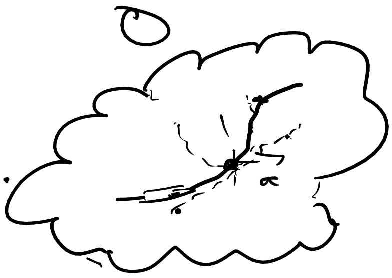
$f \quad f' \quad f'' \quad f''' \quad f^{IV}$

2. DER.   3.   . . . .

$\frac{df}{dx} \quad \frac{d^2f}{dx^2} \quad \frac{d^nf}{dx^n}$

$f(x)$

$$f(x+a) = f(x) + f'(x)\cdot a + f''(x)\cdot\frac{a^2}{2} + f'''(x)\cdot\frac{a^3}{3!} + \dots$$



$$f(x+a) = f(x) + \sum_{n=1}^{\infty} f^{(n)}(x)\cdot\frac{a^n}{n!}$$

n-tá DERIVACE

$1\cdot2\cdot3\cdot\ldots\cdot n$

$$\underbrace{f(x+a)} = \sum_{n=0}^{\infty} c_n \cdot a^n$$

$$= c_0 + c_1 a + c_2 a^2 + c_3 a^3 \dots$$

$$f(x+0) = c_0$$

$$f'(x+a) = c_1 + 2c_2 a + 3c_3 a^2 + \dots$$

$$f'(x) = c_1$$

$$f^{(n)}(x+a) = n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot1\, c_n + \dots a$$

$$f^{(n)}(x) = n!\, c_n \implies \underbrace{c_n = \frac{f^{(n)}(x)}{n!}}$$

$$f(x) = \frac{1}{1-x} \quad \frac{+1}{(1-x)^2} \quad \frac{2}{(1-x)^3} \quad \frac{2\cdot3}{(1-x)^4} \quad \dots$$

$$f^{(n)}(x) = \frac{n!}{(1-x)^{n+1}}$$

$$f(a) = \sum_{n=0}^{\infty} \frac{n!}{n!}\, a^n = \sum_{n=0}^{\infty} a^n$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad (|x|<1)$$

$x=1$

"$\frac{1}{1-1} = 1+1+1+\dots$" $\to \infty$

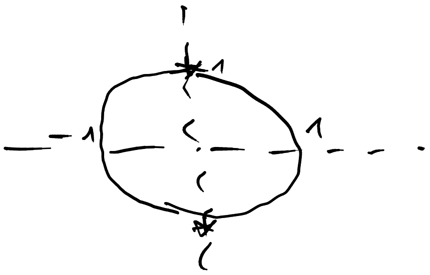
$x=y^2$

$-1 \quad 0 \quad 1$

"$\frac{1}{1-2} = -1 = 1+2+4+8+\dots$" $\to \infty$ ✗

$$\frac{1}{1+y^2} = \sum_{n=0}^{\infty} (-y^2)^n = \sum_{n=0}^{\infty} (-1)^n y^{2n} \quad (|y|<1)$$

$\pm i$



$$\boxed{e^{-1/x^2} = 0+0+0+\dots = 0 \;!}$$

$\alpha \in \mathbb{R}$

$$\boxed{(1+x)^\alpha = 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!} x^n} \quad (|x|<1)$$

$\alpha(1+x)^{\alpha-1} \cdot 1$

$\alpha(\alpha-1)(1+x)^{\alpha-2}$

$\alpha(\alpha-1)(\alpha-2)(1+x)^{\alpha-3}$

$\vdots$

$x=0$

$x=0$

$$\boxed{e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \quad (x\in\mathbb{R})}$$

e^x	$\sin x$	0
e^x	$\cos x$	1
e^x	$-\sin x$	0
e^x	$-\cos x$	-1
$\vdots$		

$$\Rightarrow \quad \sin x = 0 + \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$(x\in\mathbb{R})$

$e^{i\varphi} = \cos\varphi + i\sin\varphi$ $\qquad \cos x = \dfrac{e^{ix}+e^{-ix}}{2} \qquad \sin x = \dfrac{e^{ix}-e^{-ix}}{2i}$

LN x KOLEM $x=1$

LN$(1+x)$ KOLEM $x=0$

I.	$\dfrac{1}{1+x}$	1
II.	$\dfrac{-1}{(1+x)^2}$	-1
III.	$\dfrac{2}{(1+x)^3}$	2
IV.	$-\dfrac{2\cdot 3}{(1+x)^4}$	-6
	$\vdots$	$\vdots$

$$(\ln(1+x))^{(n)}(0) = (-1)^{n-1}\cdot(n-1)!$$

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}.$$

$$(|x|<1)$$

$$\sqrt[2]{16+1} \approx \sqrt[2]{16} = 4$$

$$\sqrt[2]{17} = \sqrt[2]{16+1} = \sqrt[2]{16}\,\sqrt[2]{1+\frac{1}{16}} =$$

$$= 4\left(1+\frac{1}{2}\cdot\frac{1}{16} - \frac{1\cdot 1}{2\cdot 4}\cdot\frac{1}{256} + \dots\right) =$$

$$= 4 + \frac{1}{8} - \frac{1}{512}$$

$$\approx 4{,}123\ldots \quad ✓$$

$$\ln(1{,}2) = \ln(1+0{,}2) = \frac{0{,}2}{1} - \frac{0{,}04}{2} + \frac{0{,}008}{3} \ldots$$

$0{,}2 \qquad 0{,}18 \qquad 0{,}183$

$$\lim_{n\to\infty} \sqrt[2]{n^2+n+1} - \sqrt[2]{n^2-2n+5}$$

$$n\left(\sqrt[2]{1+\underbrace{\frac{1}{n}+\frac{1}{n^2}}} - \sqrt[2]{1\underbrace{-\frac{2}{n}+\frac{5}{n^2}}}\right)$$

$$1+\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n^2}\right) - \frac{1\cdot 1}{2\cdot 4}\left(\frac{1}{n}+\frac{1}{n^2}\right)^2 + \dots$$

$$-\left[1+\frac{1}{2}\left(-\frac{2}{n}+\frac{5}{n^2}\right) - \frac{1\cdot 1}{2\cdot 4}\left(-\frac{2}{n}+\frac{5}{n^2}\right)^2 \dots\right]$$

$$= n\left(\cancel{1} + \frac{1}{2n} - \cancel{1} + \frac{1}{2}\frac{2}{n}\right) = \frac{1}{2} + 1 = \frac{3}{2}$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2} + \dots}{g(a) + g'(a)(x-a) + g''(a)\frac{(x-a)^2}{2} + \dots}$$

$$= \lim_{x\to a} \frac{f'(a)\cancel{(x-a)} + f''(a)\frac{(x-a)^{\cancel{2}}}{2} + \dots}{g'(a)\cancel{(x-a)} + g''(a)\frac{(x-a)^{\cancel{2}}}{2} + \dots} =$$

$$= \frac{f'(a)}{g'(a)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

$$\boxed{\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}}$$

JEN $f = g = 0$
NEBO $f = g = \infty$
V BODĚ $x = a$.

$$\lim_{x\to 1} \frac{2x+3}{3x+2} = \frac{5}{5} = 1$$

$$\cancel{\lim_{x\to 1} \frac{2}{3} = \frac{2}{3}}$$

$$\lim_{x\to 0^+} \underbrace{\ln x}_{-\infty}\ \underbrace{\sin x}_{0} = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sin x}} = \lim_{x\to 0^+} \frac{1/x}{\frac{\cos x}{\sin^2 x}} =$$

$$= \lim_{x\to 0^+} \underbrace{\frac{\sin^2 x}{x^2}}_{1^2} \cdot x = \lim_{x\to 0^+} x = 0$$

$$\lim_{x\to\infty} \frac{\sqrt[2]{x^2+1}}{x+1} = \lim_{x\to\infty} \frac{\frac{1}{2\sqrt[2]{x^2+1}} \cdot 2x}{1} = \lim_{x\to\infty} \frac{x}{\sqrt[2]{x^2+1}}$$

$$= \lim_{x\to\infty} \frac{1}{\frac{2x}{2\sqrt[2]{x^2+1}}} = \lim_{x\to\infty} \frac{\sqrt[2]{x^2+1}}{x} = \dots$$

$$\lim_{x\to\infty} \frac{\sqrt[2]{1+\frac{1}{x^2}}\to 0}{1+\frac{1}{x}\to 0} = \frac{\sqrt[2]{1}}{1} = 1$$

$$\lim_{x\to\infty} \frac{x - \cos x}{x} = \lim_{x\to\infty} \frac{1-\sin x}{1} = 1 - \lim_{x\to\infty} \sin x$$

NEEXISTUJE!

$$\| $$

$$1 + \lim_{x\to\infty} \frac{\cos x^{-1 \text{ až } 1}}{x} = \underset{\to 0}{}$$

$$= 1$$

$$\lim_{x\to 0} \frac{\cos x - e^{-x^2/2}}{x^4} = \lim_{x\to 0} \frac{\cos x^{\nearrow 1} - (\dots)^{\nearrow 3}}{4\cdot 3\cdot 2\cdot 1} = \frac{-1}{12}$$

$$-x e^{-x^2/2}$$

$$e^{-x^2/2}(-1 + x^2)$$

$$e^{-x^2/2}(3x - x^3)$$

$$\underset{\to 1}{e^{-x^2/2}}(3 - \underbrace{3x^2 - 3x^2 + x^4}_{\to 0})$$

$$\lim_{x\to 0} \frac{1 - \cancel{\frac{x^2}{2}} + \frac{x^4}{24} - \frac{x^6}{720} \dots - (\cancel{1} - \cancel{\frac{x^2}{2}} + \frac{x^4}{4\cdot 2} \dots}{}$$

$$= \lim_{x\to 0} \frac{-\frac{x^4}{8} + \frac{x^4}{24} + (x^6 \dots)}{x^4} =$$

$$= \left(-\frac{1}{8} + \frac{1}{24}\right) = \frac{-3+1}{24} = \frac{-1}{12}$$

$$\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right) \quad \text{"}\infty - \infty\text{"} = \lim_{x\to 0} \frac{\overbrace{\sin x - x}^{0}}{\underbrace{x \sin x}_{0}} = \lim_{x\to 0} \frac{\cos x - 1}{\sin x + x\cos x} =$$

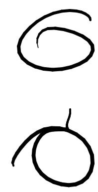
$$= \lim_{x\to 0} \frac{-\sin x^{\nearrow 0}}{2\cos x - x\sin x} = 0$$

$$2\cdot 1 \quad -0\cdot 0$$

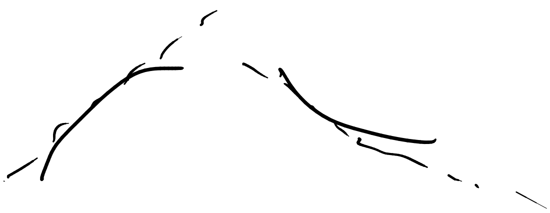
$$\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{x - \frac{x^3}{6} + \dots}\right) = \lim_{x\to 0}\left(\cancel{\frac{1}{x}} - \cancel{\frac{1}{x}}\left(1 + \frac{x^2}{6} + \dots\right)\right)$$

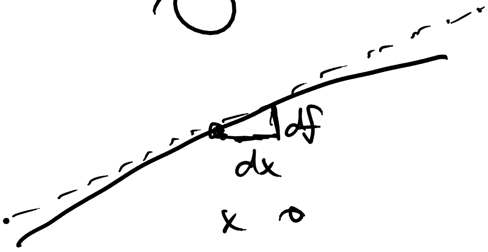
$$\frac{1}{x} \cdot \frac{1}{1 - \frac{x^2}{6} + \dots}$$

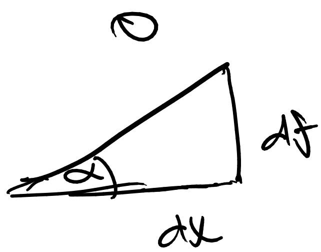
$$= \lim_{x\to 0} -\frac{x}{6} + \dots = 0$$



$$\operatorname{tg}\alpha = \frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

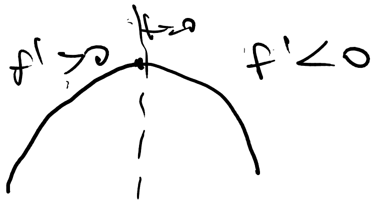






PŘI $f' > 0$ f ROSTE

PŘI $f' < 0$ f KLESÁ



20 m PLOTU
OPLOTIT OBDÉLNÍK
S NEJVĚTŠÍ PLOCHOU

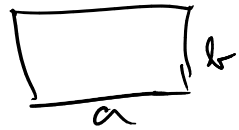
a, b

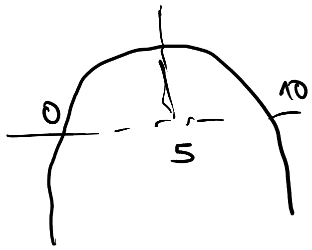
$2a + 2b = 20$

ab MAX.

$\Downarrow$

$b = 10 - a$





$\Downarrow$

$f(a) = a(10-a)$ MAX.

$f' = -a + 10 - a = 10 - 2a$

$f' = 0 = 10 - 2a \Rightarrow a = 5$

KONVEXNÍ

$f'' > 0$

$\Downarrow$

f' ROSTOUCÍ

KONKÁVNÍ

$f'' < 0$

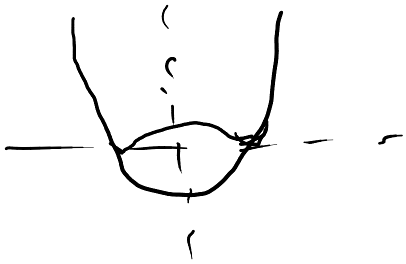
$\Downarrow$

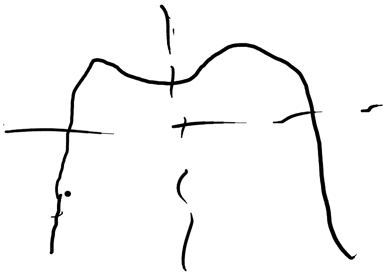
f' KLESAJÍCÍ

„DO KONKÁVNÍ KÁVY NENALEJEŠ"

$f'' = 0 \Rightarrow$ INFLEXNÍ BOD

$$f(x) = 1 + x^2 - \frac{x^4}{2} = -\frac{1}{2}\left(x^4 - 2x^2 - 2\right)$$
$$\left((x^2-1)^2 - 3\right)$$





$f(x) = 0$

$(x^2-1)^2 = 3$

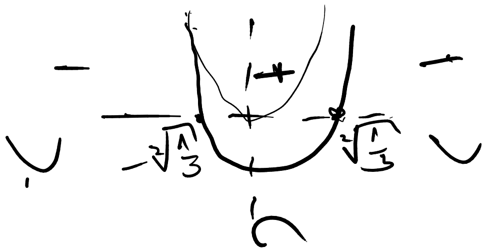
$x^2 = 1 \pm \sqrt{3}$

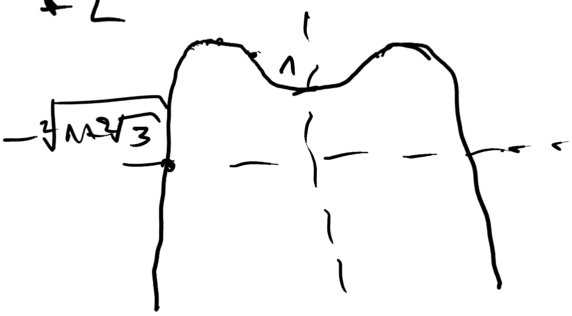
$x = \pm\sqrt{1+\sqrt{3}}$

$f' = 1 + 2x - 2x^3$

$-2x^3 + 2x + 1 = 0$

$f'' = -6x^2 + 2$

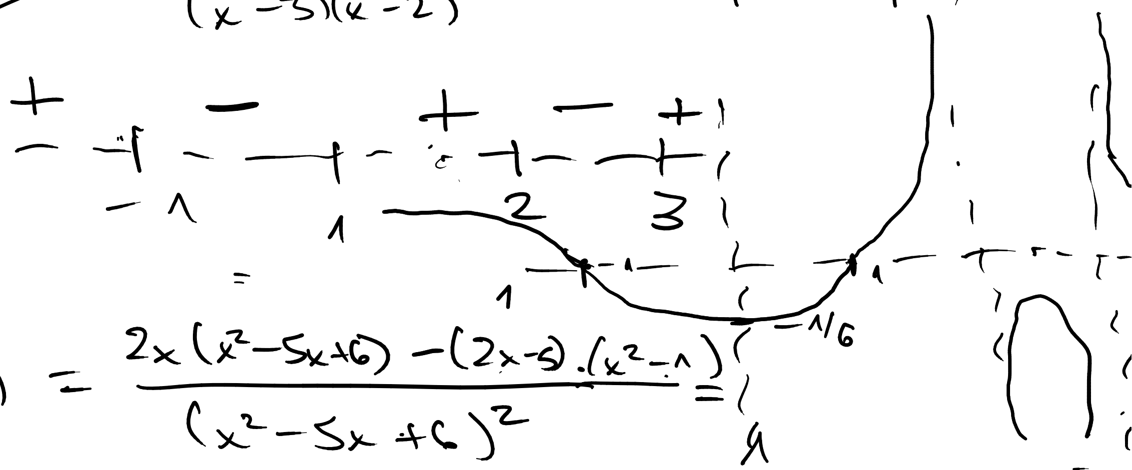




$(x-1)(x+1)$

$$f(x) = \frac{x^2-1}{x^2-5x+6} = \frac{(x-1)(x+1)}{(x-3)(x-2)}$$

$(x-3)(x-2)$



$$f'(x) = \frac{2x(x^2-5x+6) - (2x-5)\cdot(x^2-1)}{(x^2-5x+6)^2} =$$

$$= \frac{\cancel{2x^3} - 10x^2 + 12x - \cancel{2x^3} + 2x + 5x^2 - 5}{(x-3)^2(x-2)^2} = \frac{-5x^2 + 14x - 5}{(x-3)^2(x-2)^2}$$

$5x^2 - 14x + 5$

$$\frac{7 \pm \sqrt{24}}{5} = \frac{14 \pm \sqrt{96}}{10} = \frac{14 \pm \sqrt{196 - 4\cdot 25}}{10}$$

$-\quad + \quad -$

$\frac{7-2\sqrt{24}}{5}$ $\qquad$ $\frac{7+2\sqrt{24}}{5}$

MIN $\qquad$ MAX

$\approx \frac{7-5}{5}$ $\qquad$ $\approx \frac{12}{5}$

$\frac{2}{5}$