

# Electrical characteristics of barrier discharge

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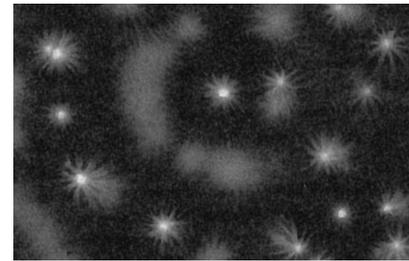
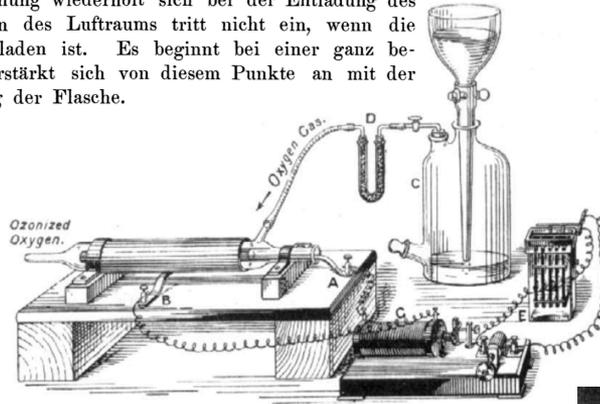
Gas Discharges lecture

# Outline

- Historical overview
- Q-V plot (aka Lissajous figure)
- Simplest equivalent circuit of the barrier discharge
- Electrical current in the discharge gap vs. electrical current measured in the external circuit
- Voltage on the gas gap and the electric field parameter
- Application of the electrical analysis for the (not only) spectroscopic plasma investigation
- Understanding the plasma chemistry of low pressure volume and for coplanar barrier discharge in atmospheric pressure air

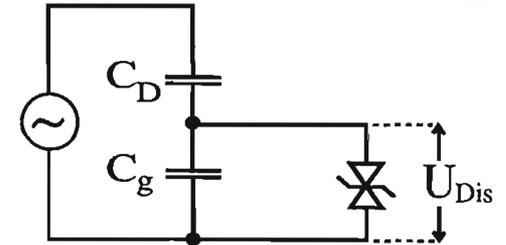
# Barrier discharge - already Siemens had done it...

Wenn man zwei dünne Glas- und Glimmerplatten einseitig mit Stanniol belegt und die nicht belegten Seiten so aufeinander legt, dass ein luftefüllter Zwischenraum von geringer aber gleichmässiger Dicke sich zwischen ihnen befindet, so erhält man bekanntlich eine Lichterscheinung in dem ganzen luftefüllten Raume, wenn man den so gebildeten Collector durch eine hinlänglich geladene Leydner Flasche ladet. Diese Lichterscheinung wiederholt sich bei der Entladung des Collectors. Das Leuchten des Luftraums tritt nicht ein, wenn die Flasche sehr schwach geladen ist. Es beginnt bei einer ganz bestimmten Ladung und verstärkt sich von diesem Punkte an mit der Vergrößerung der Ladung der Flasche.



W. Siemens 1857 Ann. Phys. Chem.  
 sein. Nimmt man an, dass die Gasmoleküle mit einer Aetherhülle umgeben sind, so muss bei der chemischen Verbindung zweier oder mehrerer solcher Moleküle auch eine veränderte Lagerung der Aetherhüllen derselben eintreten. Die hierdurch bedingte Bewegung der Aethertheilchen muss sich durch Schwingungen ausgleichen, welche die Ausgangspunkte der Licht- und Wärmewellenzüge bilden können. In

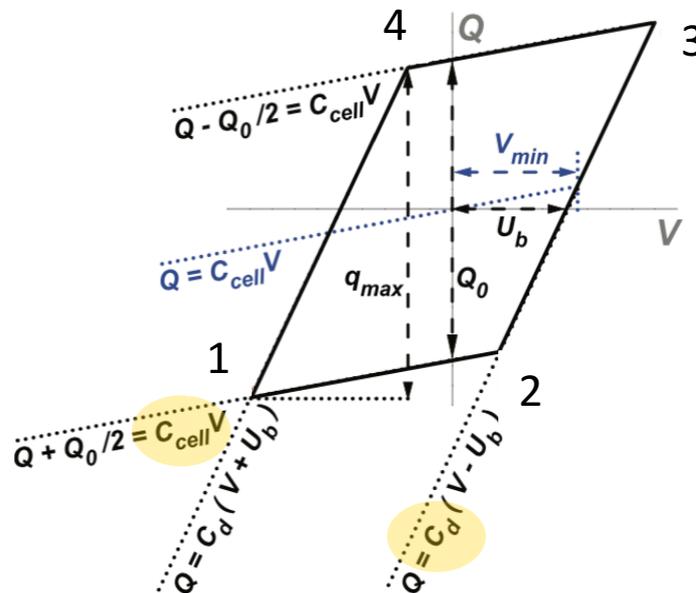
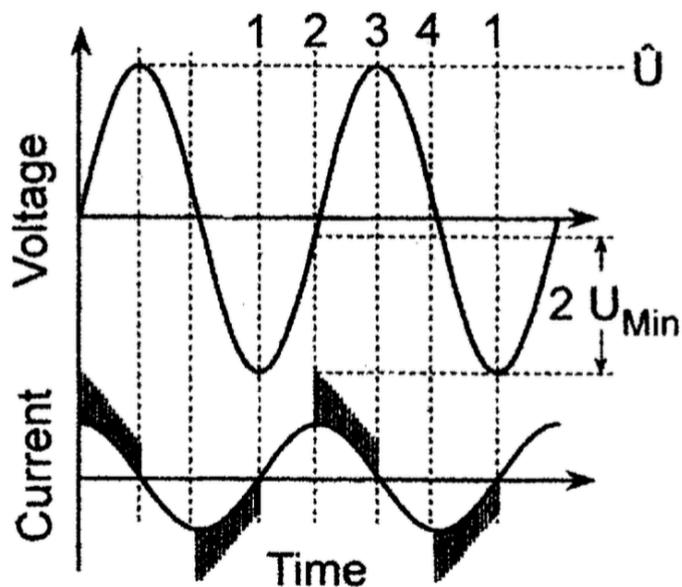
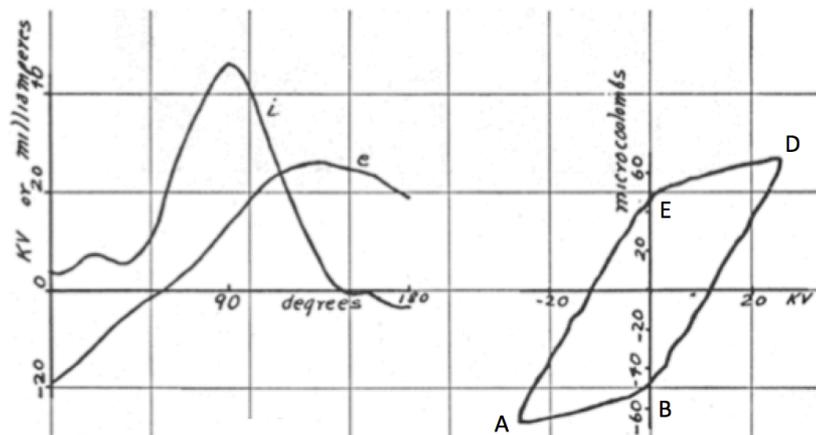
*A Dynamical Theory of the Electromagnetic Field.*  
 Received October 27,—Read December 8, 1864.  
 By J. CLERK MAXWELL, F.R.S.



W. Siemens 1857 and

Buss 1932, Klemenc 1937, Manley 1943, Samoilovich 1966, Gibalov 1981, Eliasson, Kogelschatz 1983, Heuser 1985, Okazaki 1993, Zhu 1996, Kozlov 2001, Stollenwerk 2007

# Manley and his Q-V plots for large scale ozonizers

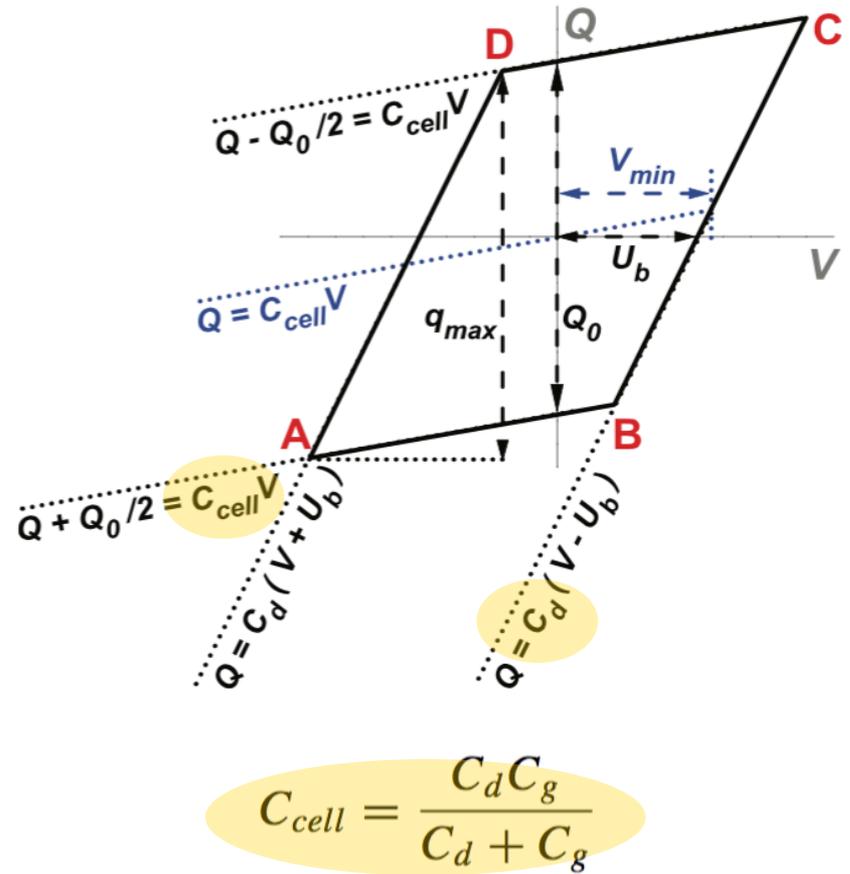
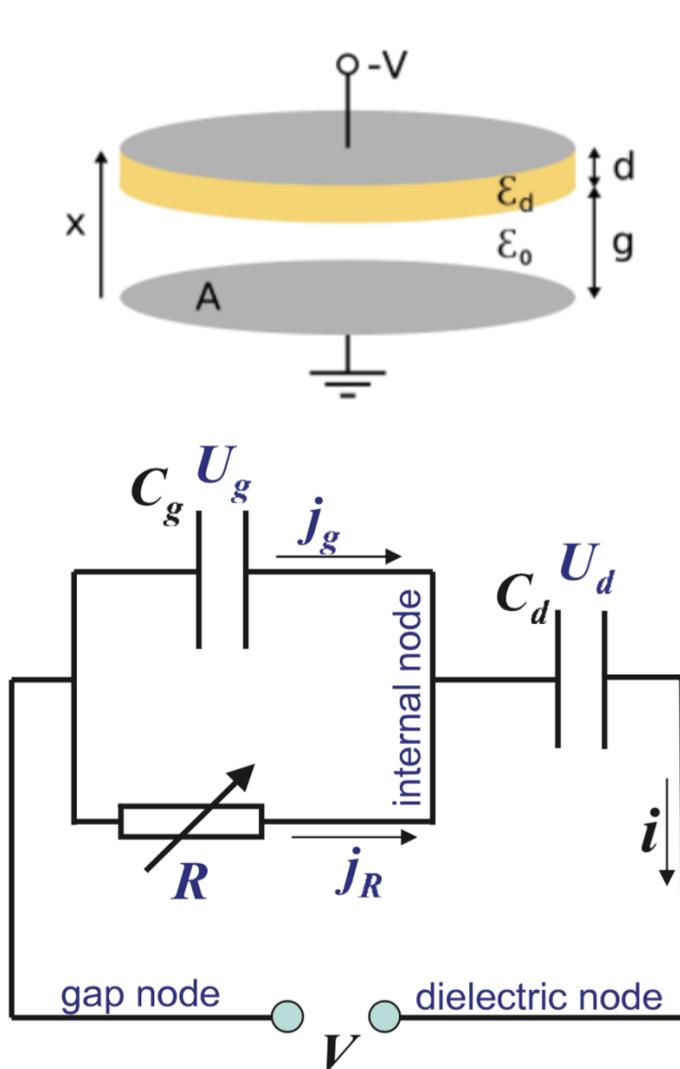


Manley 1943 Trans. Electrochem. Soc.

Kogelschatz 2003 Plasma Chem. Plasma Process.

# Simplest equivalent circuit of barrier discharge

... a macroscopic point of view



$$C_{cell} = \frac{C_d C_g}{C_d + C_g}$$

Liu et al. 2003 J. Phys. D: Appl. Phys.  
Pipa et al. 2012 Rev. Sci. Instrum.

# Kirchhoff's circuit equations and the result

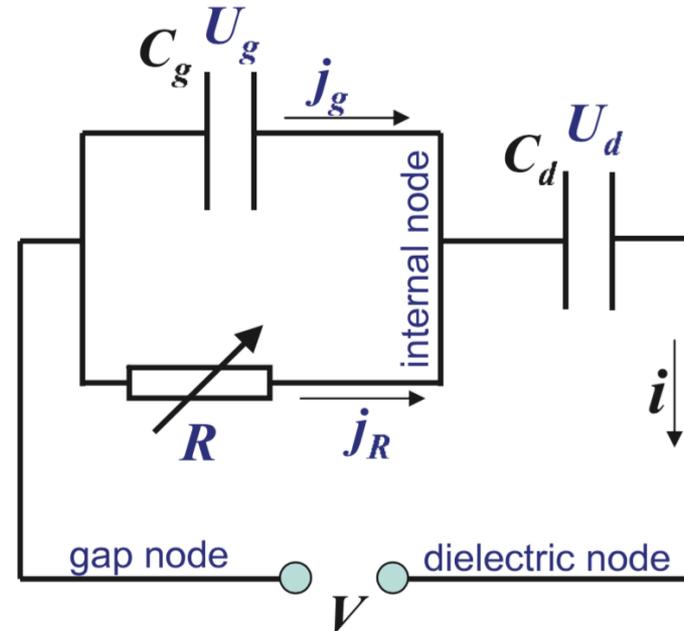
... a macroscopic point of view

$$U_d(t) = \frac{Q(t)}{C_d}$$

$$U_g(t) = V(t) - U_d(t)$$

$$j_R = i(t) - j_g(t)$$

$$j_g(t) = C_g \frac{dU_g(t)}{dt}$$



$$U_g(t) = V(t) - \frac{Q(t)}{C_d}$$

$$j_R(t) = \left[ 1 + \frac{C_g}{C_d} \right] i(t) - C_g \frac{dV(t)}{dt}$$

$$C_{cell} = \frac{C_d C_g}{C_d + C_g}$$



$$j_R(t) = \frac{1}{1 - \frac{C_{cell}}{C_d}} \left[ i(t) - C_{cell} \frac{dV(t)}{dt} \right]$$

$$q(t) = \frac{C_{cell}}{1 - \frac{C_{cell}}{C_d}} \left[ \frac{Q(t)}{C_{cell}} - V(t) \right] + q_0$$

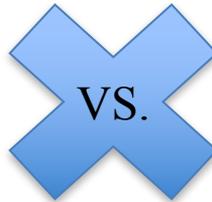
Liu et al. 2003 J. Phys. D: Appl. Phys.

Pipa et al. 2012 Rev. Sci. Instrum.

# Discharge current correct value?

- Tschiersch et al. 2017 J. Phys. D: Appl. Phys.  
Peeters et al. 2015 Plasma Sources Sci. Technol.  
Pipa et al. 2012 Rev. Sci. Instrum.  
Williamson et al. 2006 J. Phys. D: Appl. Phys.  
Merbahi et al. 2004 J. Phys. D: Appl. Phys.  
Liu et al. 2003 J. Phys. D: Appl. Phys.  
Bibinov et al. 2001 J. Phys D: Appl. Phys.

$$j_R(t) = \left[ 1 + \frac{C_g}{C_d} \right] i(t) - C_g \frac{dV(t)}{dt}$$

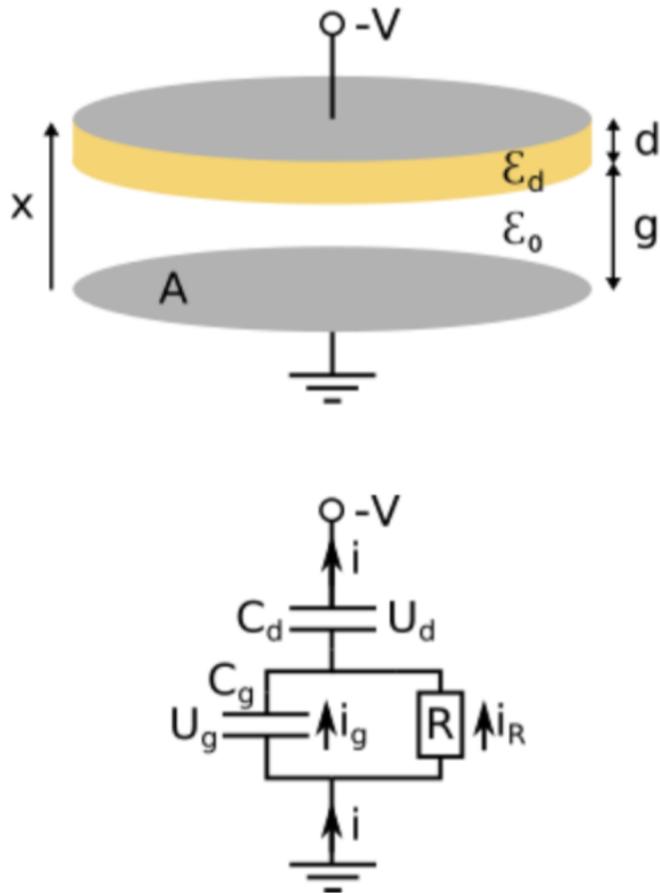


$$I_{\text{discharge}}(t) = I_{\text{meas}}(t) - I_{C_{\text{gas}}} = \overset{?}{I_{\text{meas}}(t)} - C_{\text{gas}} \frac{dV_{\text{gas}}(t)}{dt}$$

- Reichen et al. 2010 J. Phys. D: Appl. Phys.  
Massines et al. 2005 Plasma Phys. Control. Fusion  
Naude et al. 2005 J. Phys. D: Appl. Phys.  
Bletzinger et al. 2003 J. Phys. D: Appl. Phys.  
Lomaev et al. 2001 Atmos. Oceanic Optic.

# Electrical current balance equation

... a microscopic point of view



$$i_t(t) = i_c(x, t) + \epsilon(x)\epsilon_0 \frac{\partial E(x, t)}{\partial t}$$



$$\frac{i_t(t)}{\epsilon_0} \int_0^{d+g} \frac{dx}{\epsilon(x)} = \int_0^{d+g} \frac{i_c(x, t)}{\epsilon(x)\epsilon_0} dx + \frac{\partial}{\partial t} \int_0^{d+g} E(x, t) dx$$

$$j_t(t) \left( \frac{1}{C_d} + \frac{1}{C_g} \right) = \frac{1}{A\epsilon_0} \int_0^{d+g} \frac{j_c(x, t)}{\epsilon(x)} dx + \frac{dV(t)}{dt}$$

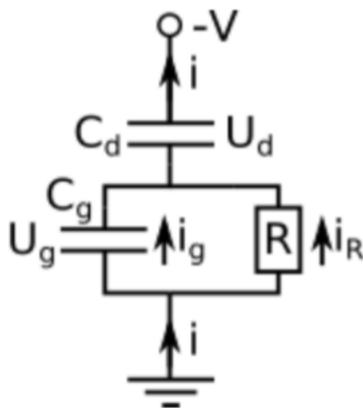
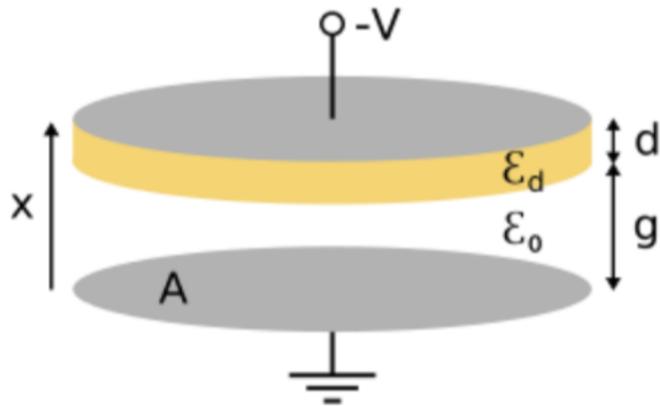


$$j_R(t) = j_c(t) = \frac{1}{1 - \frac{C_{cell}}{C_d}} \left[ i(t) - C_{cell} \frac{dV(t)}{dt} \right]$$

Kulikovsky 1994 J. Phys. D: Appl. Phys.  
 Wang et al. 2006 J. Appl. Phys.  
 Hoder, Bonaventura et al. 2016

# Electrical current balance equation + surface charge

... a microscopic point of view



$$i_t(t) = i_c(x, t) + \varepsilon(x) \frac{\partial E_g(t)}{\partial t} = \varepsilon(x) \varepsilon_0 \frac{\partial E_d(t)}{\partial t}$$



$$E_g = \varepsilon_r E_d - \sigma / \varepsilon_0$$

$$\frac{\partial \sigma}{\partial t} = i_c$$

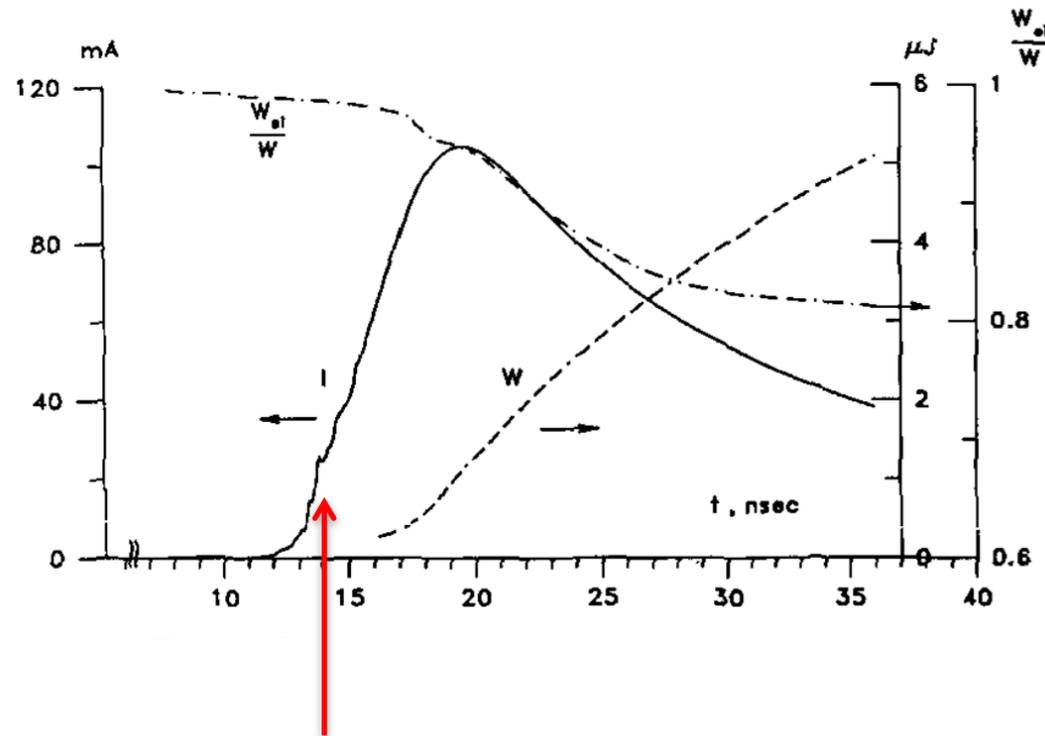
$$\begin{aligned} \frac{j_t(t)}{A} &= \int_0^g j_c(x, t) dx + \left( \frac{\partial \sigma}{\partial t} \right) \left( \frac{dg(\varepsilon_r - 1)}{\varepsilon_r g + d} \right) - \frac{g + d}{\varepsilon_r g + d} \varepsilon_r \varepsilon_0 \frac{\partial V}{\partial t} \\ &= \frac{\partial \sigma}{\partial t} \left[ \frac{\varepsilon_r g (g + d)}{\varepsilon_r g + d} \right] + \frac{g + d}{\varepsilon_r g + d} \varepsilon_r \varepsilon_0 \frac{\partial V}{\partial t} \end{aligned}$$



$$j_R(t) = j_c(t) = \frac{1}{1 - \frac{C_{cell}}{C_d}} \left[ i(t) - C_{cell} \frac{dV(t)}{dt} \right]$$

# Limitations: net charge in streamer head and sheath

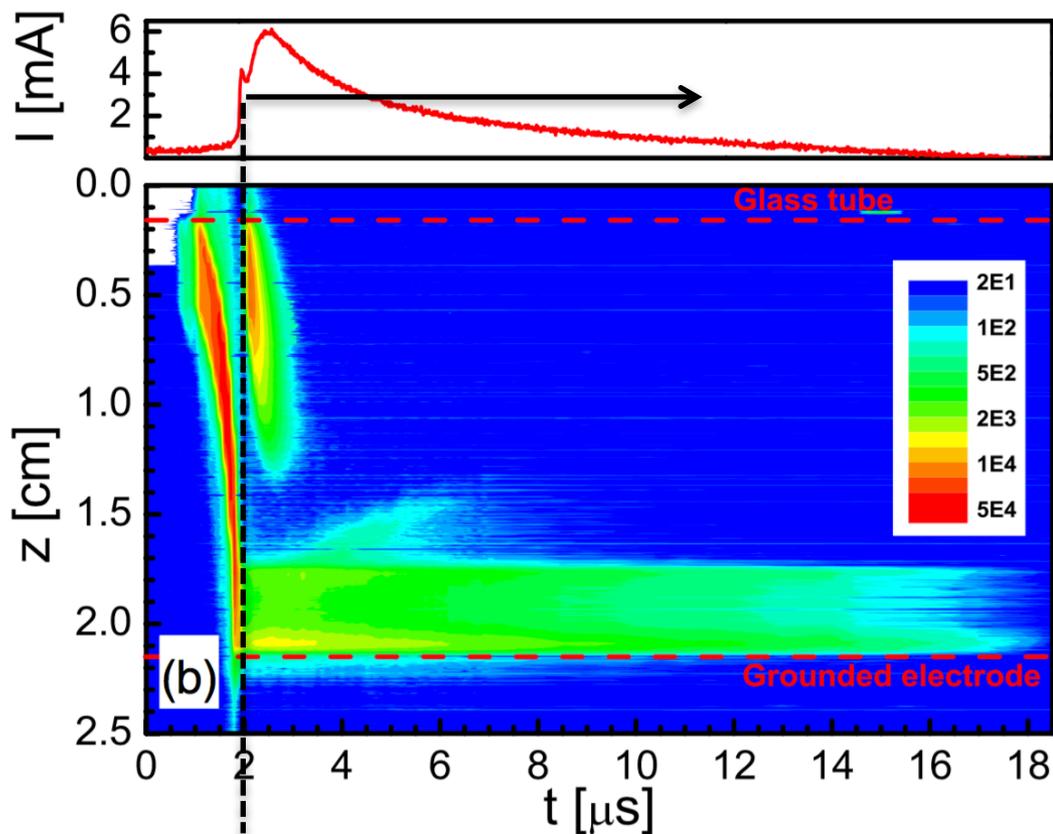
2D simulation of the volume barrier discharge  
in atmospheric pressure air



red arrow denotes the streamer impact onto the cathode  
creating the conductive channel

# Limitations: net charge in streamer head and sheath

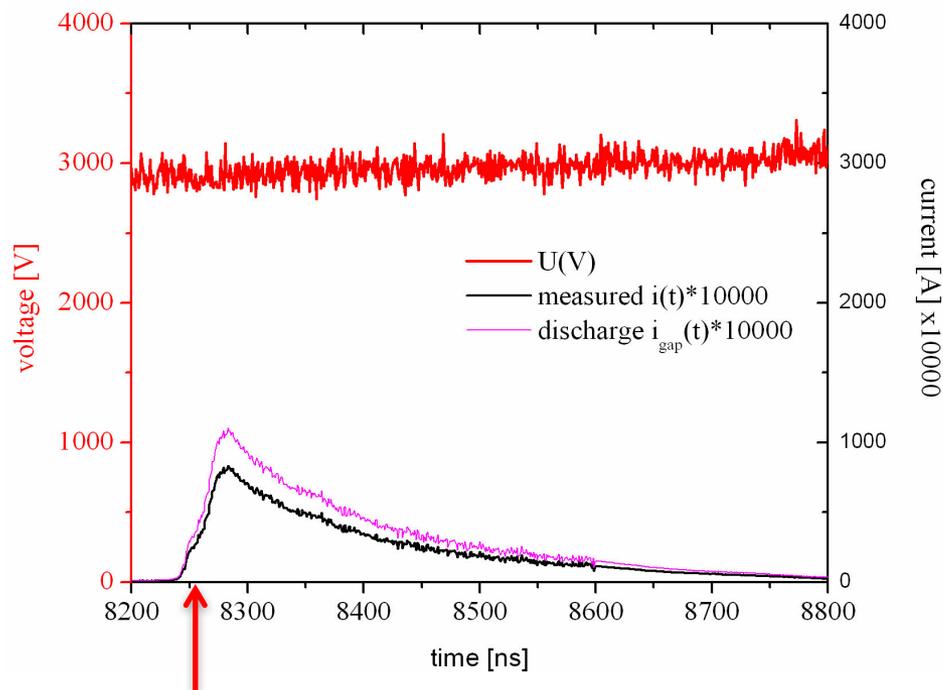
Correlated current and spatiotemporal development of helium line in barrier discharge plasma jet in atmospheric pressure helium



the streamer impact creating the conductive channel

# Limitations of current determination

Coplanar barrier discharge in air  
at 30 kPa pressure



red arrow denoting the  
impact of streamers on  
the electrodes

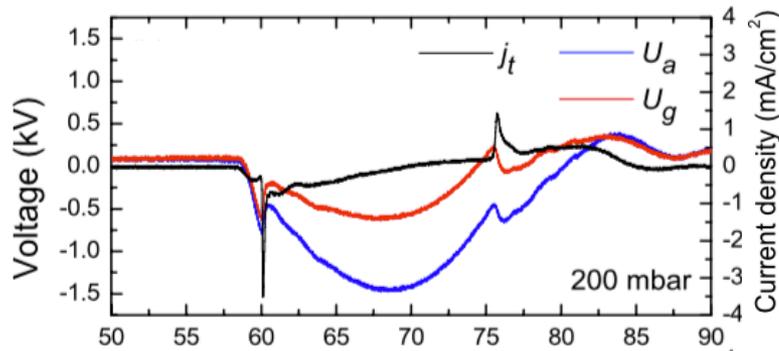
$$j_R(t) = j_c(t) = \frac{1}{1 - \frac{C_{\text{cell}}}{C_d}} \left[ i(t) - C_{\text{cell}} \frac{dV(t)}{dt} \right]$$

$$U_g(t) = V(t) - \frac{Q(t)}{C_d}$$

$$E(t) = U_g(t)/g$$

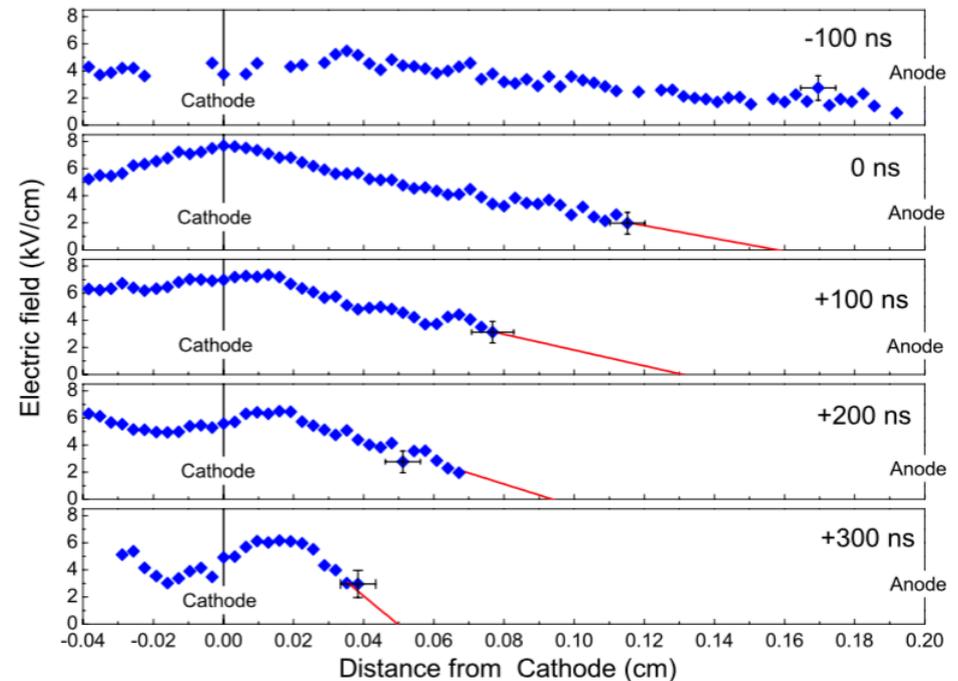
# Spectroscopic comparison – helium barrier discharge at 20kPa

Spatiotemporally resolved direct electric field measurement using Stark polarization emission spectroscopy in helium volume barrier discharge



$$I_g(t) = \left(1 + \frac{C_g}{C_d}\right) I_t(t) - C_g \frac{dU_a(t)}{dt},$$

$$U_g(t) = \frac{C_d}{C_g + C_d} U_a(t) - \frac{1}{C_g + C_d} \int_0^t I_g(\tau) d\tau$$



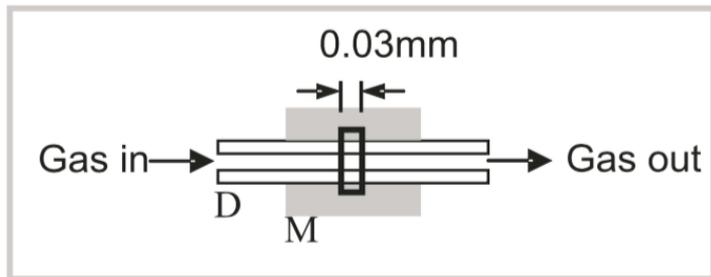
**Figure 8.** Development of the electric field spatial distribution in DBD in helium at 200 mbar.

Ivkovic et al. 2009 J. Phys. D: Appl. Phys.

Liu et al. 2003 J. Phys. D: Appl. Phys.

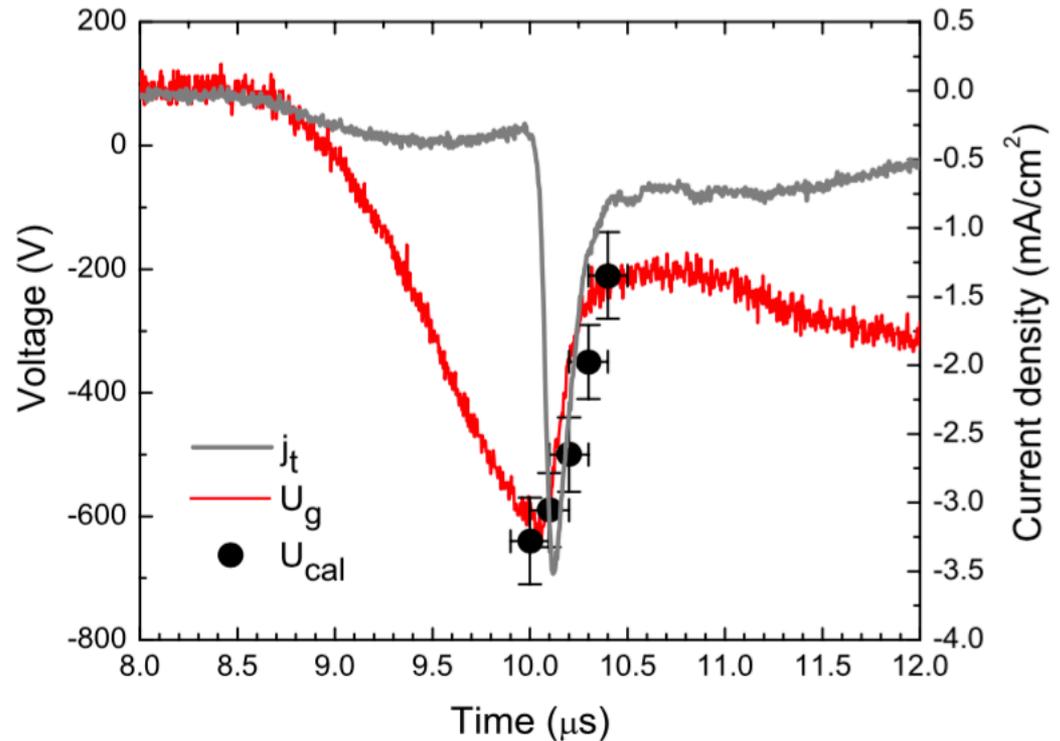
# Spectroscopic comparison – helium at 20kPa

Spatiotemporally resolved direct electric field measurement using Stark polarization emission spectroscopy - comparison



$$I_g(t) = \left(1 + \frac{C_g}{C_d}\right) I_t(t) - C_g \frac{dU_a(t)}{dt},$$

$$U_g(t) = \frac{C_d}{C_g + C_d} U_a(t) - \frac{1}{C_g + C_d} \int_0^t I_g(\tau) d\tau$$



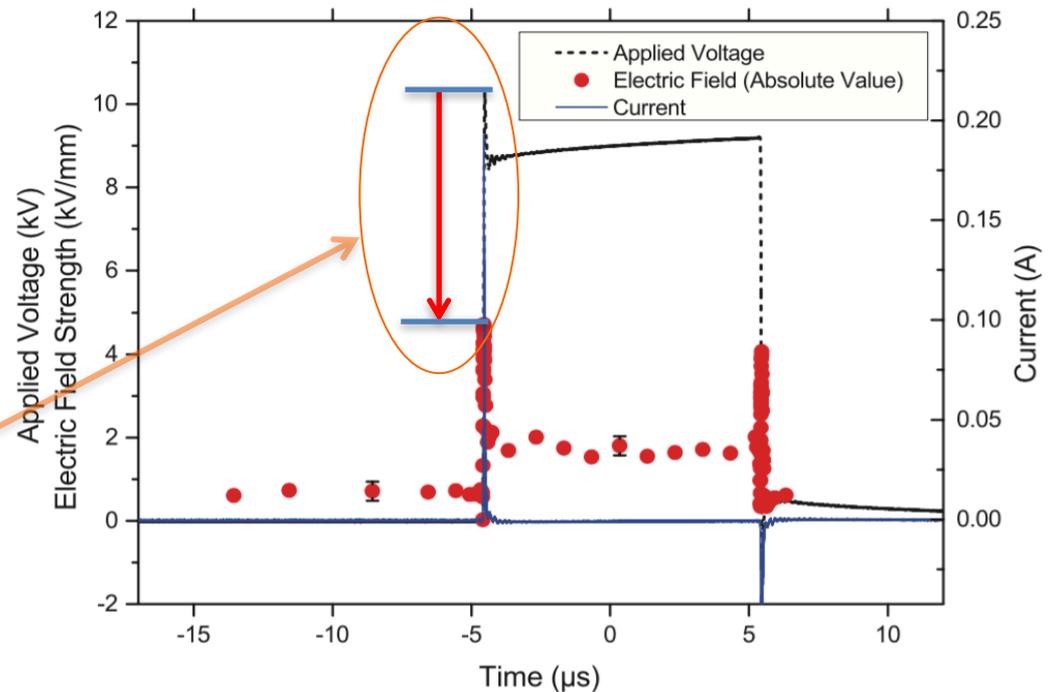
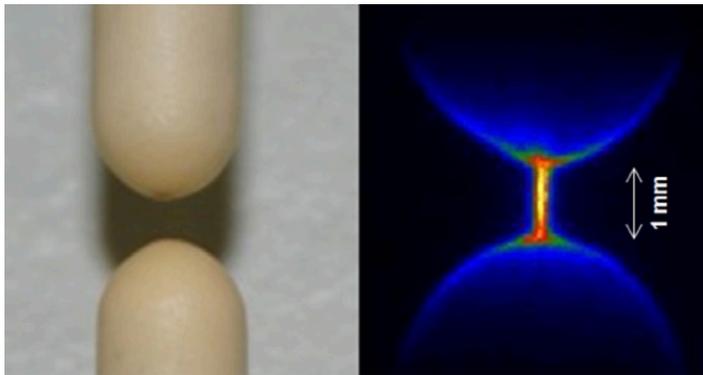
**Figure 10.** Comparison of calculated cathode fall voltage  $U_{cal}$  and measured gap voltage  $U_g$  in the DBD at 200 mbar pressure.

Ivkovic et al. 2009 J. Phys. D: Appl. Phys.

Liu et al. 2003 J. Phys. D: Appl. Phys.

# Spectroscopic comparison – N<sub>2</sub>/H<sub>2</sub> mixture at atmospheric pressure in ns-pulsed barrier discharge

Direct electric field measurement in the discharge gap based on coherent anti-Stokes Raman spectroscopy four-wave mixing method



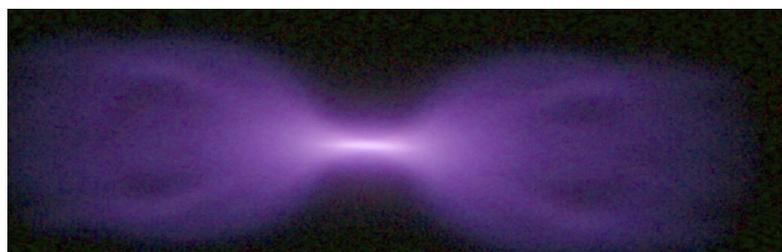
$$U_g(t) = V(t) - \frac{Q(t)}{C_d}$$

derived within the simplest equivalent circuit approach

Boehm et al. 2016 Plasma Sources Sci. Technol.  
Kettlitz et al. 2012 J. Phys. D: Appl. Phys.  
Pipa et al. 2012 Rev. Sci. Instrum.

# Spectroscopic comparison – air 30kPa Townsend phase of coplanar barrier discharge prior the breakdown

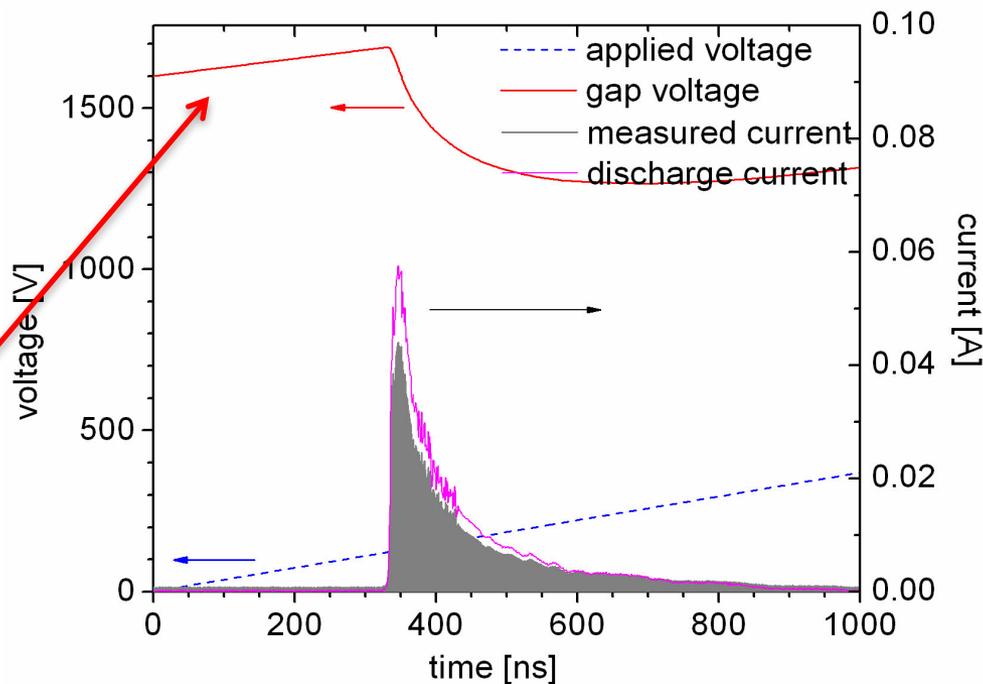
Effective electric field determined by Townsend alpha coefficient fitting of  $\alpha(E/N)$  from high-resolution emission of  $N_2(C-B)$  in coplanar barrier discharge



1 mm

$$U_g(t) = V(t) - \frac{Q(t)}{C_d}$$

derived within the simplest equivalent circuit approach



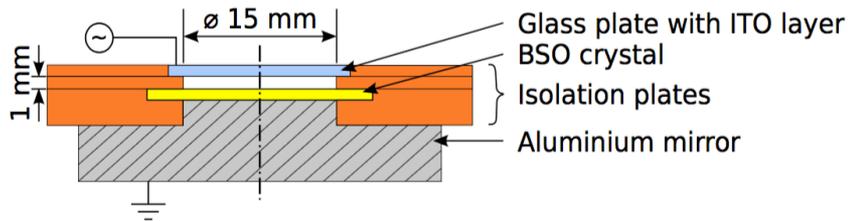
Electric field at the breakdown instant:  
 $190 \pm 30$  Td (electrics) and  $220 \pm 20$  Td (fitting)  
and  $210 \pm 40$  Td from FNS/SPS(E/N)

Pipa et al. 2012 Rev. Sci. Instrum.

Hoder, Synek et al. 2016 Plasma Phys. Control. Fusion

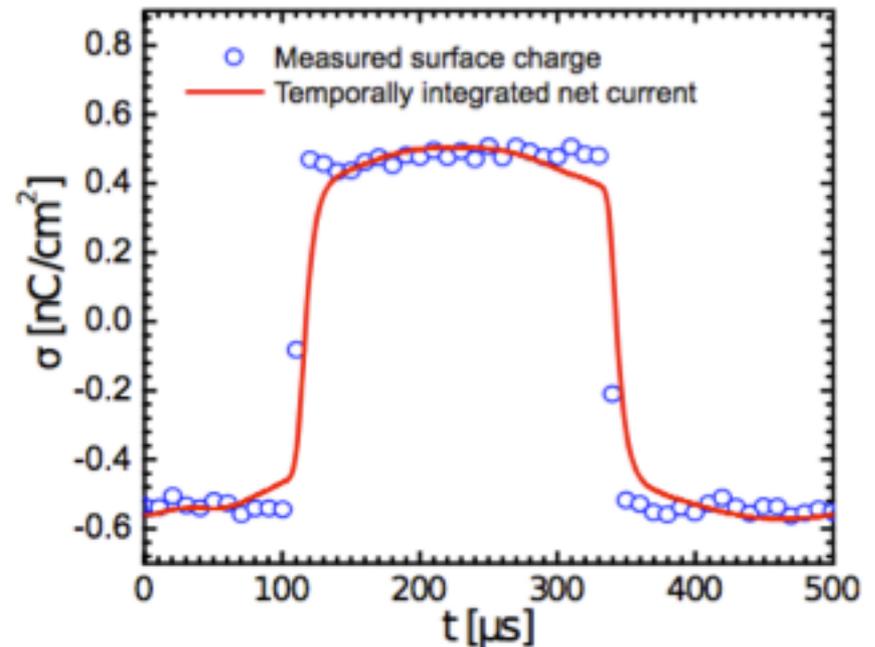
# Pockels effect comparison – helium barrier discharge at atmospheric pressure

Electric field measurement induced by Pockels effect on deposited surface charge



$$I_g(t) = \left(1 + \frac{C_g}{C_d}\right) I_t(t) - C_g \frac{dU_a(t)}{dt}$$

$$q(t) = \frac{C_{cell}}{1 - \frac{C_{cell}}{C_d}} \left[ \frac{Q(t)}{C_{cell}} - V(t) \right] + q_0$$



Bogaczyk, Sretenović et al. 2012 Eur. Phys. J. D

Liu et al. 2003 J. Phys. D: Appl. Phys.

# Determination of capacitances – fully powered large scale reactors (DCSBD, ozonizers, ...)

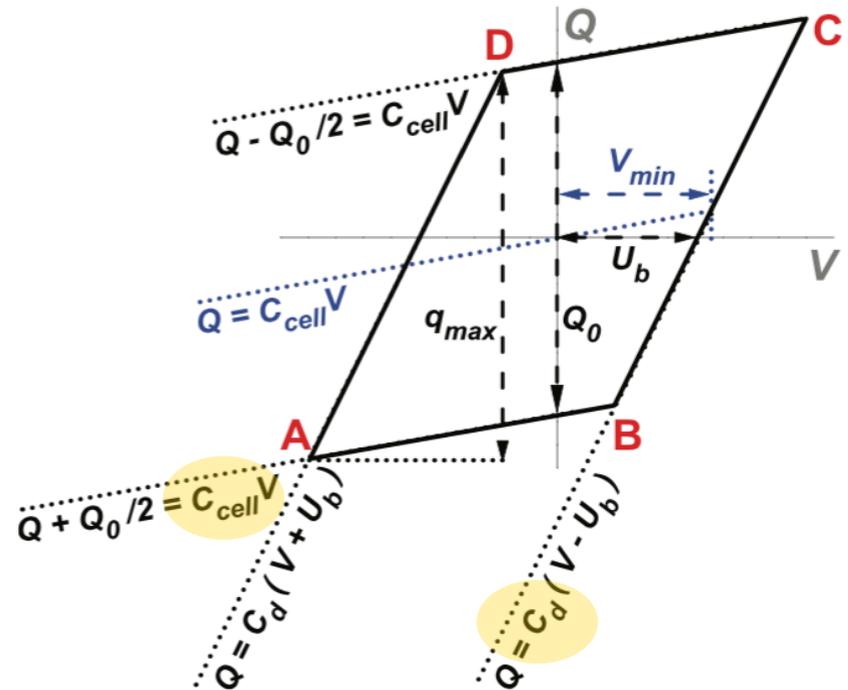
$$j_R(t) = \frac{1}{1 - \frac{C_{cell}}{C_d}} \left[ i(t) - C_{cell} \frac{dV(t)}{dt} \right]$$

$$U_g(t) = V(t) - \frac{Q(t)}{C_d}$$

$$q(t) = \frac{C_{cell}}{1 - \frac{C_{cell}}{C_d}} \left[ \frac{Q(t)}{C_{cell}} - V(t) \right] + q_0$$

Limited just for full electrode surface coverage by plasma!

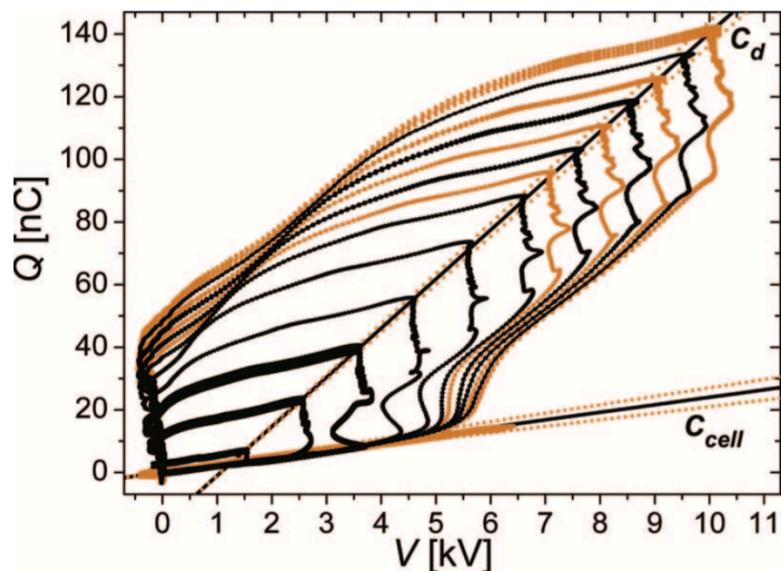
E.g. DCSBD at power with full coverage of electrodes by plasma filaments!



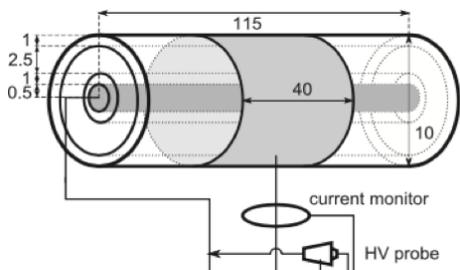
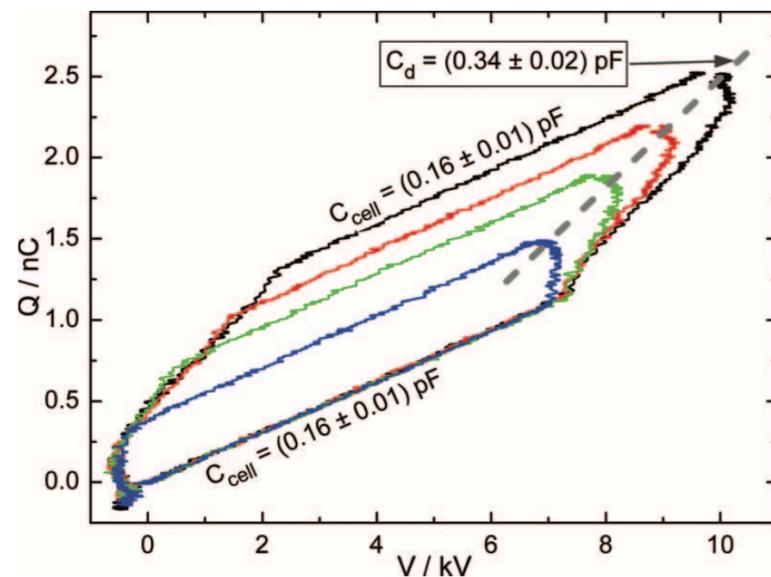
Falkenstein et al. 1997 J. Phys. D: appl. Phys.  
 Manley 1943 Trans. Electrochem. Soc.  
 Peeters et al. 2015 Plasma Sources Sci. Technol.  
 Pipa et al. 2012 Rev. Sci. Instrum.

# Determination of capacitances – pulsed reactors

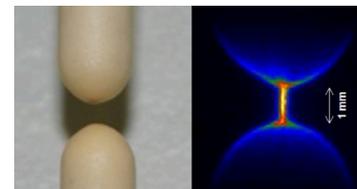
Low pressure asymmetric barrier discharge in argon at 100 mbar



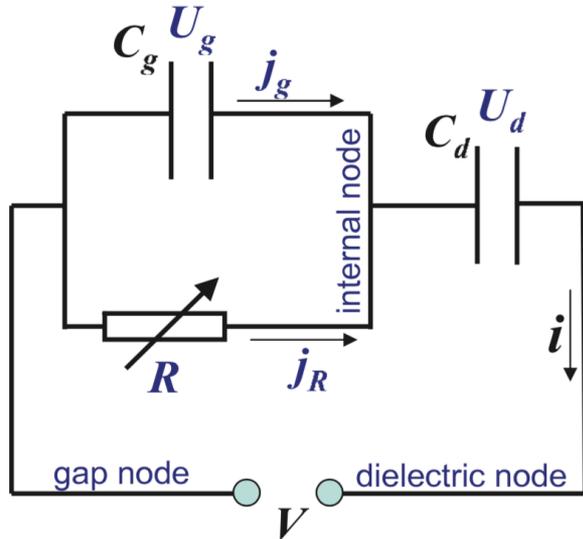
Atmospheric pressure symmetric barrier discharge in N<sub>2</sub>-O<sub>2</sub> mixture



$$Q_{\max} = C_d (V_{\max} - U_{res})$$



# Applicability of the approach - limitations



Limited to barrier discharges which can be described within a single node approximation – i.e. the radial structure is negligible for given spatial- and temporal-scale:

1. Homogeneous barrier discharges (pulsed or sine applied voltage)
2. Nanosecond pulsed barrier discharges
3. Spatially confined single-filament barrier discharges
4. Multi-filament plasma sources with full electrode coverage

$$j_R(t) = \frac{1}{1 - \frac{C_{cell}}{C_d}} \left[ i(t) - C_{cell} \frac{dV(t)}{dt} \right]$$

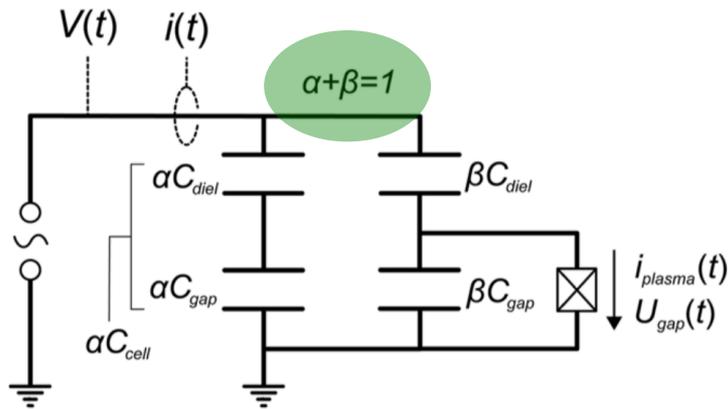
$$q(t) = \frac{C_{cell}}{1 - \frac{C_{cell}}{C_d}} \left[ \frac{Q(t)}{C_{cell}} - V(t) \right] + q_0$$

$$U_g(t) = V(t) - \frac{Q(t)}{C_d}$$

Liu et al. 2003 J. Phys. D: Appl. Phys.  
Pipa et al. 2012 Rev. Sci. Instrum.

# What about not fully powered barrier discharge reactors ... ?

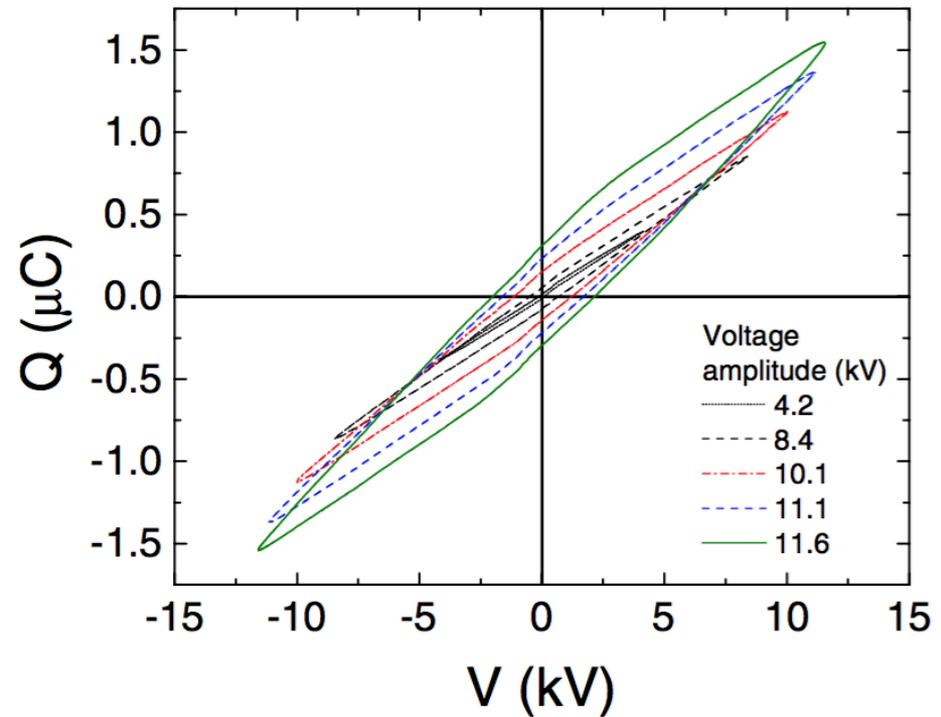
... or middle-sized multifilament discharges without full surface coverage by plasma – what to do?



$$i_{\text{plasma}}(t) = \frac{1}{1 - C_{\text{cell}}/C_{\text{diel}}} \left[ \frac{dQ(t)}{dt} - C_{\text{cell}} \frac{dV(t)}{dt} \right]$$

$$Q(t) = \left( 1 - \frac{C_{\text{cell}}}{C_{\text{diel}}} \right) Q_{\text{plasma}}(t) + C_{\text{cell}} V(t)$$

$$U_{\text{gap}}(t) = \left( 1 + \frac{\alpha C_{\text{cell}}}{\beta C_{\text{diel}}} \right) V(t) - \frac{1}{\beta C_{\text{diel}}} Q(t).$$



# How to use this?

## What parameters can be approached...

... besides the cases for mentioned spectroscopy  
and Pockels effect cases

$$j_R(t) = \frac{1}{1 - \frac{C_{cell}}{C_d}} \left[ i(t) - C_{cell} \frac{dV(t)}{dt} \right]$$

The upper estimate of the electron  
density development within the  
established discharge channel

$$n_e = \frac{j_R(t)}{eE(t)\mu_e(E(t)/N)}$$

$$U_g(t) = V(t) - \frac{Q(t)}{C_d}$$

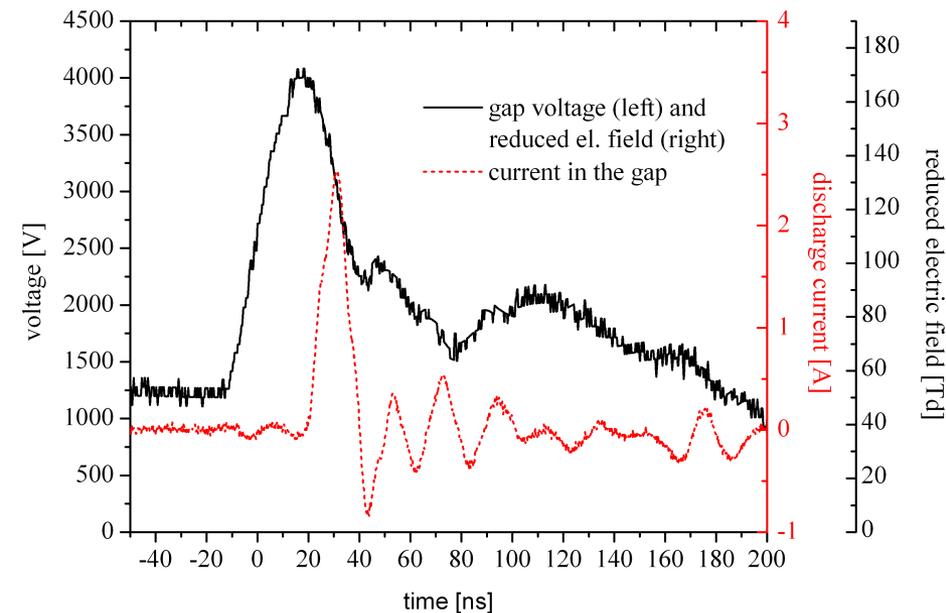
$P(t) = j_R(t)U_g(t)$   
Instantaneous discharge  
power development

Averaged, spatially unresolved,  
electric field strength development

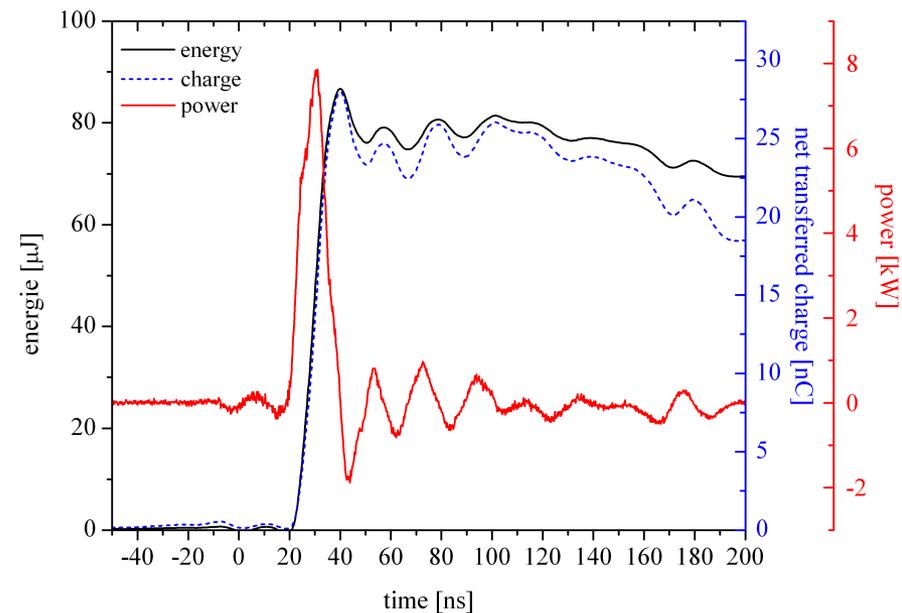
$$E(t) = U_g(t)/g$$

# Effective electric field and power in ns-pulsed single-filament coplanar barrier discharge

Complete analysis of macroscopic parameters of nanosecond pulsed plasma in atmospheric pressure argon



Gap voltage, effective reduced electric field and internal discharge current development



Instantaneous development of the internal transferred charge, energy and power in the discharge

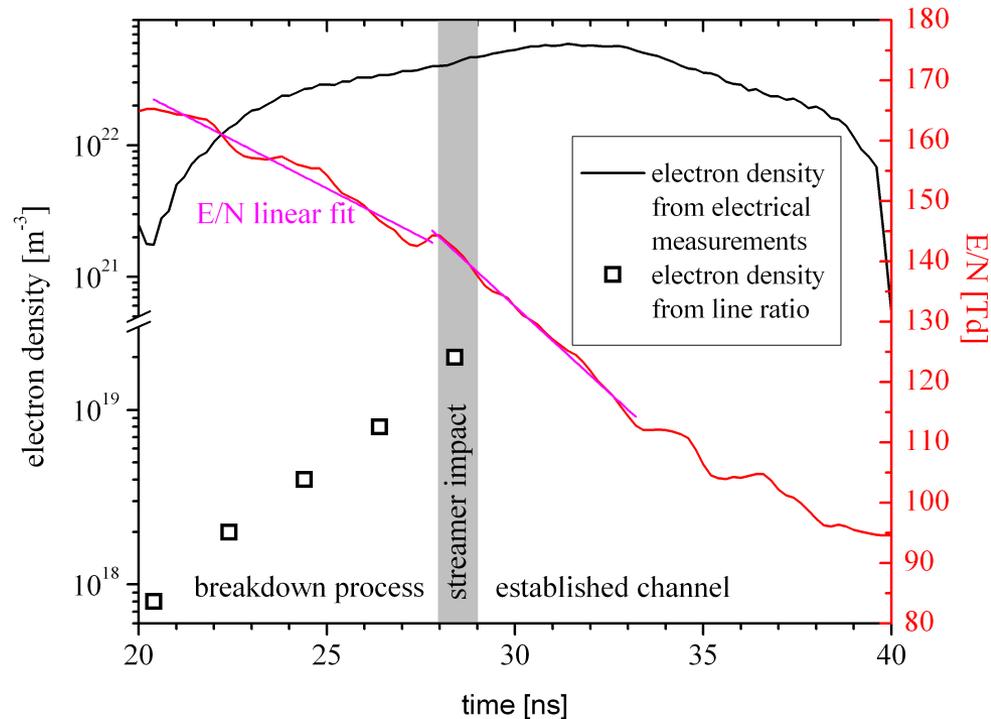
Compare to Leiweke et al 2013 J. Appl. Phys.

Dedrick et al. 2012 Plasma Sources Sci. Technol

Hoder, Šimek et al. 2017

# Electron density in ns-pulsed single-filament coplanar barrier discharge

Rough estimate of lower and upper limit of electron density by line-ratio and electrical methods



Other limitation of the method probably reached – plasma channel with high electron density would have less capacitive behaviour as  $C_g$

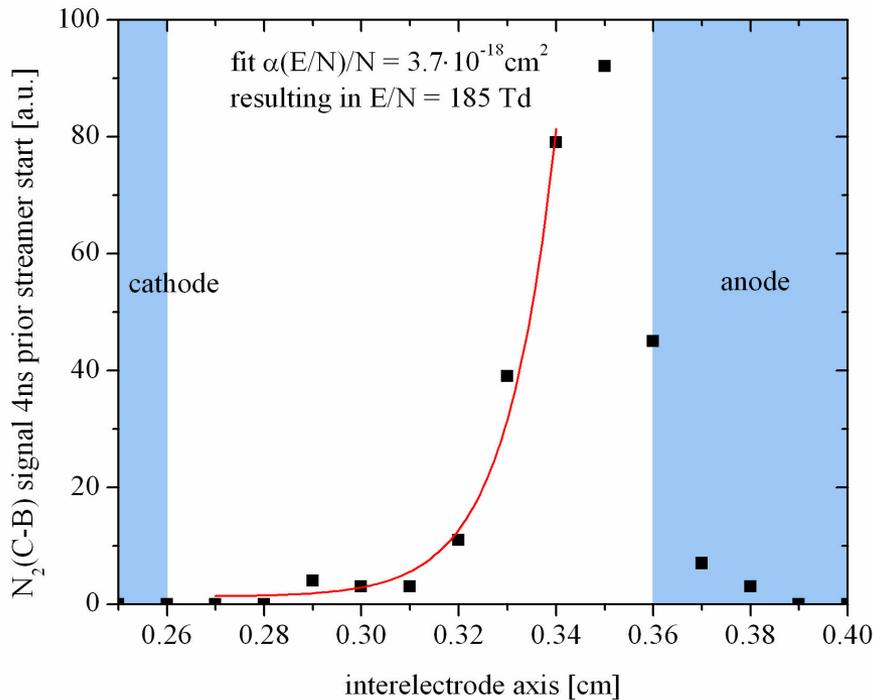
Hoder, Šimek et al. 2017

Compare to Walsh et al 2010 Eur. Phys. J. D

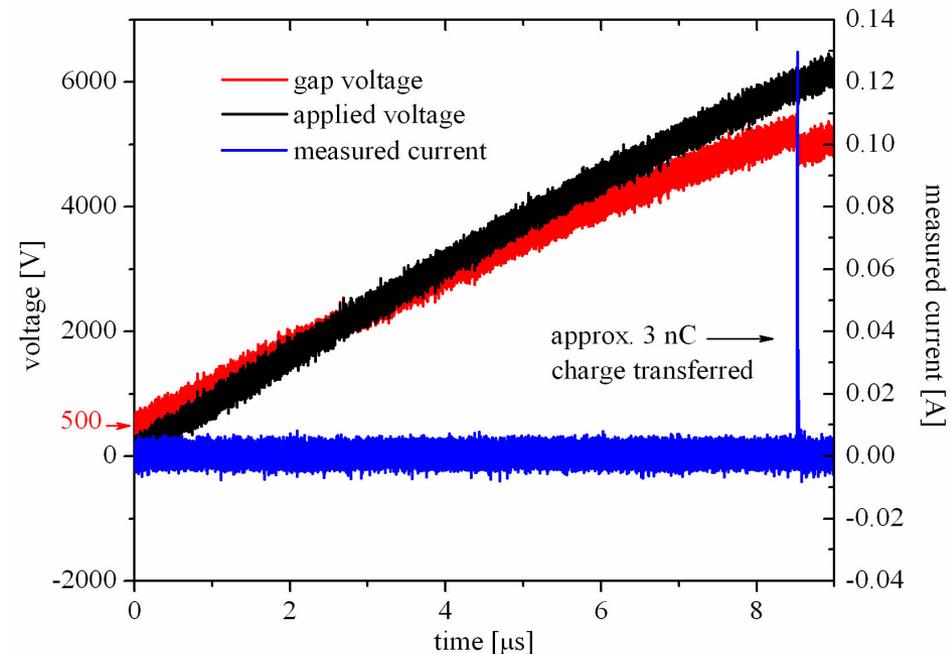
Zhu and Pu 2010 J. Phys. D: Appl. Phys.

# Electric field in Townsend phase of coplanar barrier discharge in atmospheric pressure air

Electric field prior the breakdown from Townsend  $\alpha(E/N)$  coefficient fitting on  $N_2(C-B)$  spectra emission



Electric field in the gap from electrical analysis

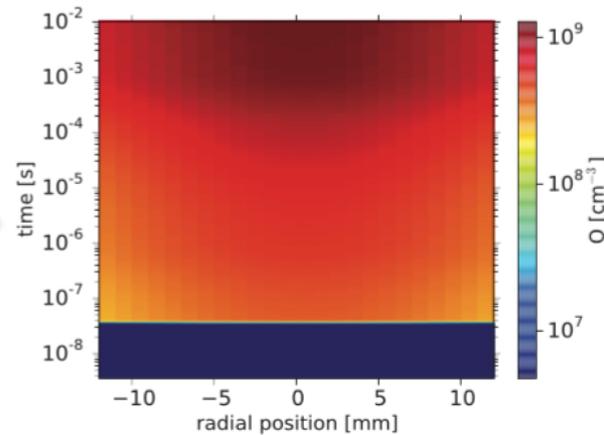
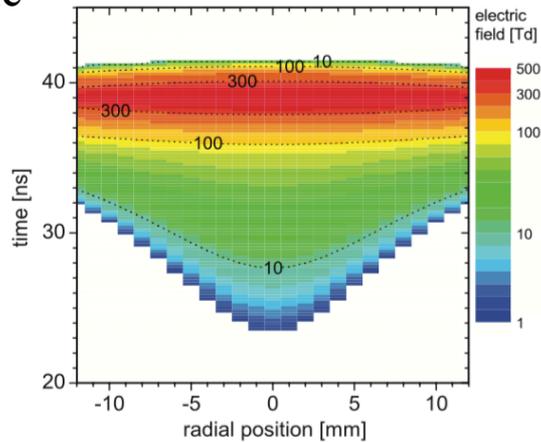


Electric field at the breakdown instant:  
 $180 \pm 30 \text{ Td}$  (electrics) and  $185 \pm 20 \text{ Td}$  (fitting)  
and  $200 \pm 40 \text{ Td}$  from FNS/SPS( $E/N$ )

# Importance of basic plasma parameters for long-term chemistry (4torr volume BD streamer)

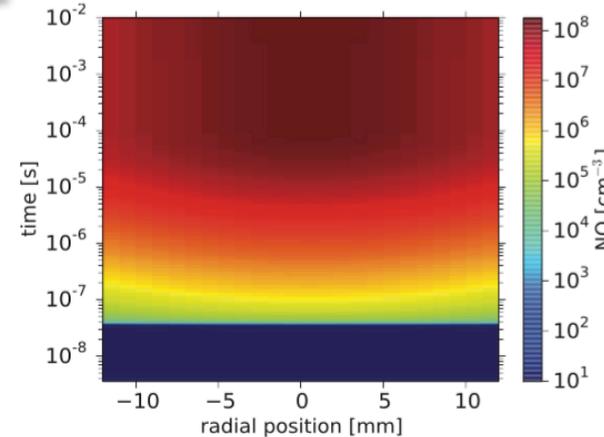
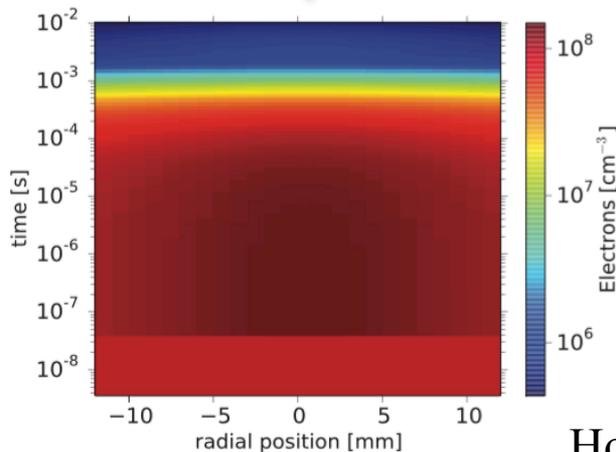
$E(x,t)$   $\rightarrow$  EVDF( $x,t$ )  $\rightarrow$  rate coefficients ( $x,t$ )  $\rightarrow$  electron, radical, metastable densities ( $x,t$ )

electric  
field



oxygen  
atoms

electrons



NO  
molecules

# Take-home message

Although spatially unresolved and approximative, the **electrical analysis** according to the simplest equivalent circuit approach **can give important informations about the plasma for (not only) low-density confined plasmas.**

It can gives information about temporal development of the **effective electric field** in the discharge gap, about the **net transferred charge** or **electron density** within the plasma channel or the instantaneous consumed **power in the plasma.**

All these derived informations can **support other methods** applied to investigation of the plasma. For precise analysis an, at least, **2D numerical model for given conditions has to be utilised.**

Single-filament coplanar barrier discharge was **studied numerically and experimentally resulting in electric field high-resolution records in quantitative agreement.** We plan to compute the generated surface gas chemistry using novel kinetic model of Zdeněk Bonaventura including usage of sensitivity analysis.

# Thank you for your attention!

**Z. Bonaventura, P. Synek,  
J. Ráhel' and M.Černák**



**M. Šimek**

**J. Jánský**



**D. Bessieres  
and J. Paillol**



**A. Pipa**



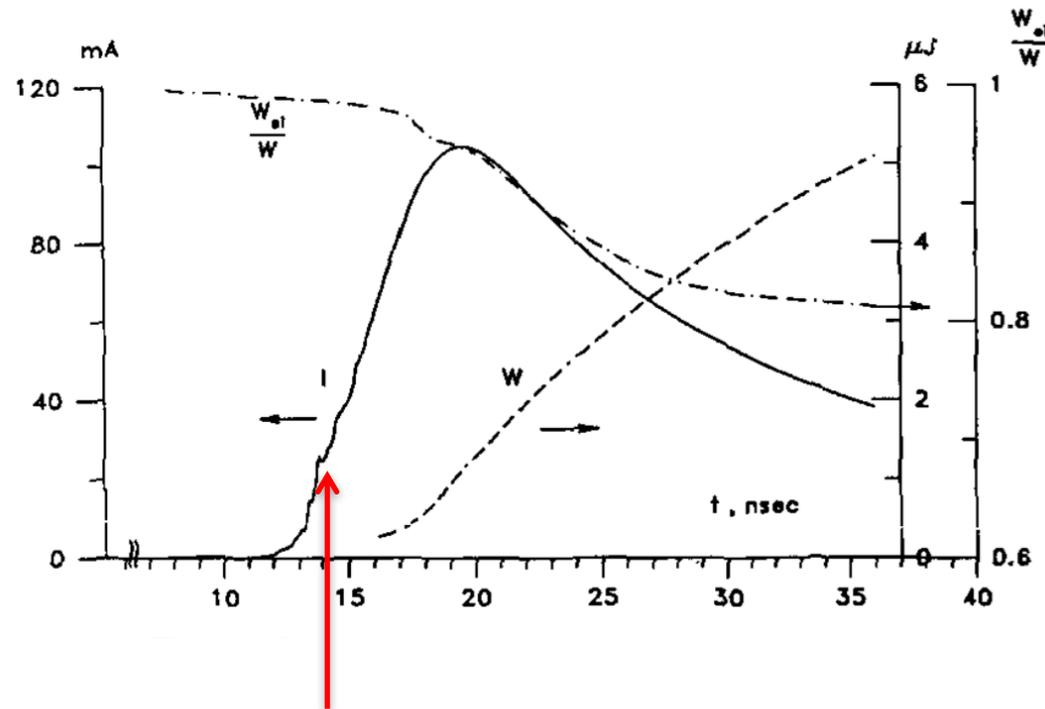
**F.J. Gordillo-Vázquez**

... and thanks to my colleagues and collaborators  
for the fruitful discussions and their contribution!



# Streamer impact and channel current

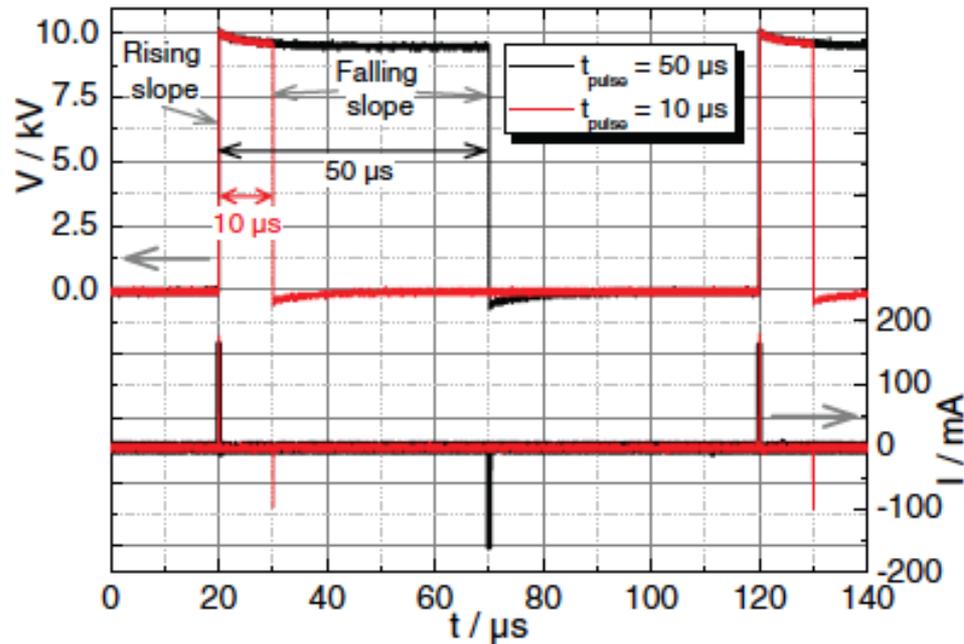
2D simulation of the volume barrier discharge  
in atmospheric pressure air



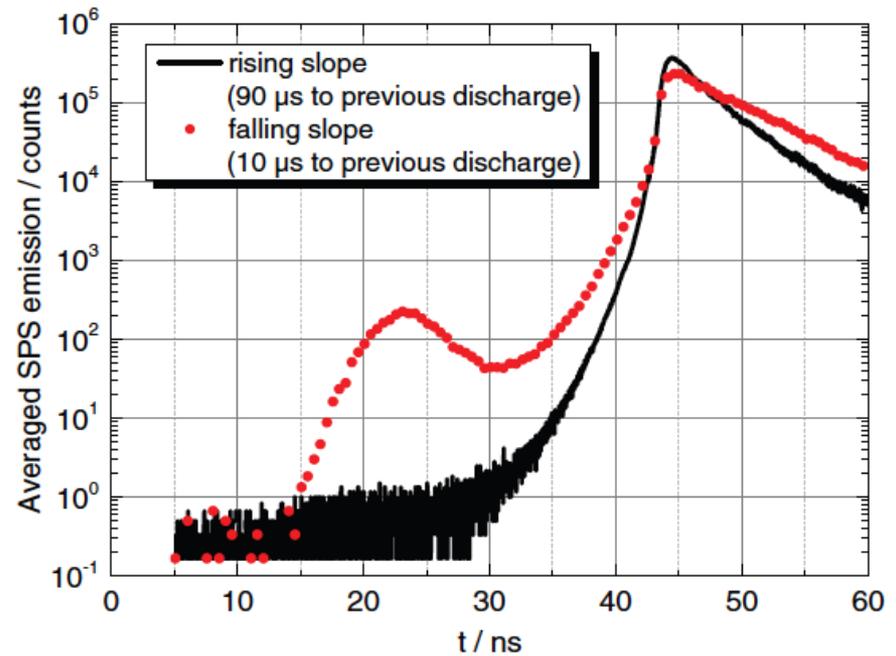
red arrow denotes the streamer impact onto the cathode creating the  
conductive channel

# Pre-breakdown phase of pulsed BDs: different pulse widths

## Electrical characteristics

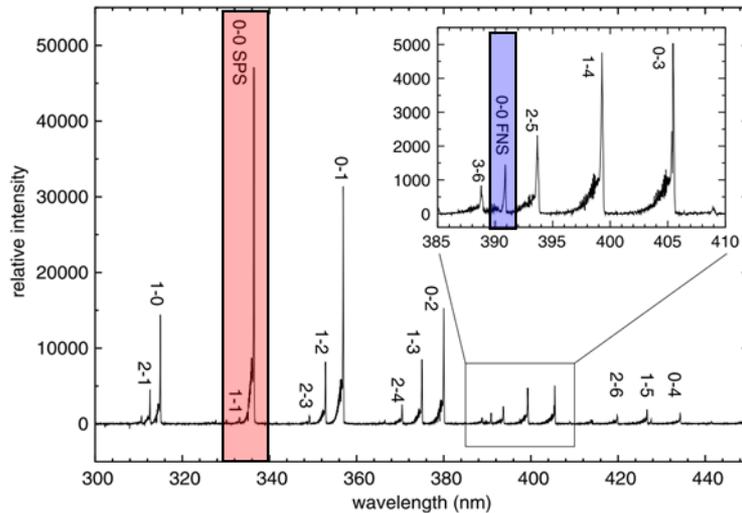


## TC-SPC recording at the anode

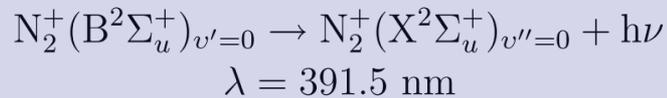


>>> new-found **local maximum** emerging prior to the breakdown of the gap

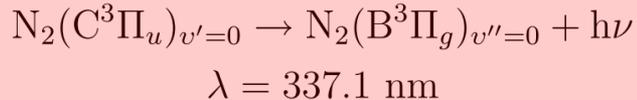
# Emission spectra and E/n determination



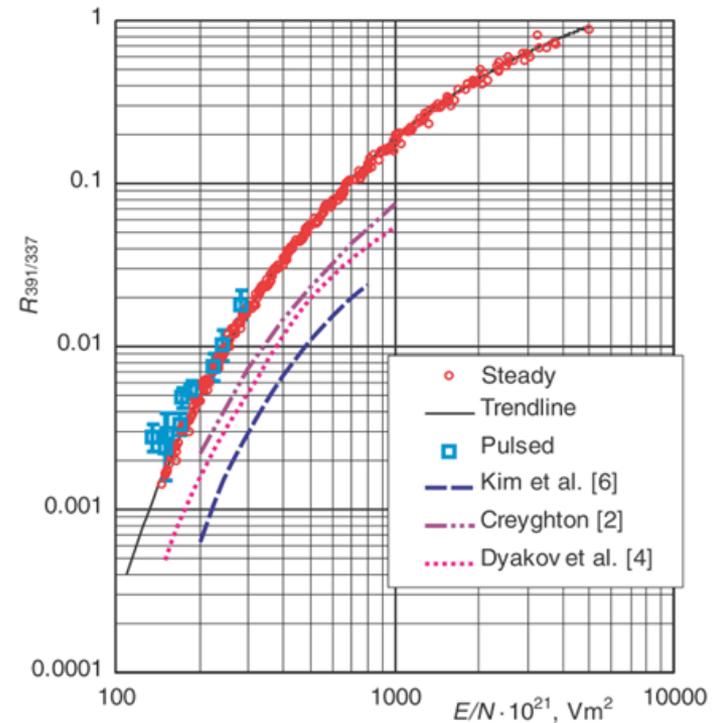
FNS



SPS



Intensity ratio FNS/SPS = f(E/n)



Paris et al. 2005 J.Phys.D:Appl.Phys.

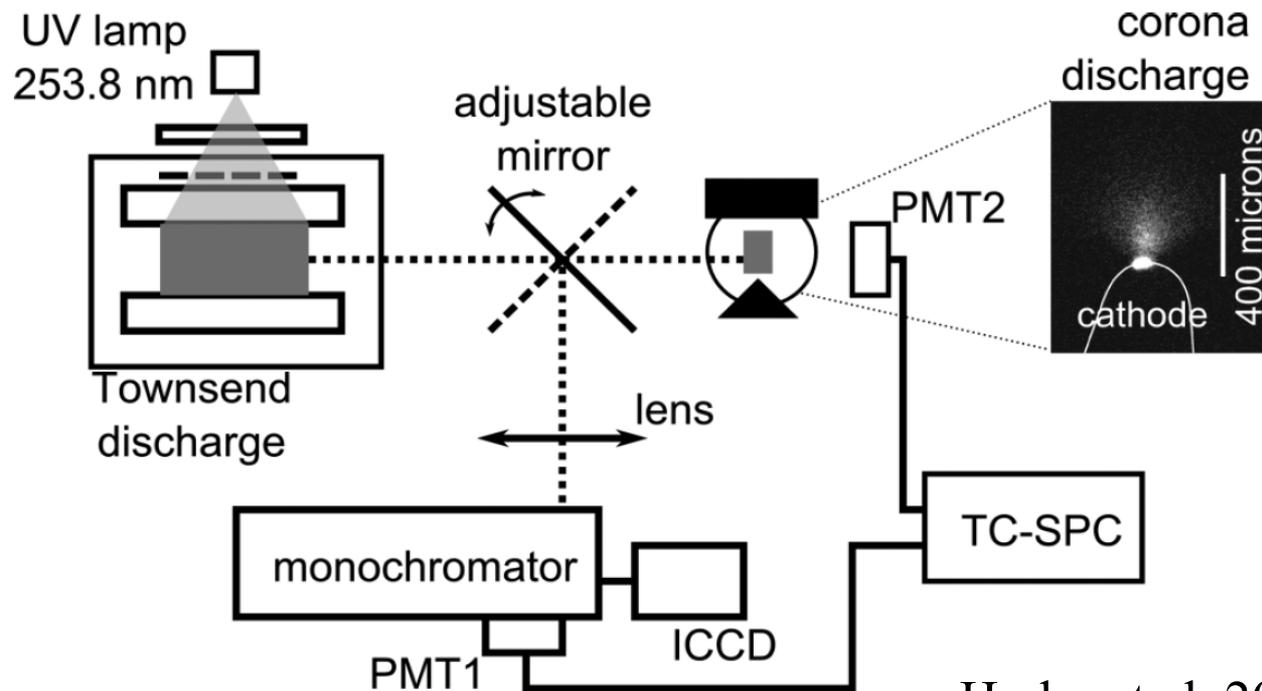
→ access to the electric field determination

without any distortion of the discharge – just from its emission

# Experimental setup

Without use of theoretical computations we used **single-table setup** including:

- ***Corona discharge*** as a subject of investigation
- Optical setup with ***monochromator, photomultipliers*** and ***TC-SPC module*** (resolution of almost 10 ps and 10  $\mu\text{m}$ )
- ***Townsend discharge*** for electric field calibration

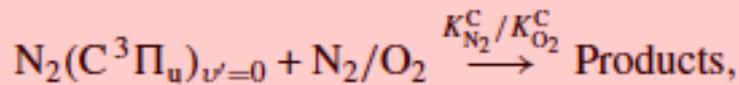
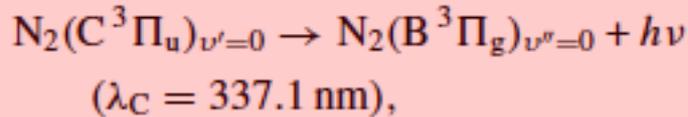
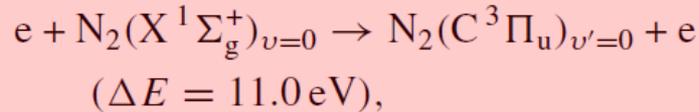


Hoder et al. 2012 Phys.Rev.E

Hoder et al. 2016 Plasma Sources Sci. Technol.

# The case of streamer discharge in air

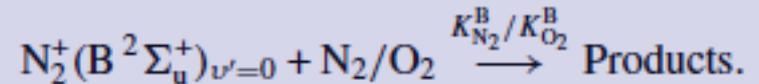
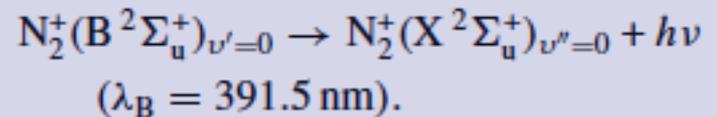
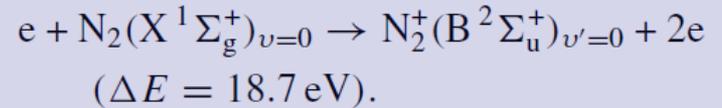
## SPS



$$\frac{dn_C(r, t)}{dt} = k_C \left( \frac{E}{n} \right) n_{N_2} n_e(r, t) - \frac{n_C(r, t)}{\tau_{\text{eff}}^C}$$

$$\begin{aligned} \frac{1}{\tau_{\text{eff}}^C} &= K_{N_2}^C n_{N_2} + K_{O_2}^C n_{O_2} + \frac{1}{\tau_0^C} \\ &= K_{N_2}^C n_{N_2} + K_{O_2}^C n_{O_2} + \sum_{v''=0}^{\infty} \frac{1}{\tau_{0v''}^C} \end{aligned}$$

## FNS



$$\frac{dn_B(r, t)}{dt} = k_B \left( \frac{E}{n} \right) n_{N_2} n_e(r, t) - \frac{n_B(r, t)}{\tau_{\text{eff}}^B}$$

$$\begin{aligned} \frac{1}{\tau_{\text{eff}}^B} &= K_{N_2}^B n_{N_2} + K_{O_2}^B n_{O_2} + \frac{1}{\tau_0^B} \\ &= K_{N_2}^B n_{N_2} + K_{O_2}^B n_{O_2} + \sum_{v''=0}^{\infty} \frac{1}{\tau_{0v''}^B} \end{aligned}$$

# Kinetic scheme, light intensities and ratio

SPS

$$\frac{dI_C(r, t)}{dt} = k_C \left( \frac{E}{n} \right) n_{N_2} n_e(r, t) T_C \frac{1}{\tau_{00}^C} \frac{hc}{\lambda_C} - \frac{I_C(r, t)}{\tau_{eff}^C}$$

FNS

$$\frac{dI_B(r, t)}{dt} = k_B \left( \frac{E}{n} \right) n_{N_2} n_e(r, t) T_B \frac{1}{\tau_{00}^B} \frac{hc}{\lambda_B} - \frac{I_B(r, t)}{\tau_{eff}^B}$$



$$\left( \frac{I_{FNS}/\tau_{eff}^{FNS} + dI_{FNS}/dt}{I_{SPS}/\tau_{eff}^{SPS} + dI_{SPS}/dt} \right) \frac{\tau_{eff}^{FNS}}{\tau_{eff}^{SPS}} =$$

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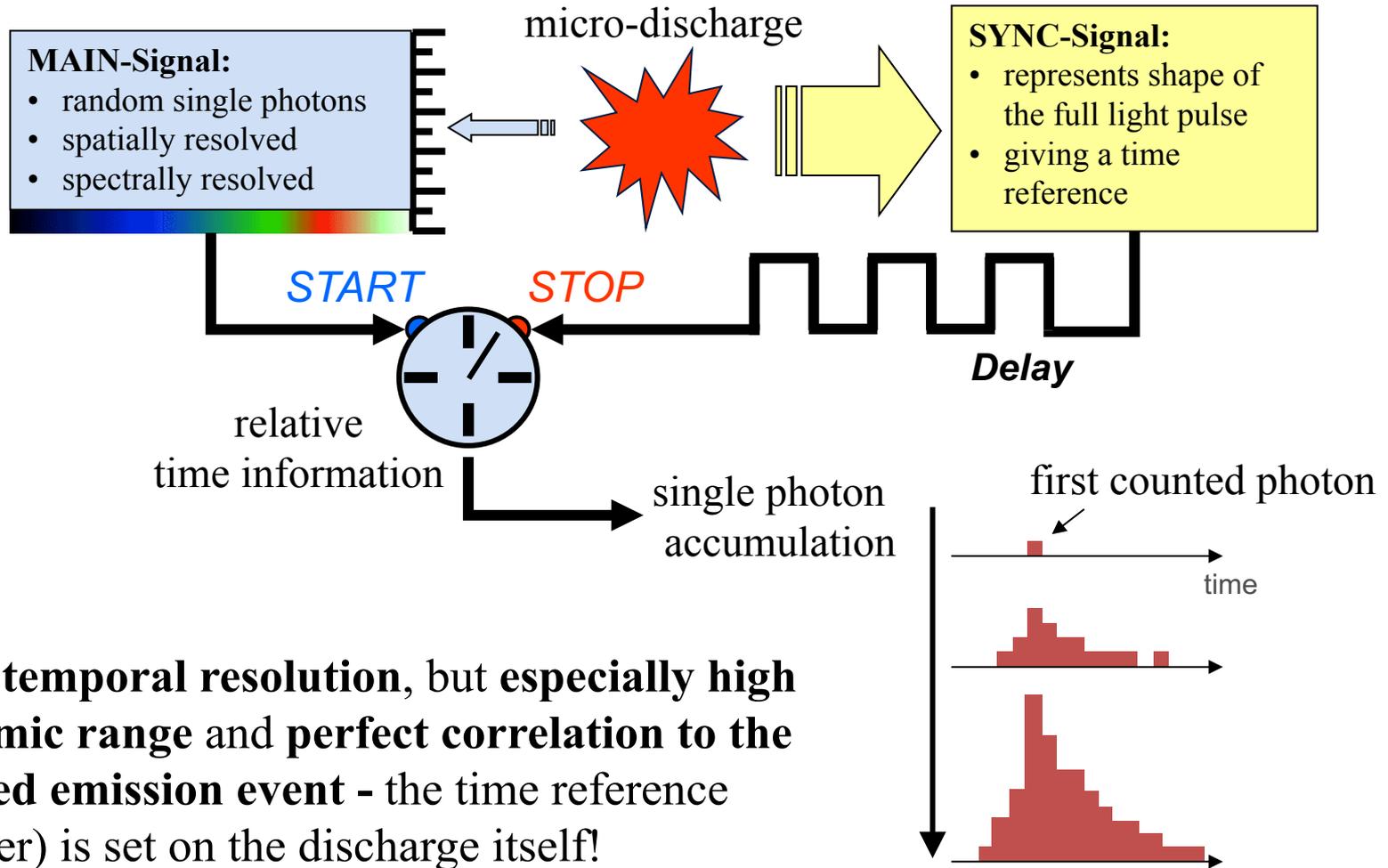
$$R(x, t) = f(E/n(x, t))$$



Paris et al. 2005  
J.Phys.D:Appl.Phys.

$$E/n(x, t)$$

# Time-correlated single photon counting technique, cross-correlation spectroscopy



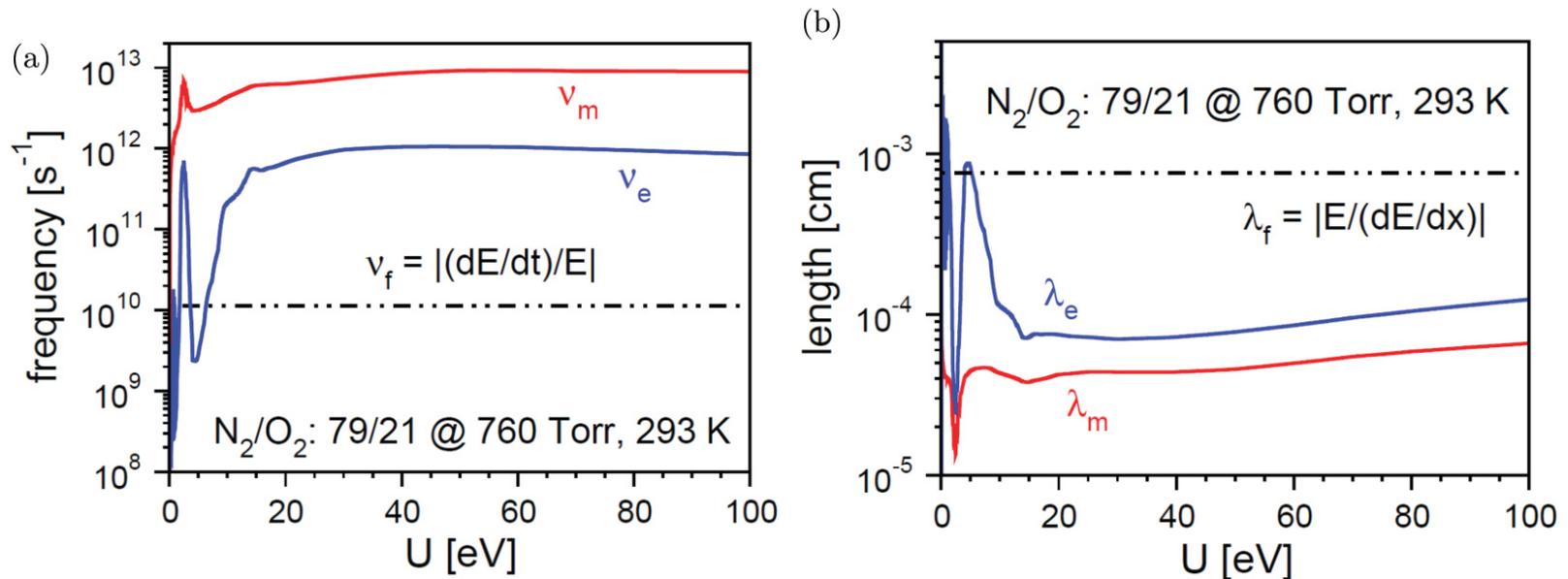
**High temporal resolution, but especially high dynamic range and perfect correlation to the studied emission event - the time reference (trigger) is set on the discharge itself!**

Ikuta and Kondo 1976 IEE

W. Becker 2005 Advanced time-correlated single-photon counting techniques

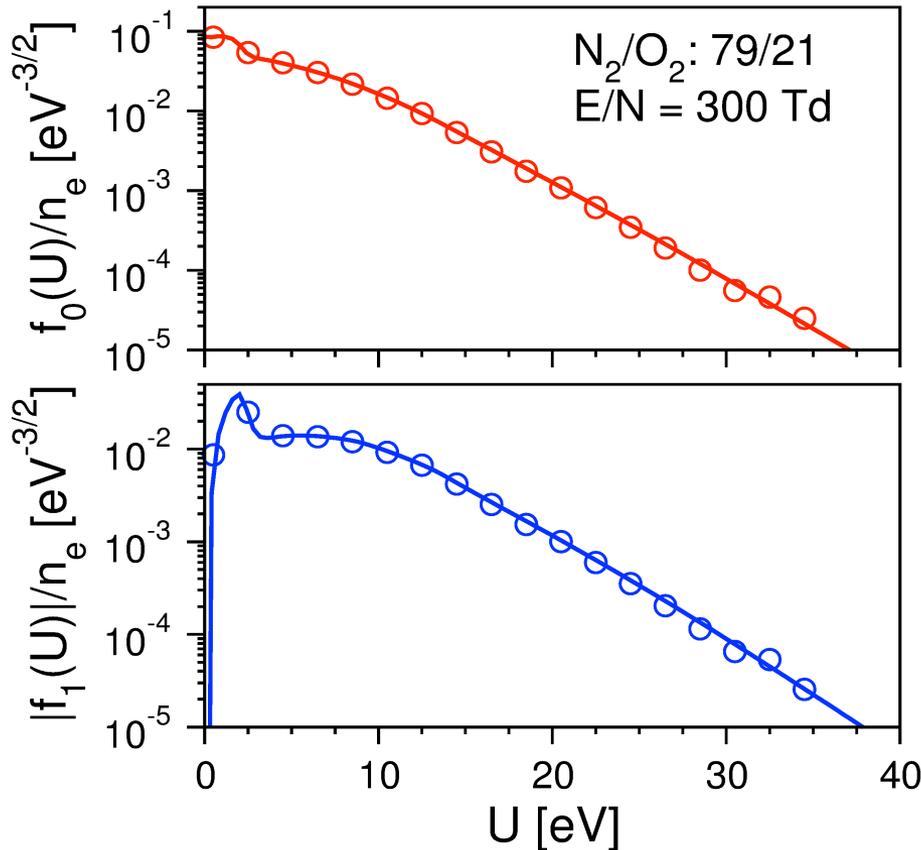
# LFA - collision frequencies

For justification of **local field approximation** the approach of analysis of the energy-resolved collision frequencies for momentum and energy dissipation as well as the energy-resolved mean free path and energy dissipation length.



Quasi-stationary evolution of the distribution function of the electrons takes place and electrons can be assumed to be in equilibrium with the local field.

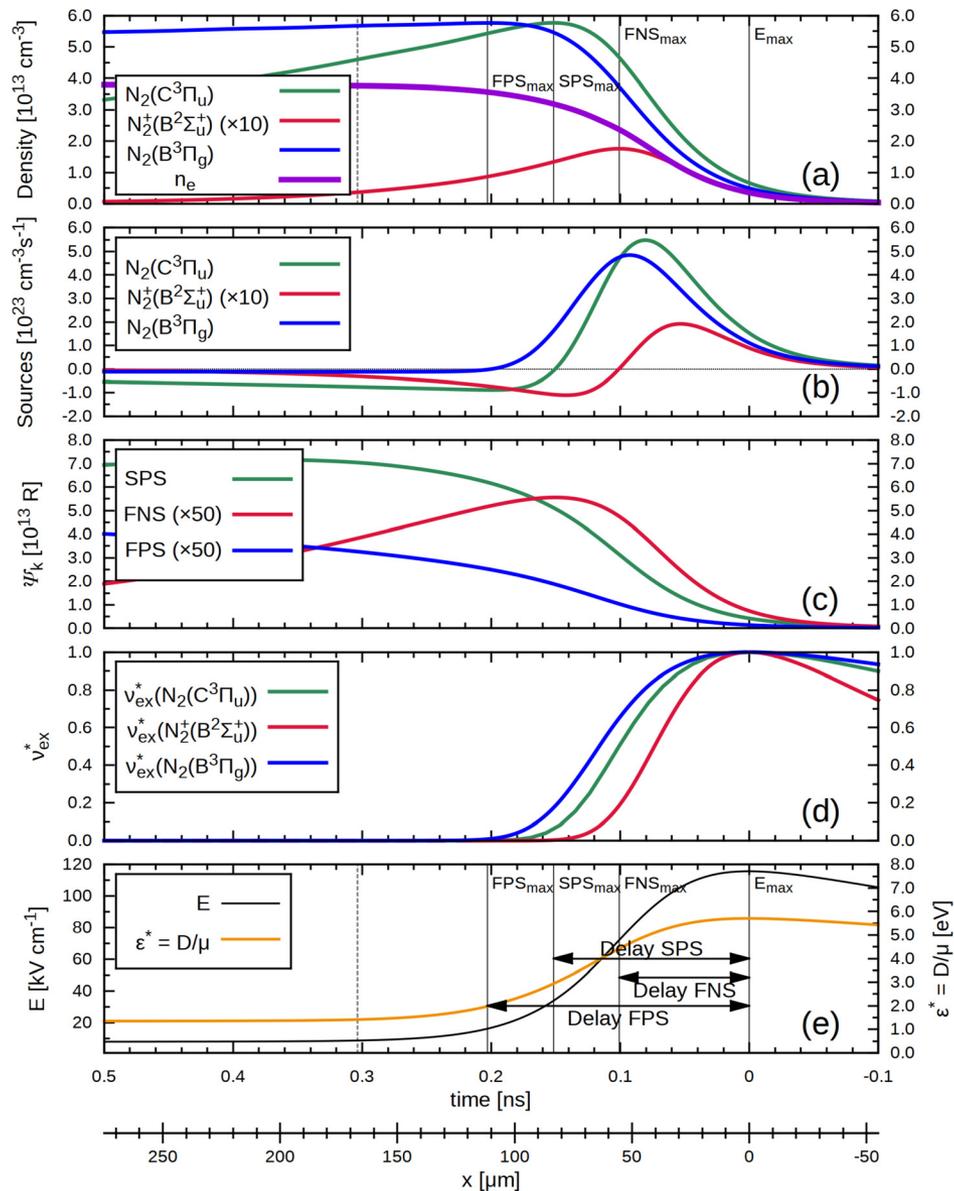
# LFA - relaxation of EVDF



EBE and MC results agree well and coincide with the corresponding steady-state electron distribution function components → **equilibrium values are reached after 10 ps and 5  $\mu\text{m}$ .**

Other approach is the study of the electron relaxation in time and space for different reduced electric field strengths  $E/N$ :

- ✧ lines represent  $f_0$ ,  $f_1$  after **10 ps** of the temporal electron relaxation (solution of the electron Boltzman equation in multi-term approximation).
- ✧ symbols denote  $f_0$ ,  $f_1$  for the 1D spatial relaxation of electrons after a distance of **5  $\mu\text{m}$**  using the Monte Carlo method.



$$\nu_m(v) = NvQ^T(v)$$

$$\lambda_m(v) = \frac{1}{NQ^T(v)}$$

$$\lambda_e(v) = \lambda_m(v) \sqrt{\frac{\nu_m(v)}{3\nu_e(v)}}$$

$$\nu_e(v) = Nv \left( 2\frac{m}{M}Q^d(v) + \sum_j Q_j^{\text{un}}(v) \frac{v_j^{\text{un}}}{v} \right)$$