3.1 (i) $c(t) = (t^2-1, -2t^2+5t, t-5)$

$c: \mathbb{R} \to \mathbb{R}^3$

Tečna v bodě $c(t_0)$ má směrový vektor $c'(t_0)$, kde

$c'(t_0) = (2t_0, -4t_0+5, 1)$

Tečna má být rovnoběžná s rovinou $\rho: 3x + y - z - 7 = 0$

norm. vektor roviny je

$n = (3, 1, -1) \Rightarrow n \perp c'(t_0)$

$\Downarrow$

určíme t_0

$c'(t_0) = (2t_0, -4t_0+5, 1) \perp (3, 1, -1)$

$6t_0 + (-4t_0+5) - 1 = 0$

$2t_0 + 4 = 0 \Rightarrow t_0 = -2$

Zápis: $c(-2) + s\,c'(-2)$

$[3, -18, -7] + s(-4, 13, 1)$

(ii) $f(x,y) = x^2 + xy + 2y^2$

⇝ tečná rovina v bodě $[1,1,?]$

$f(1,1) = 1 + 1 + 2 = 4$ ⇝ bod $[1,1,4]$

Dále uvažíme křivky

$f(x,1)$ a $f(1,y)$

$\downarrow$ $\qquad\qquad$ $\downarrow$

$f_x(1,1)$ $\qquad\qquad$ $f_y(1,1)$ $\qquad$ ~~"generují" zaměření tečné roviny~~

$f(x,y) = x^2 + xy + 2y^2$

$f_x(x,y) = 2x + 1 \qquad f_x(1,4) = 3$

$f_y(x,y) = 1 + 4y \qquad f_y(1,1) = 5$

$\varphi(x,y) = (x, y, f(x,y))$

$\mathbb{R}^2 \to \mathbb{R}^3$

$\varphi_x(1,1) = (1,0,3) \qquad \varphi_y(1,4) = (0,1,5)$

Závěr: $[1,1,4] + s(1,0,3) + t(0,1,5)$

3.2 (i) $F(x,y,z) = (x^2+y^2+z^2, xyz)$

$F: \mathbb{R}^3 \to \mathbb{R}^2 \quad \sim [1,2,3]$

$G(s,t) = \frac{t}{s}$

$G: \mathbb{R}^2 \to \mathbb{R} \quad \sim [14,6]$

$dF(1,2,3): \mathbb{R}^3 \to \mathbb{R}^2 \to$ lin. Zal

$dG(14,6): \mathbb{R}^2 \to \mathbb{R} \to$

$F_x(x,y,z) = (2x, yz)$ | $F_x(1,2,3) = (2,6)$

$F_y(x,y,z) = (2y, xz)$ | $F_y(1,2,3) = (4,3)$

$F_z(x,y,z) = (2z, xy)$ | $F_z(1,2,3) = (6,2)$

$$dF(1,2,3) = \begin{pmatrix} 2 & 4 & 6 \\ 6 & 3 & 2 \end{pmatrix}$$

$G_s(s,t) = -\frac{t}{s^2}$ | $G_s(14,6) = -\frac{6}{14^2}$

$G_t(s,t) = \frac{1}{s}$ | $G_t(14,6) = \frac{1}{14}$

$$dG(14,6) = \left(-\frac{6}{14^2}, \frac{1}{14}\right)$$

(ii) $F(1,2,3) = (14,6)$

$$\varphi(x,y,z) = \frac{xyz}{x^2+y^2+z^2} = (G \circ F)(1,2,3)$$

$d\varphi = dG \circ dF$ ve správných bodech

$$d\varphi(1,2,3) = \underbrace{dG(14,6)}_{} \circ dF(1,2,3)$$

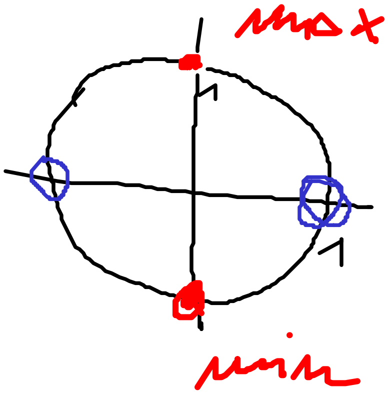
$$\left(-\frac{6}{14^2}, \frac{1}{14}\right) \cdot \begin{pmatrix} 2 & 4 & 6 \\ 6 & 3 & 2 \end{pmatrix} \cdot$$

$$= \left(-\frac{12}{14^2} + \frac{6}{14}, -\frac{24}{14^2} + \frac{3}{14}, -\frac{36}{14^2} + \frac{2}{14}\right)$$

$$d\varphi(1,2,3) : \mathbb{R}^3 \to \mathbb{R}$$

lze zjednodušit –.

3.3 $x^2+y^2=1$ zadává implicitně funkci $y=f(x)$



$y=y(x)$

$$x^2+y^2-1=0 \;\Big/ \frac{d}{dx}$$

$$\boxed{2x+2y\cdot y'=0}$$

$$y'=-\frac{x}{y}$$

$y\neq 0$

stacionární body $y'=0$

$\hookrightarrow$ $x=0$ $\rightsquigarrow$ $y^2=1$

$y=\pm 1$

$[0,\pm 1]$

$$x+y\cdot y'=0 \;\Big/ \frac{d}{dx}$$

$$1+y'\cdot y'+y\cdot y''=0$$

$$y''=\frac{-1-(y')^2}{y}=\frac{-1-\left(\frac{x}{y}\right)^2}{y}=\frac{-y^2-x^2}{y^3}$$

bod $[0,1]$ $\rightsquigarrow$ $y''=-\frac{1}{1}=1<0$ $\rightsquigarrow$ max

$[0,-1]$ $\rightsquigarrow$ $y''=-\frac{1}{-1}=1>0$ $\to$ min

3.4 Funkce $y(x)$ je zadána implicitně vztahem $\underline{xy^2} - 2xy + x^3 - 3y^2 + 5 = 0$

Derivováno: $y = y(x)$ $\quad / \frac{d}{dx}$

$$(y^2 + x \cdot \underline{2y \cdot y'}) - 2(y + x\underline{y'}) + 3x^2 - \underline{6y \cdot y'} = 0$$

$$y'(2xy - 2x - 6y) + y^2 - 2y + 3x^2 = 0$$

$$\boxed{y' = \frac{-y^2 + 2y - 3x^2}{2xy - 2x - 6y}}$$

$$F(x,y) = xy^2 - 2xy + x^3 - 3y^2 + 5$$

$$F_x(x,y) = y^2 - 2y + 3x^2$$

$$F_y(x,y) = 2xy - 2x - 6y$$

$$\to y' = -\frac{F_x}{F_y}, \quad F_y \neq 0$$

↳ body, kde implicitní popis zadává funkci

3.5 $F(x,y) = x^3 - y^3 + 2xy = 0$

zadává křivku (to je graf funkce $y = y(x)$)

Rozhodněte, zda leží v okolí bodu $[1,-1]$ nad/pod tečnou

$$x^3 - y^3 + 2xy = 0 \quad \Big/ \frac{d}{dx} \qquad y = y(x)$$

$$\boxed{3x^2 - 3y^2 \cdot y' + 2(y + xy')} = 0 \quad \Big/ \frac{d}{dx}$$

$$y'(-3y^2 + 2x) + (3x^2 + 2y) = 0 \qquad y = y(x)$$

$$y' = \frac{-(3x^2 + 2y)}{-3y^2 + 2x} \quad \leftarrow \quad F_y(x,y) \neq 0$$

$$6x - 3(2y \cdot y' \cdot y' + y^2 \cdot y'') + 2y' + 2(y' + xy'') = 0$$

$$y''(-3y^2 + 2x) + 6x - 6yy'^2 + 2y' + 2y' = 0$$

$$y'' = \frac{-6x + 6yy'^2 - 4y'}{-3y^2 + 2x}$$

V bodě $[\underset{x}{\underset{\parallel}{1}}, \underset{y}{\underset{\parallel}{-1}}]$: $y' = \dfrac{-(3-2)}{-3+2} = 1$

$$y'' = \frac{-6+6\cdot(-1)\cdot 1-4\cdot 1}{-3+2} = \frac{-16}{-1} = 16 > 0$$

$\Rightarrow$ nad tečnou

3.6 $z = f(x,y)$ zad. maí implicitne

$$F(x,y,z) = x^2+y^2+z^2 - xz - \sqrt{2}\,yz - 1 = 0$$

v bode $[1, \sqrt{2}, 2]$ $\quad F(1,\sqrt{2},2) =$

$= 1+2+4-2-4-1 = 0$

$$x^2+y^2+z^2 - xz - \sqrt{2}\,yz - 1 = 0 \quad / \frac{d}{dx} \quad \frac{d}{dy}$$

$$2x + 2z \cdot z_x - (z + x z_x) - \sqrt{2}\,y z_x = 0 \qquad z = z(x,y)$$

$$z_x(2z - x - \sqrt{2}\,y) + 2x - z = 0$$

$$z_x = \frac{z - 2x}{\boxed{2z - x - \sqrt{2}\,y}} \qquad z_x(1,\sqrt{2}) = \frac{2-2}{4-1-2} = 0$$

$x = 1,\ y = \sqrt{2}$

$z = 2$

$$\frac{\partial F}{\partial z} = F_z = 2z - x - \sqrt{2}\,y$$

$$2y + 2z z_y - x z_y - \sqrt{2}(z + y z_y) = 0 \quad \frac{d}{dy}$$

$$z_y(2z - x - \sqrt{2}\,y) + 2y - \sqrt{2}\,z = 0$$

$$z_y = \frac{\sqrt{2}\,z - 2y}{2z - x - \sqrt{2}\,y} \qquad z_y(1,\sqrt{2}) = \frac{2\sqrt{2} - 2\sqrt{2}}{4-1-2} = 0$$

$x = 1,\ y = \sqrt{2}$

$\Rightarrow [1, \sqrt{2}, 2]$ je stacionárny bod ($z = 2$)

$$r_x(2r - x - \sqrt{2}\,y) + 2x - r = 0 \quad \Big/ \frac{d}{dx} \quad \frac{d}{dy}$$

$$r_{xx}(2r - x - \sqrt{2}\,y) + r_x(2r_x - 1) + 2 - r_x = 0$$

$$r_{xx} = \frac{-2r_x^2 + 2r_x - 2}{2r - x - \sqrt{2}\,y}$$

$$\boxed{\begin{array}{l} r_{xx}(\underset{\overset{\shortparallel}{x}}{1}, \underset{\overset{\shortparallel}{y}}{\sqrt{2}}) \\ r = 2, \ r_x = 0 \end{array}} = \frac{-2}{4-1-2} = -2$$

$$\Rightarrow r_{xy}(2r - x - \sqrt{2}\,y) + \underbrace{r_x}_{=0}(2r_y - \sqrt{2}) - \underbrace{r_y}_{=0} = 0$$

$$r_{xy}(\underbrace{4-1-2}_{\neq 0}) = 0 \Rightarrow r_{xy}(1, \sqrt{2}) = 0$$

$\vdots$

$$r_{yy}(1, \sqrt{2}) = -2$$

matice druhých derivací

Hessián v bodě $[1, \sqrt{2}, 2]$ funkce $r = r(x, y)$ je

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$ neg. definitní

$\Rightarrow$ lok. maximum