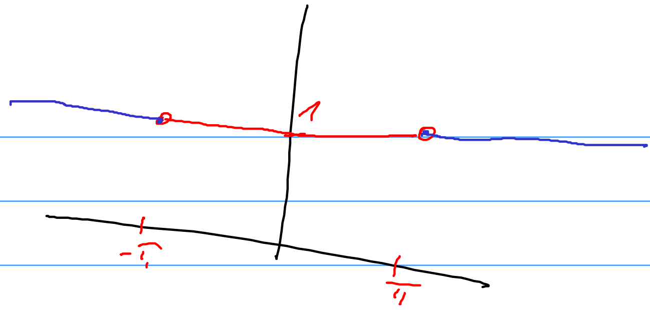
(1) $f(x) = 1$ na $[-\pi, \pi]$



$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} 1\,dx = \frac{1}{\pi}\cdot 2\pi = 2$$

$n: 1, 2, 3, 4, \dots$

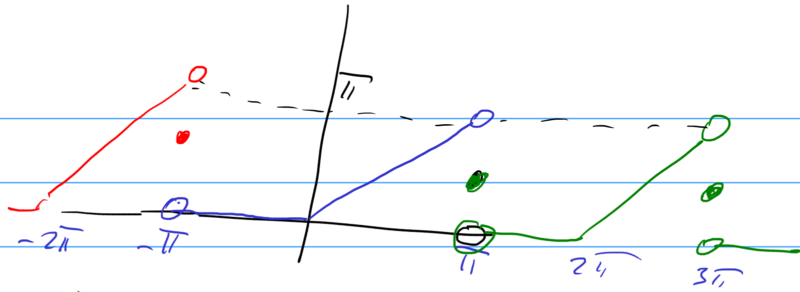
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos nx\,dx = \frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left(\frac{\cancelto{0}{\sin n\pi}}{n} + \frac{\cancelto{0}{\sin(+n\pi)}}{n}\right)$$

$$= \underline{0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx\,dx = \underline{0}$$

F.R. $\frac{2}{2} + \sum_{n=1}^{\infty} 0 = \underline{\underline{1}}$

② $f(x) = \begin{cases} x, & x \in [0, \pi] \\ 0, & x \in [-\pi, 0] \end{cases}$



$$\pi a_0 = \int_{-\pi}^{\pi} f(x)\,dx = \int_0^{\pi} x\,dx = \left[\frac{x^2}{2}\right]_0^{\pi} = \frac{\pi^2}{2}$$

$$a_0 = \frac{\pi}{2}$$

$$\pi a_n = \int_0^{\pi} x\cos(nx)\,dx = \left| \begin{matrix} u = x & u' = 1 \\ v' = \cos(nx) & v = \frac{\sin(nx)}{n} \end{matrix} \right| = \ldots =$$

$$= \left[\frac{\cos(nx)}{n^2} + \frac{x\sin(nx)}{n}\right]_0^{\pi} = \frac{\overbrace{\cos(n\pi)}^{=(-1)^n}}{n^2} + \frac{\overbrace{\pi\sin(n\pi)}^{\to 0}}{n} - \left(\frac{1}{n^2} + 0\right) =$$

$$= \frac{(-1)^n}{n^2} - \frac{1}{n^2}$$

$$\pi b_n = \int_0^{\pi} x\sin(nx)\,dx = \left| \begin{matrix} u = x & u' = 1 \\ v' = \sin(nx) & v = -\frac{\cos(nx)}{n} \end{matrix} \right| = \ldots =$$

$$= \left[\frac{\sin nx}{n^2} - \frac{x\cos nx}{n}\right]_0^{\pi} = \frac{\overbrace{\sin n\pi}^{\to 0}}{n^2} - \frac{\pi\cos n\pi}{n} = -\frac{\pi(-1)^n}{n} =$$

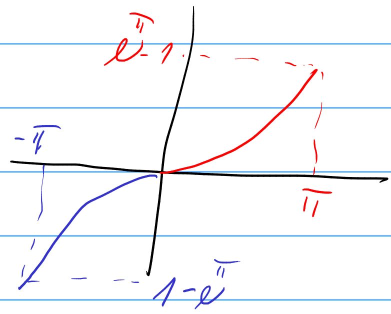
$$= \frac{(-1)^{n+1}}{n}\pi$$

F.Ř. $\displaystyle \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2}\cos(nx) + \frac{(-1)^{n+1}}{n}\sin(nx)$

(3) $f(x) = e^x - 1$ na $[0,\pi]$ sinovou řadu

$$g(x) = \begin{cases} e^x - 1, & \text{na } (0,\pi] \\ 1 - e^{-x}, & \text{na } [-\pi, 0] \end{cases}$$

$f(x) = -f(-x) = -e^{-x} + 1$



$a_0 = a_n = 0$

$$\frac{\pi}{2} b_n = \int_0^\pi (e^x - 1)\sin(nx)\,dx = \int_0^\pi e^x \sin(nx)\,dx - \int_0^\pi \sin(nx)\,dx =$$

2 × per partes + převedení na druhou stranu

$$= \left[\frac{e^x}{1+n^2}(\sin nx - n\cos nx)\right]_0^\pi - \left[-\frac{\cos nx}{n}\right]_0^\pi =$$

$$= \frac{e^\pi}{1+n^2}\left(\cancel{\sin n\pi}^{\,0} - n\underbrace{\cos n\pi}_{(-1)^n} - \cancel{\sin 0}^{\,0} + n\cos 0\right) + \left(\frac{\cos n\pi}{n} - \frac{\cos 0}{n}\right) =$$

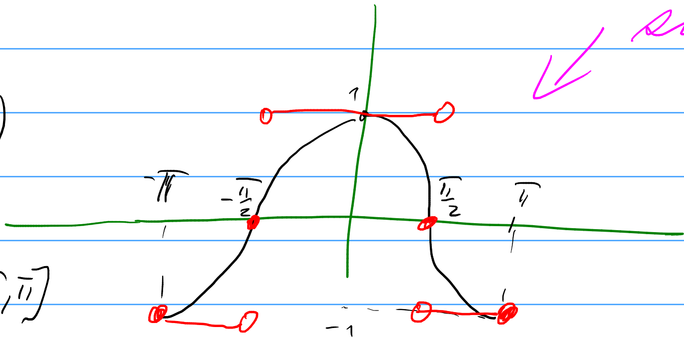
$$= \frac{e^\pi n}{1+n^2}\left[(-1)^n + 1\right] + \frac{1}{n}\left((-1)^n - 1\right)$$

sinová řada: $$\sum_{n=1}^{\infty}\left[\frac{2}{\pi}\,\frac{e^\pi n\left[(-1)^n+1\right]}{1+n^2} + \frac{(-1)^n - 1}{n}\right]\sin(nx)$$

④ $f(x) = \operatorname{sgn}(\cos x)$ na $[-\pi, \pi]$

snadá

$$f(x) = \begin{cases} 1 & , x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ 0 & , x \in \left\{\frac{\pi}{2}, -\frac{\pi}{2}\right\} \\ -1 & , \text{jinak } \cap [-\pi, \pi] \end{cases}$$



$b_n = 0$

$$\frac{\pi}{2} a_0 = \int_0^{\pi} f(x)\,dx = \int_0^{\frac{\pi}{2}} 1\,dx + \int_{\frac{\pi}{2}}^{\pi} -1\,dx = \frac{\pi}{2} - \frac{\pi}{2} = 0$$

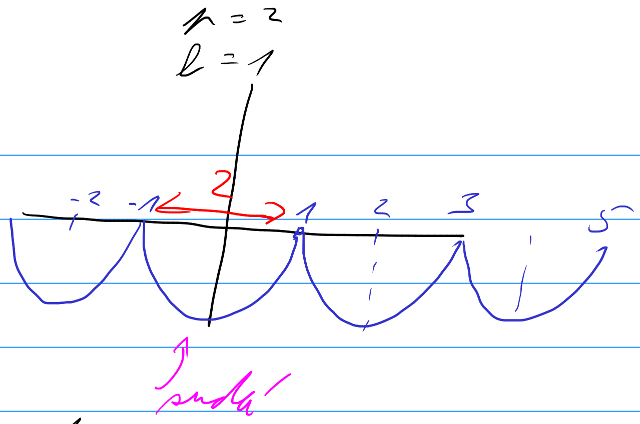
$$\frac{\pi}{2} a_n = \int_0^{\frac{\pi}{2}} \cos nx\,dx - \int_{\frac{\pi}{2}}^{\pi} \cos(nx)\,dx = \left[\frac{\sin nx}{n}\right]_0^{\frac{\pi}{2}} - \left[\frac{\sin nx}{n}\right]_{\frac{\pi}{2}}^{\pi} =$$

$$= \frac{\sin\frac{n\pi}{2}}{n} - 0 - \left(\frac{\cancelto{0}{\sin n\pi}}{n} - \frac{\sin\frac{n\pi}{2}}{n}\right) = \frac{2}{n}\sin\frac{n\pi}{2}$$

$$a_n = \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

F.Ř. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n\pi}\sin\frac{n\pi}{2}\cos(nx)$

(5) $f(x) = x^2 - 1 \quad [-1,1]$



$b_n = 0$

$$a_0 = \frac{1}{1}\int_{-1}^{1} x^2 - 1\,dx = 2\int_0^1 x^2 - 1\,dx = 2\left[\frac{x^3}{3} - x\right]_0^1 =$$

$$= 2\left(\frac{1}{3} - 1\right) = -\frac{4}{3}$$

$$a_n = 2\int_0^1 (x^2 - 1)\cos(n\pi x)\,dx = \ldots$$

$$\ldots = \frac{4}{n^2\pi^2}(-1)^n$$

F.Ř. $-\frac{2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}(-1)^n \cos(n\pi x)$

⑥ $f(x) = x$ na $(0, 2\pi)$ $\qquad 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

$$\pi a_0 = \int_0^{2\pi} x\,dx = \left[\frac{x^2}{2}\right]_0^{2\pi} = \frac{4\pi^2}{2}$$

$$a_0 = 2\pi$$

$$\pi a_n = \int_0^{2\pi} x \cos nx\,dx = \left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{2\pi} =$$

$$= \frac{2\pi \cancel{\sin(n 2\pi)}}{n} + \frac{\cos(n2\pi)}{n^2} - 0 - \frac{\cos 0}{n^2} = \frac{1}{n^2} - \frac{1}{n^2} = 0$$

$$\pi b_n = \int_0^{2\pi} x \sin nx\,dx = \left[\frac{\cancel{\sin nx}}{n^2} - \frac{x \cos nx}{n}\right]_0^{2\pi} = -\frac{2\pi \cos(n2\pi)}{n} =$$

$$= -\frac{2\pi}{n} \qquad b_n = -\frac{2}{n}$$

na $(0, 2\pi)$

$$x = \pi + \sum_{n=1}^{\infty} -\frac{2}{n} \sin(nx)$$

$\sin\frac{\pi}{2}$, $\sin\pi$, $\sin\frac{3\pi}{2}$, $\sin 2\pi$, $\sin\frac{5\pi}{2}$

$1, 0, -1, 0, 1, 0, -1, 0, 1 \dots$

$x = \frac{\pi}{2}$

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

$$1 + \frac{0}{2} - \frac{1}{3} + \frac{0}{4} + \frac{1}{5} + \frac{0}{6} - \frac{1}{7} + 0 \dots$$

$$\frac{\pi}{2} = \pi - 2 \underbrace{\sum_{n=1}^{\infty} \frac{\sin n\frac{\pi}{2}}{n}}_{S}$$

$$-2S = -\frac{\pi}{2}$$

$$S = \frac{\pi}{4} \qquad 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$