Př $f: R^2 \to R^1$

$$f(x) = \begin{cases} 0 & x_1 = 0 \\ \dfrac{2x_2 e^{-x_1^{-2}}}{x_2^2 + e^{-2x_1^{-2}}} & x_1 \neq 0 \end{cases}$$

f má v bodě Θ G-derivaci

Řešení Hledáme matici $A = (a_1, a_2)$ takovou, aby platilo

$$(*) \quad \lim_{t \to 0} \left| \left(\frac{2t h_2 e^{-t^{-2}h_1^{-2}}}{t^2 h_2^2 + e^{-2t^{-2}h_1^{-2}}} - f(0,0) \right) - a_1 h_1 - a_2 h_2 \right| = 0$$

$\forall h_1, h_2 \quad h_1 \neq 0$ $\qquad f(0,0) = 0$

pro $h_2 = 0 \Rightarrow a_1 = 0$

Nechť $h_1 h_2 \neq 0$ položme $z = t^{-2} h_1^{-2}$

$$\lim_{t \to 0} 2h_2 \frac{e^{-t^{-2}h_1^{-2}}}{t^2 h_2^2 + e^{-2t^{-2}h_1^{-2}}} =$$

$$= \lim_{t \to 0} 2h_2 \frac{e^{-z}}{t^2 h_2^2 + e^{-2z}} = \left| t^2 = \frac{h_1^{-2}}{z} \Rightarrow t^2 h_2^2 = \left(\frac{h_2}{h_1}\right)^2 \frac{1}{z} \right|$$

$$= \lim_{z \to \infty} 2h_2 \frac{e^{-z}}{\frac{1}{z}\left(\frac{h_2}{h_1}\right)^2 + e^{-2z}} = \lim_{z \to \infty} \frac{2h_2}{\left(\frac{e^z}{z}\right)\left(\frac{h_2}{h_1}\right)^2 + e^{-z}} =$$

$$= 0 \quad \left(\lim_{z \to \infty} \frac{e^z}{z} = 0 \right) \qquad = \underline{0}$$