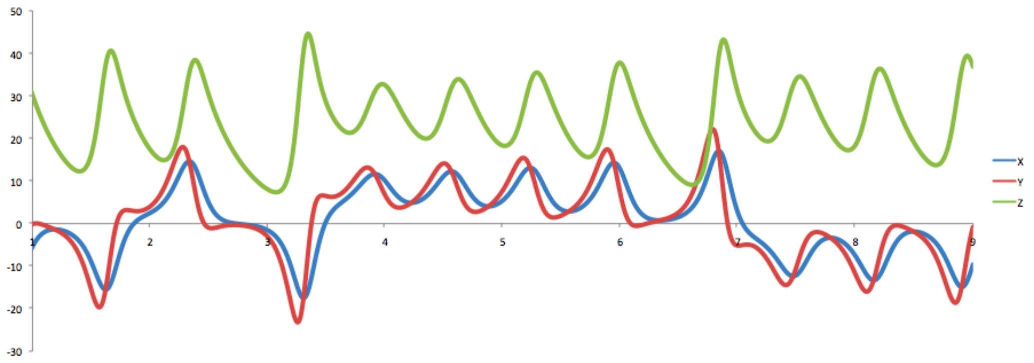
A closed hydrodynamical system of finite mass may ostensibly be treated mathematically as a finite collection of molecules—usually a very large finite collection —in which case the governing laws are expressible as a

Deterministický chaos

Butterfly effect

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$

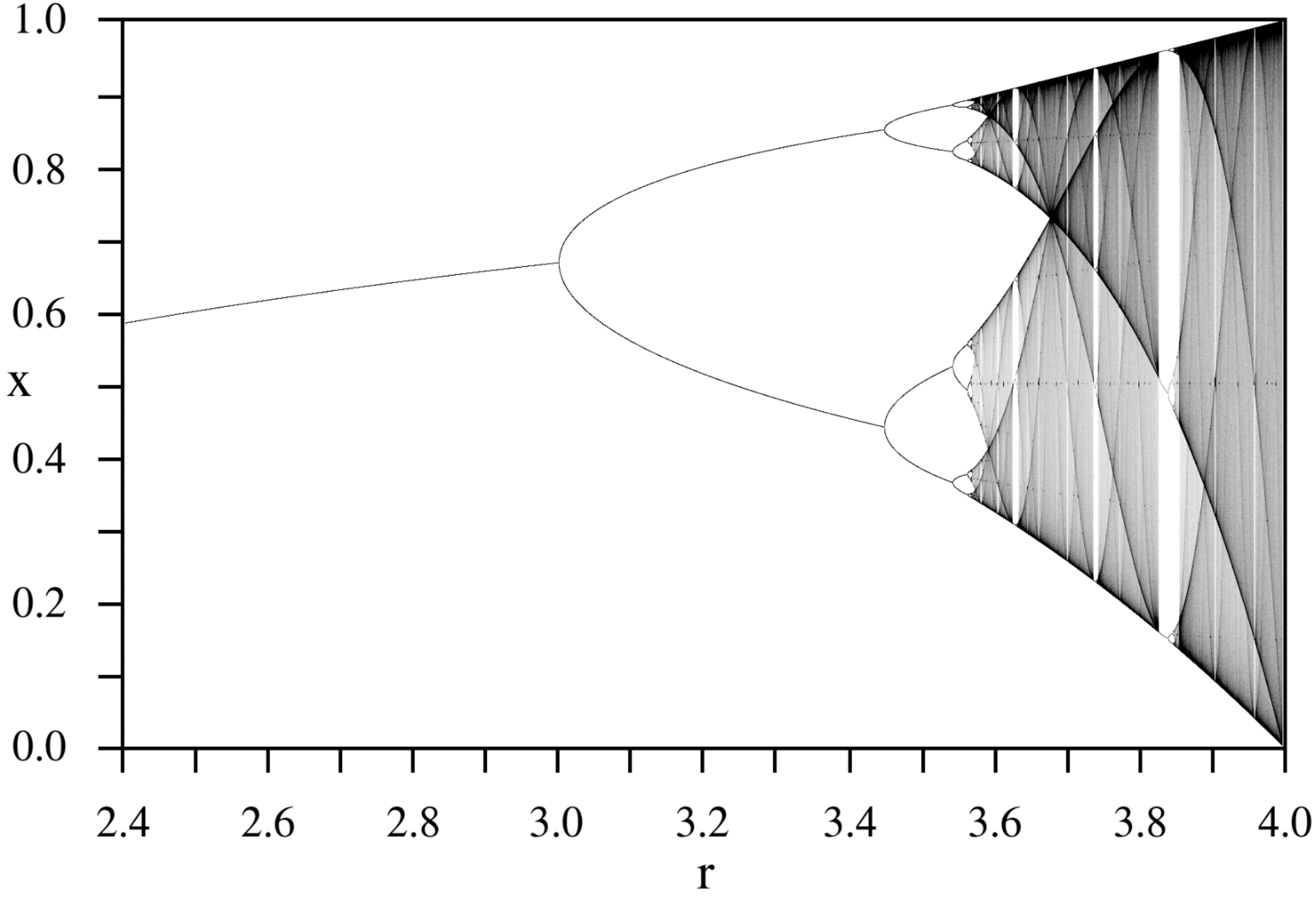




Deterministický chaos

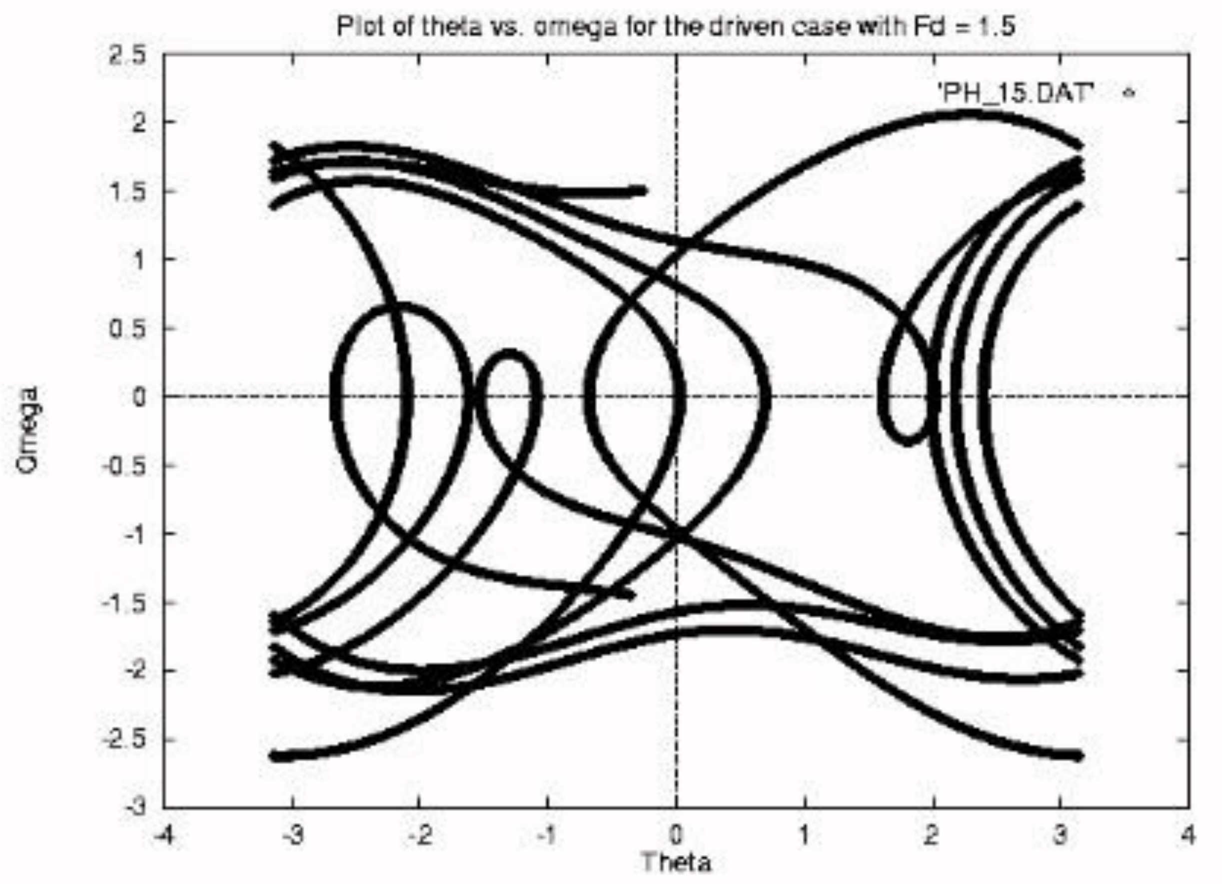
Logistická rovnice – diskrétní

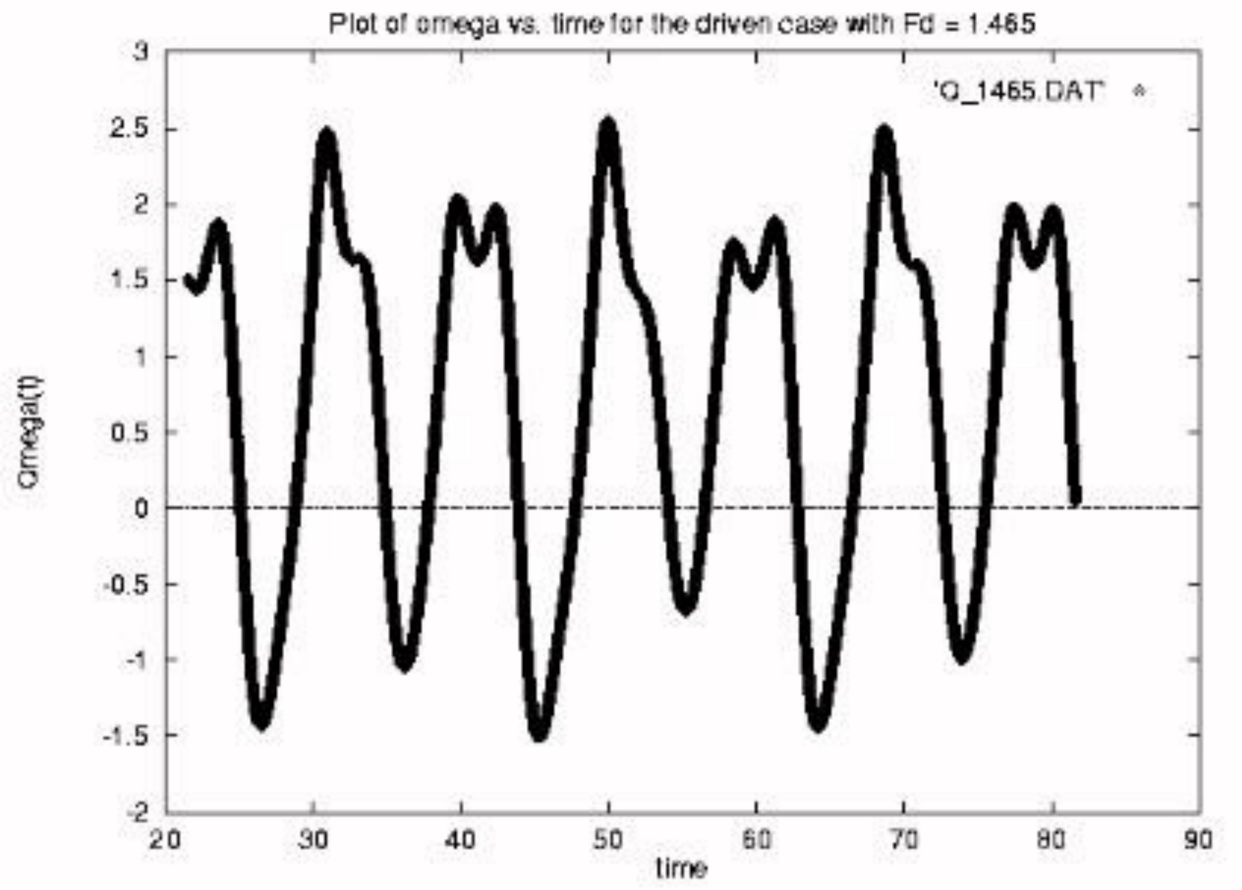
$x \rightarrow r\,x\,(1 - x)$



Deterministický chaos

Chaotické kyvadlo





Fraktály

How long is the coast of Britain?
Statistical self-similarity and fractional dimension

Science: 156, 1967, 636-638

B. B. Mandelbrot

Geographical curves are so involved in their detail that their lengths are often infinite or more accurately, undefinable. However, many are statistically "self-similar," meaning that each portion can be considered a reduced-scale image of the whole. In that case, the degree of complication can be described by a quantity D that has many properties of a "dimension," though it is fractional. In particular, it exceeds the value unity associated with ordinary curves.

1. Introduction

Seacoast shapes are examples of highly involved curves with the property that — in a statistical sense — each portion can be considered a reduced-scale image of the whole. This property will be referred to as "statistical self-similarity." The concept of "length" is usually meaningless for geographical curves. They can be considered superpositions of features of widely scattered characteristic sizes; as even finer features are taken into account, the total measured length increases, and there is usually no clear-cut gap or crossover, between the realm of geography and details with which geography need not be concerned.

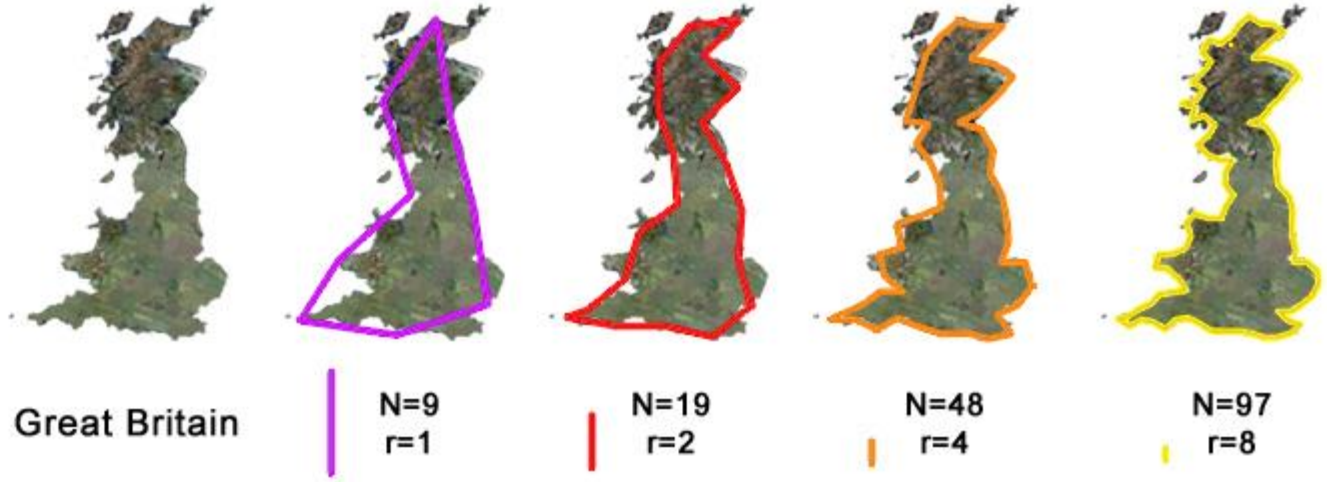
Quantities other than length are therefore needed to discriminate between various degrees of complication for a geographical curve. When a curve is self-similar, it is characterized by an exponent of similarity, *D*, which possesses many properties of a dimension, though it is usually a fraction greater that the dimension 1 commonly attributed to curves. I propose to reexamine in this light, some empirical observations in Richardson 1961 and interpret them as implying, for example, that the dimension of the west coast of Great Britain is $D = 1.25$. Thus, the so far esoteric concept of a "random figure of fractional dimension" is shown to have simple and concrete applications of great usefulness.

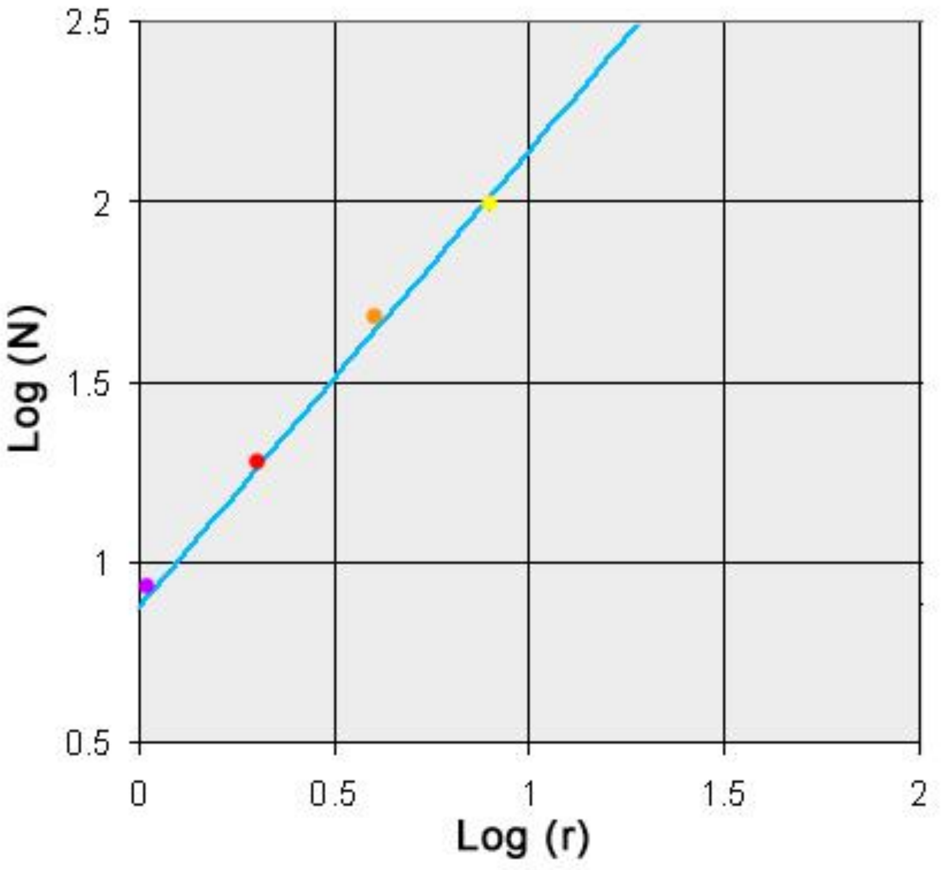
Fraktály

$$D = \frac{\log(N)}{\log(r)}$$

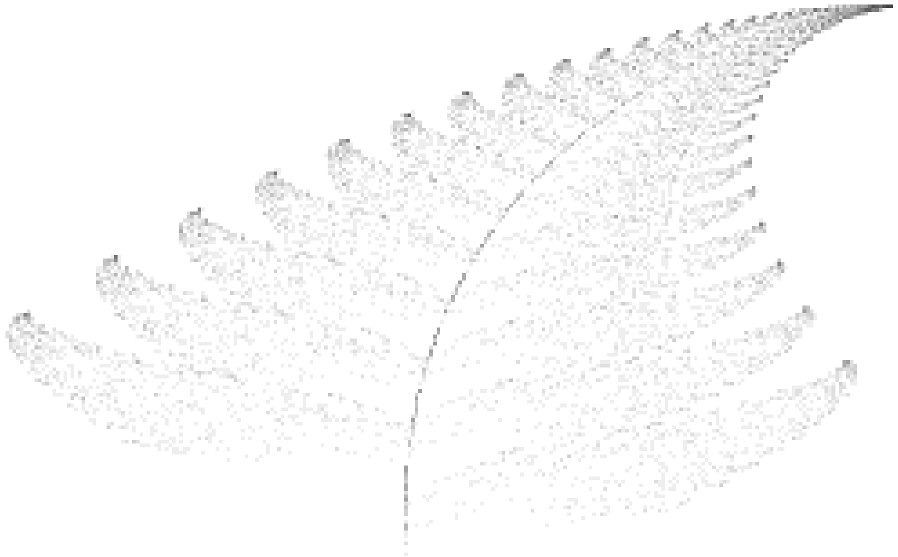
fraktální dimenze

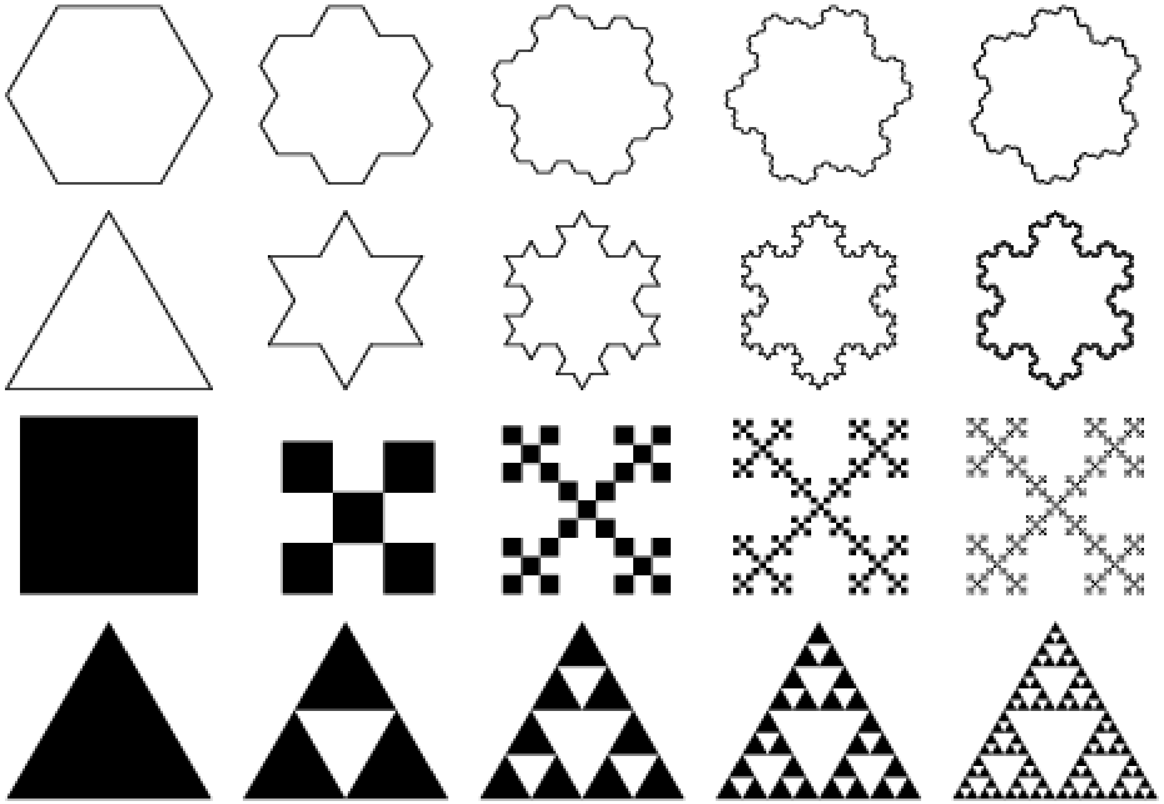
D = 1.21





Fraktály





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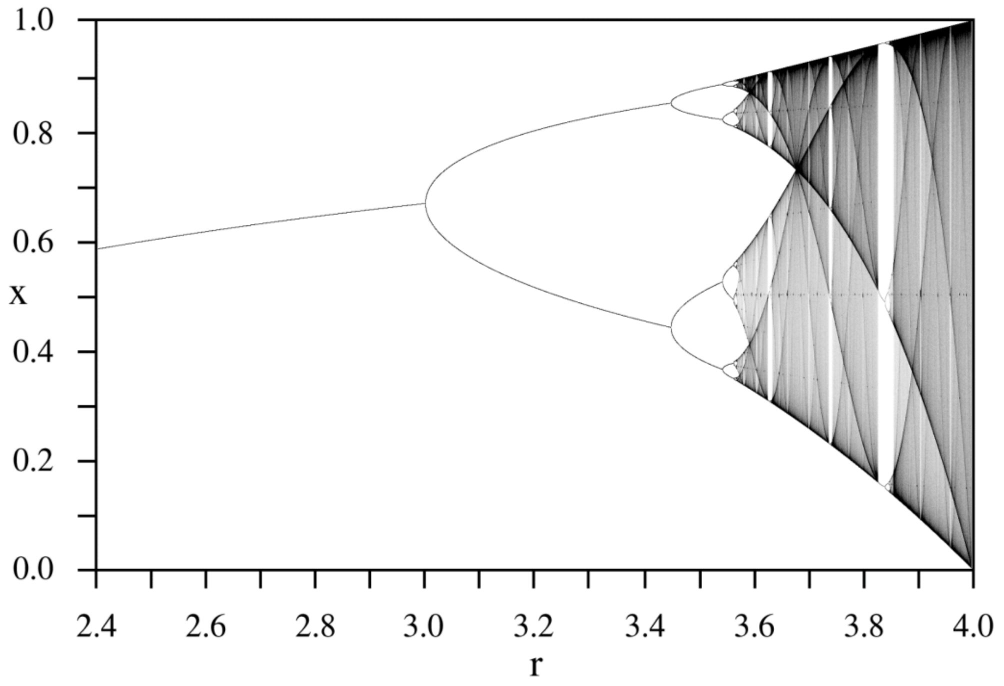
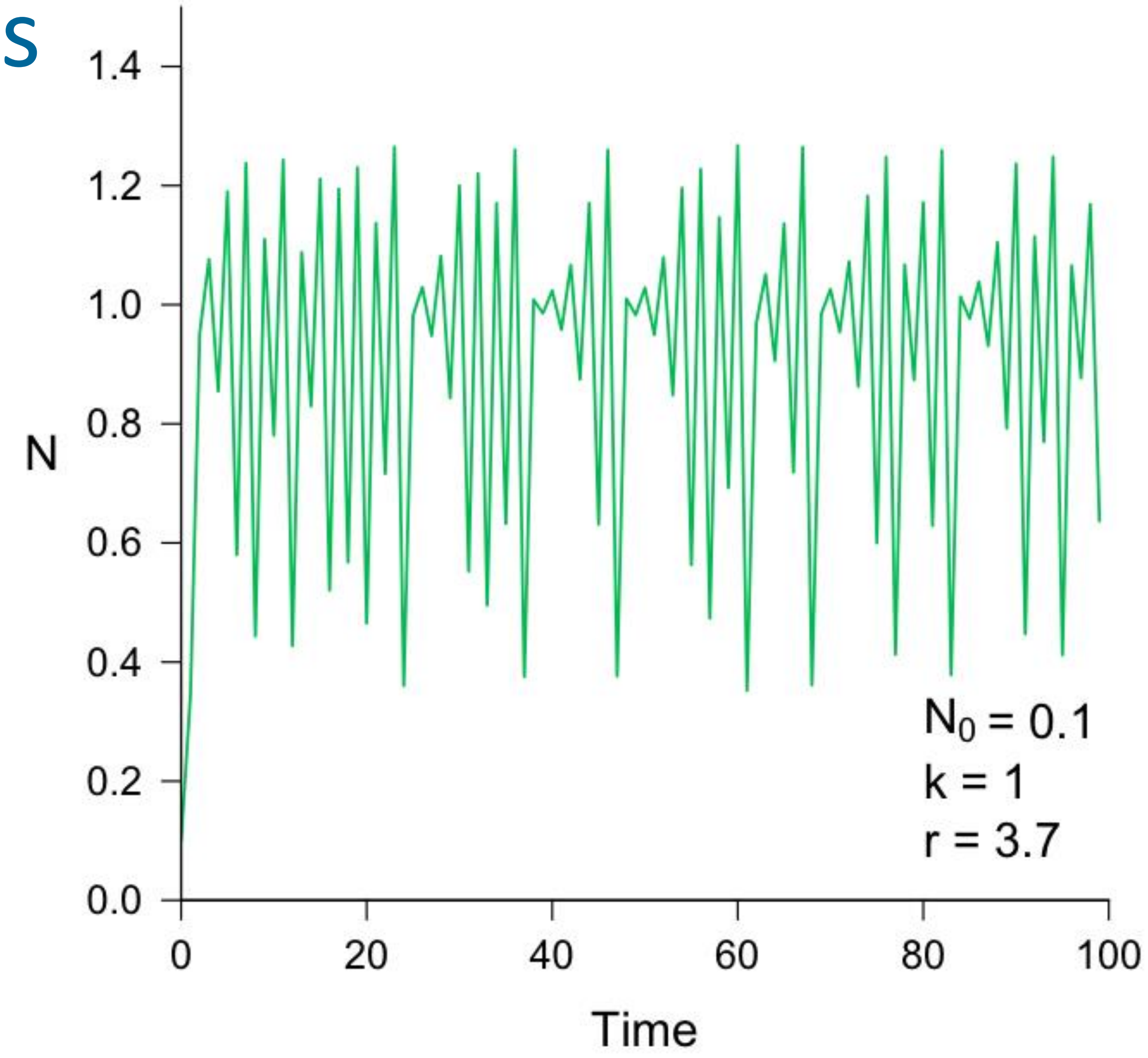
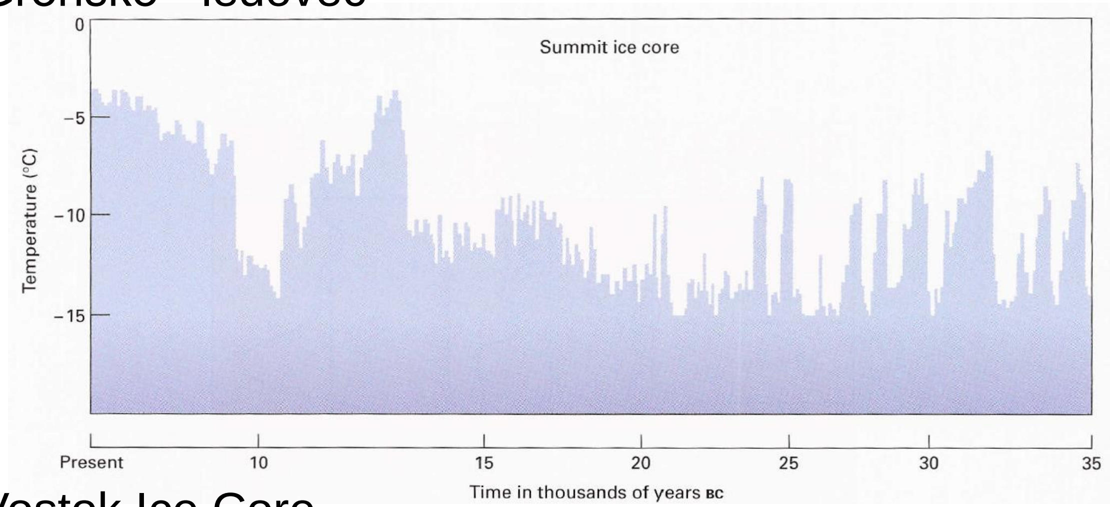




Oxid uhličitý v atmosféře

Grónsko - ledovec



Vostok Ice Core