Cviceni 4: Inverzni matice; linearni prostory

Monday, October 26, 2020 8:10 AM

Inverzni matice: pokus o rozdeleni matic

A, X: $n \times n$

Chledame X: $AX = I$ (pravo-inv.)

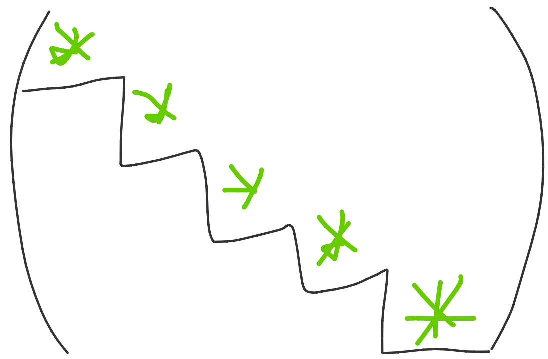
—||— X: $XA = I$ (levo-inv)

Veta: $\exists$ levo-inv. $\Leftrightarrow \exists$ pravo-inv, a v tom připade jsou stejni

Takto: $A^{-1} = X$: $\boxed{AX = XA = I}$

Kdy $A^{-1} \exists$?

Veta: $A^{-1} \exists \Leftrightarrow$ Schod tvar vypada (∄ volny prom.)



(J rving plom.) ($\ast$) (n schodu)

$$\boxed{(AB)^{-1} = B^{-1}A^{-1}}$$

$$\boxed{Ax = B \iff X = A^{-1}B}$$

$$A = \begin{pmatrix} a_{11} & & 0 \\ & \ddots & \\ 0 & & a_{nn} \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} \frac{1}{a_{11}} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{a_{nn}} \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & & \ast \\ & \ddots & \\ 0 & & a_{nn} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{1}{a_{11}} & & \ast \\ & \ddots & \\ 0 & & \frac{1}{a_{nn}} \end{pmatrix}$$

$$\boxed{A^{-1} \exists \iff a_{11}, \dots, a_{nn} \neq 0}$$ – Zeimena

$$\begin{pmatrix} 1 & \cdots & 1 \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix} ? \qquad \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 2 & 1 & 0 & \cdots & \vdots \\ \vdots & \vdots & 1 & & \vdots \\ \vdots & \vdots & \vdots & & 0 \\ 50 & 49 & 48 & \cdots & 1 \end{pmatrix} ?$$

Algoritm hledani A^{-1}

Zapišime $(A|I)$ $n \times 2n$

Budeme delat' elem. opravi. $(A|I) \to (I|X)$

Pak: $\boxed{X = A^{-1}}$

Proč? $AX = I$, $E_{ij}AX = E_{ij}$, ...

... $IX = \{\text{neco}\}$

Priklad: $A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & -1 & -3 \\ 1 & 5 & 1 \end{pmatrix}$

$$(A|E) = \left(\begin{array}{ccc|ccc} 3 & 4 & 2 & 1 & 0 & 0 \\ 2 & -1 & -3 & 0 & 1 & 0 \\ 1 & 5 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{(1)} \left(\begin{array}{ccc|ccc} 1 & 5 & 1 & 0 & 0 & 1 \\ 2 & -1 & -3 & 0 & 1 & 0 \\ 3 & 4 & 2 & 1 & 0 & 0 \end{array}\right) \xrightarrow{(2)} \left(\begin{array}{ccc|ccc} 1 & 5 & 1 & 0 & 0 & 1 \\ 0 & -11 & -5 & 0 & 1 & -2 \\ 0 & -11 & -1 & 1 & 0 & -3 \end{array}\right) \xrightarrow{(3)}$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 5 & 1 & 0 & 0 & 1 \\ 0 & -11 & -5 & 0 & 1 & -2 \\ 0 & 0 & 4 & 1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 5 & 1 & 0 & 0 & 1\\ 0 & -11 & -5 & 0 & 1 & -2\\ 0 & 0 & 4 & 1 & -1 & -1\end{array}\right) \xrightarrow{(4)} \left(\begin{array}{ccc|ccc} -20 & -100 & -20 & 0 & 0 & -20\\ 0 & -44 & -20 & 0 & 4 & -8\\ 0 & 0 & 20 & 5 & -5 & -5\end{array}\right) \xrightarrow{(5)} \left(\begin{array}{ccc|ccc} -20 & -100 & 0 & 5 & -5 & -25\\ 0 & -44 & 0 & 5 & -1 & -13\\ 0 & 0 & 20 & 5 & -5 & -5\end{array}\right) \xrightarrow{(6)}$$

$$\to \left(\begin{array}{ccc|ccc} -4 & -20 & 0 & 1 & -1 & -5\\ 0 & 44 & 0 & -5 & 1 & 13\\ 0 & 0 & 4 & 1 & -1 & -1\end{array}\right) \xrightarrow{(7)} \left(\begin{array}{ccc|ccc} -44 & -220 & 0 & 11 & -11 & -55\\ 0 & 220 & 0 & -25 & 5 & 65\\ 0 & 0 & 4 & 1 & -1 & -1\end{array}\right) \xrightarrow{(8)}$$

$$\to \left(\begin{array}{ccc|ccc} -44 & 0 & 0 & -14 & -6 & 10\\ 0 & 220 & 0 & -25 & 5 & 65\\ 0 & 0 & 4 & 1 & -1 & -1\end{array}\right) \xrightarrow{(9)} \left(\begin{array}{ccc|ccc} 44 & 0 & 0 & 14 & 6 & -10\\ 0 & 44 & 0 & -5 & 1 & 13\\ 0 & 0 & 4 & 1 & -1 & -1\end{array}\right) \xrightarrow{(10)}$$

$$\to \left(\begin{array}{ccc|ccc} 44 & 0 & 0 & 14 & 6 & -10\\ 0 & 44 & 0 & -5 & 1 & 13\\ 0 & 0 & 44 & 11 & -11 & -11\end{array}\right) \xrightarrow{(11)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{14}{44} & \frac{6}{44} & -\frac{10}{44}\\ 0 & 1 & 0 & -\frac{5}{44} & \frac{1}{44} & \frac{13}{44}\\ 0 & 0 & 1 & \frac{11}{44} & -\frac{11}{44} & -\frac{11}{44}\end{array}\right) = (E|A^{-1})$$

__Příklad__ A 100×100, $A = \begin{pmatrix} 1 & 1 & \cdots & 1\\ & 1 & \cdots & 1\\ & & \ddots & \vdots\\ 0 & & & 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & \cdots & 1 & 1 & & 0\\ & \ddots & \vdots & & \ddots & \\ 0 & & 1 & 0 & & 1\end{array}\right) \to$$

$$\left(\begin{array}{cccc|cccc} 1 & \cdots & 1 & 0 & 1 & & 0 & -1\\ & \ddots & \vdots & \vdots & & \ddots & & \vdots\\ 0 & & 1 & 0 & 0 & & 1 & -1\\ & & & 1 & 0 & \cdots & 0 & 1\end{array}\right) \to$$

$$\left(\begin{array}{ccccc|ccccc} 1 & \cdots & 1 & 0 & 0 & 1 & & 0 & -1 & 0\\ & \ddots & \vdots & \vdots & \vdots & & \ddots & \vdots & & \vdots\\ 0 & & 1 & 0 & 0 & 0 & \cdots & 1 & -1 & 0\end{array}\right)$$

$$\left(\begin{array}{ccccc|ccccc} 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 1 & -1 & 0 \\ 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & -1 \\ 0 & \cdots & \cdots & 0 & 1 & 0 & \cdots & \cdots & 0 & 1 \end{array}\right) \longrightarrow \cdots$$

$$\cdots \longrightarrow \left(\begin{array}{ccc|cccccc} 1 & & 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ & \ddots & & 0 & 1 & \ddots & & \vdots & \vdots \\ & & & \vdots & & \ddots & \ddots & 0 & \vdots \\ & & & & & & \ddots & -1 & 0 \\ 0 & & & & 0 & & \ddots & 1 & -1 \\ & & 1 & 0 & \cdots & \cdots & & 0 & 1 \end{array}\right)$$

Další - praktika ...

Linearni prostory

V - množina

R - realni čisla

2 typu operaci.

(1) $a \in V$,, $k \in R$

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„vektor"

$$a \to k \cdot a$$

(2) $a, b \in V$

$$\{a; b\} \to a + b$$

„plus"

Musí platit „aksiomy":

1) $k(a+b) = ka + kb$

2) $(\lambda + \mu)a = \lambda a + \mu a$

3) $a + b = b + a$ (komutativita)

4) $(a+b) + c = a + (b+c)$

4) $(a+b)+c = a+(b+c)$

(distributivita)

5) $\lambda \cdot (\mu a) = \lambda \mu \cdot a$

6) $\exists\, 0 \in V: a + 0 = a$

7) $\forall a\ \exists (-a): a + (-a) = 0$

(fakt: $-a = (-1) \cdot a$)

8) $1 \cdot a = a$

Jasny příklady:

1) $R^2, R^3, \ldots, R^n$

2) Polynomy od x

(nebo $x_1, \ldots, x_k$)

$a_0 + a_1x + a_2x^2 + \dots + a_nx^n$

n neni opraveno!

Veta: V - lin prostor,

$W \subset V$: $\forall a, b \in W \rightarrow a+b \in W$

$\forall a \in W, \forall k \in \mathbb{R} \rightarrow ka \in W$

$\Rightarrow W$ - lin. prostor.

W se nazyva podprostor v V

1) $V = \{\text{pol } 3 \text{ stupnu}\}$

$$a_0 + a_1x + a_2x^2 + a_3x^3$$

$$a_3 \neq 0$$

Ne! $x^3 + (-x^3) \notin V$

2) $V = \{$

2) $V = \{\text{pol stupňu} \leq 3\}$

$a_0 + a_1 x + a_2 x^2 + a_3 x^3$

Ano!

3) $V = \{(x,y) \in \mathbb{R}^2 : x + y = 0\}$

Ano! $(a_1, a_2) \in V \Rightarrow (ka_1, ka_2) \in V$

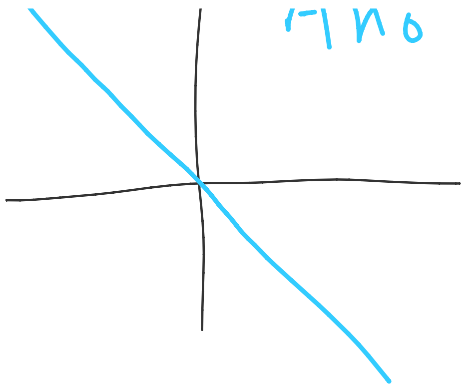
$(a_1, a_2) + (b_1, b_2) \in V$

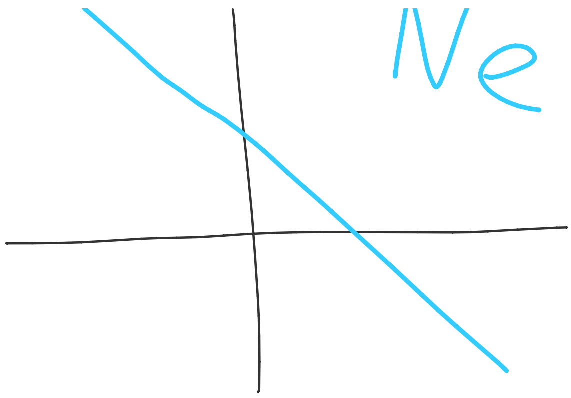
4) $V = \{(x,y) \in \mathbb{R}^2 : x + y = 1\}$

Ne! $2 \cdot (1,0) \notin V$

\| Ano

\| Ne





5) Horní-troj. matici

Ano

6) Diag. matici: Ano

7) $V = \left\{ \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \right\}$ Ne

8) $V = \{ \text{pol} \leq 3 \text{ stupnu}: f(1) + f(2) = 0 \}$

$$a_0 + \dots + a_3 + a_0 + 2a_1 + 4a_2 + 8a_3 = 0$$

$$2a_0 + 3a_1 + 5a_2 + 9a_3 = 0$$

$2a\cdot 6+3a_1+5a_2+9a_3=0$

Ano

8) $V=\{F(1)+F(2)=2\}$

Ne $10f \notin V$

Vlastnost: $W \subset V$, W

je popsane soustavem

homog. lin. rovnic $\Rightarrow$

W - podprostor $\boxed{AX=0}$

W — — // — nehomog.

$\Rightarrow W$ - není podprostor!

$AX=B$

$AX = B$

$B \neq 0$

9) $V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \ \lambda \cdot A = \begin{pmatrix} \lambda a & \lambda b \\ c & d \end{pmatrix} \right\}$

Ne! $(0+0)\cdot\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \neq 0 \cdot \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$

Jaky axiomy platí?...

10) $f(x), \ x \in \mathbb{R}$

$k \cdot f$ – jak obvykle

$f \oplus g := f(x) \cdot g(x)$

<u>Ne!</u> $\nexists \, (-f)$ pro

$f(x) = x$

$f(x) = x$

11) $V = \{ f(x) : f(x) > 0 \}$

$k \cdot f := (f(x))^k$

$f \oplus g := f(x) \cdot g(x)$

Ano! $\ln f(x)$

12) $V = R$

čísla bereme z Q

13) $V = R^3$, $\alpha \oplus \beta :=$

13) $V = \mathbb{R}^2$, $a \oplus b :=$
$= a \times b$

Ne! $a \oplus a = 2a = 0$.