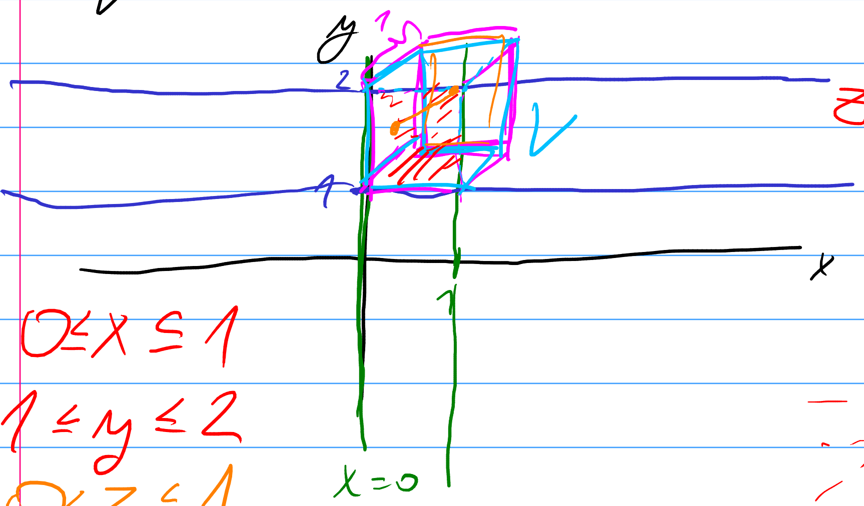
① $\iiint_V x+y \, dx\, dy\, dz = \quad V: \; x=0, \; y=1, \; \boxed{z=0}$

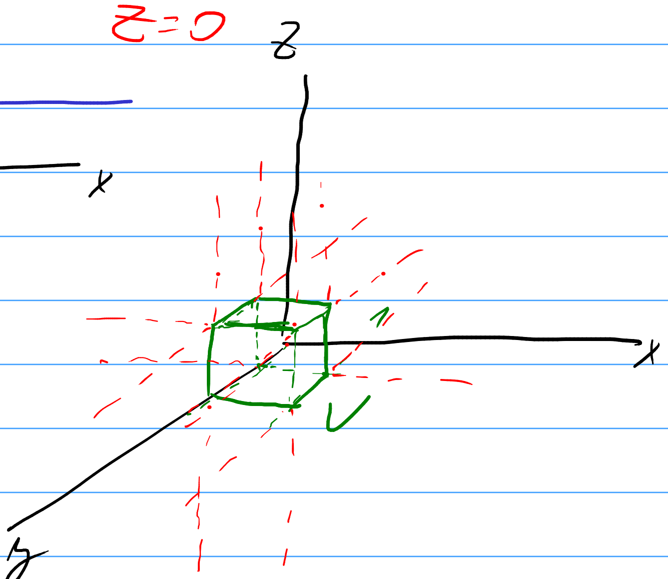
$\qquad\qquad\qquad\qquad\qquad\qquad\quad x=1, \; y=2, \; \boxed{z=1}$



$0 \le x \le 1$

$1 \le y \le 2$

$0 \le z \le 1$



$$= \int_0^1 \int_1^2 \int_0^1 x+y \, dz\, dy\, dx = \int_0^1 \int_1^2 (x+y)\cdot 1 \, dy\, dx =$$

$$= \int_0^1 \left[xy + \frac{y^2}{2} \right]_1^2 dx = \int_0^1 2x + 2 - x - \frac{1}{2} \, dx =$$

$$= \int_0^1 x + \frac{3}{2} \, dx = \left[\frac{x^2}{2} \right]_0^1 + \frac{3}{2}\cdot 1 = \frac{1}{2} - 0 + \frac{3}{2} = \underline{\underline{2}}$$

② $I=\iiint\limits_V (x+y)z\,dx\,dy\,dz$ V: $x\ge 0,\ y\ge 0,\ z\ge 0$

$\boxed{x+y+z=1}$

I. Kv.

$x+y+z=1$

$\boxed{z=1-x-y}$

$z=0:\quad x+y=1$

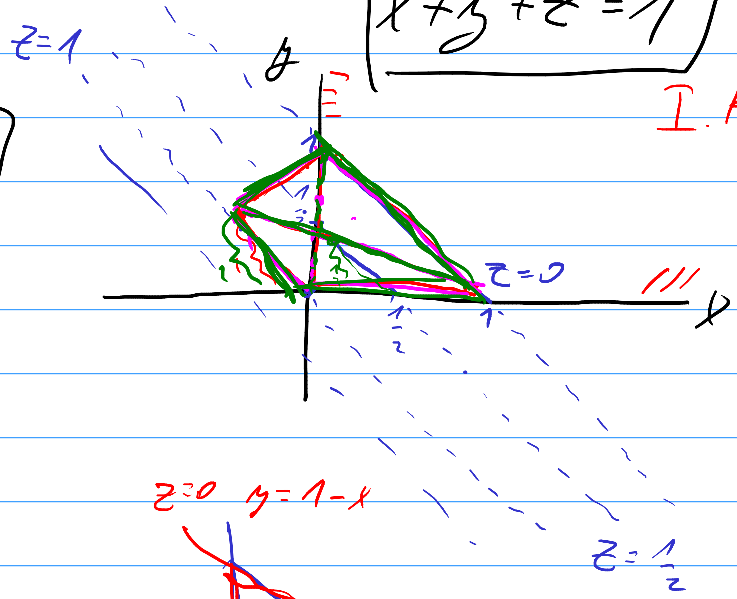
$\qquad y=1-x$

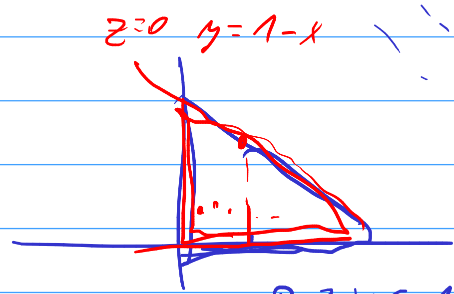
$z=\frac{1}{2}:\quad x+y=\frac{1}{2}$

$\qquad y=\frac{1}{2}-x$

$z=1:\quad x+y=0$

$\qquad y=-x$





$0\le x\le 1$

$0\le y\le 1-x$

$0\le z\le 1-x-y$

$$I=\int_0^1\int_0^{1-x}\int_0^{1-x-y}(x+y)\cdot z\,dz\,dy\,dx=\int_0^1\int_0^{1-x}(x+y)\,\frac{(1-x-y)^2}{2}\,dy\,dx=$$

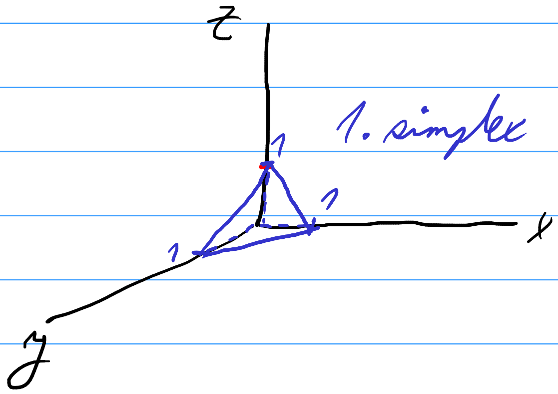
$$(1-x-y)^2=(x+y-1)^2=(x+y)^2-2(x+y)+1$$

$$\int (x+y)^n dy=\begin{vmatrix}u=x+y\\ du=dy\end{vmatrix}=\int u^n du=\frac{u^{n+1}}{n+1}=\frac{(x+y)^{n+1}}{n+1}$$

$$=\frac{1}{2}\int_0^1\int_0^{1-x}(x+y)^3-2(x+y)^2+(x+y)\,dy\,dx=\frac{1}{2}\int_0^1\left[\frac{(x+y)^4}{4}-2\frac{(x+y)^3}{3}+\frac{(x+y)^2}{2}\right]_0^{1-x}dx=$$

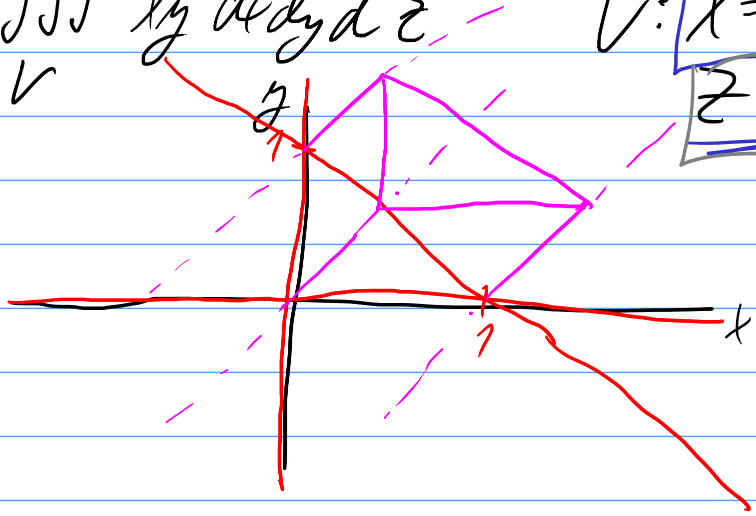
$$=\frac{1}{2}\int_0^1\underbrace{\frac{1}{4}-\frac{2}{3}+\frac{1}{2}}-\frac{x^4}{4}+\frac{2x^3}{3}-\frac{x^2}{2}\,dx=\frac{1}{2}\cdot\frac{3-8+6}{12}+\frac{1}{2}\left[-\frac{x^5}{20}+\frac{2}{3}\frac{x^4}{4}-\frac{x^3}{6}\right]_0^1=$$

$$= \frac{1}{24} + \frac{1}{2}\left[-\frac{1}{20} + \cancel{\frac{1}{6} - \frac{1}{6}}\right] = \frac{1}{8}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{1}{8}\,\frac{5-3}{15} = \frac{1}{4}\cdot\frac{1}{15} = \underline{\underline{\frac{1}{60}}}$$

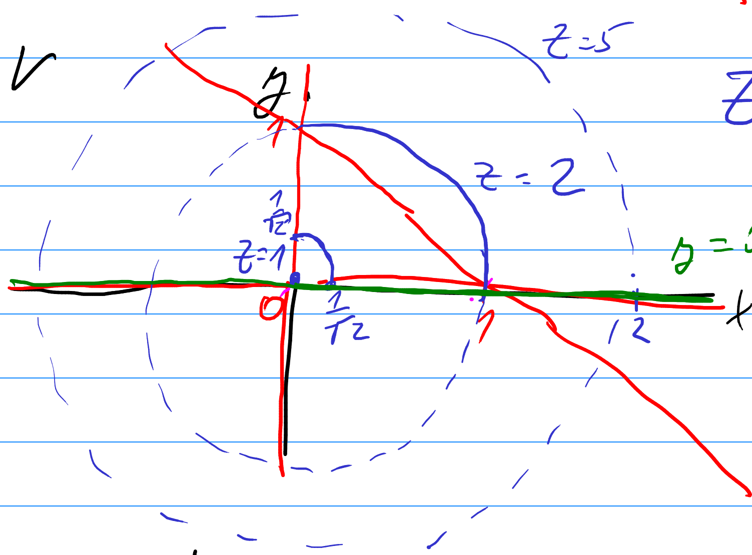


③ $\iiint_V xy\,dx\,dy\,dz$ $\quad V: \boxed{x=0,\ y=0,\ x+y=1}$, $\boxed{z=0}$, $\boxed{x^2+y^2+1=z}$

NEZÁVISÍ NA Z



V



poloměr r^2

$z-1 = x^2+y^2$

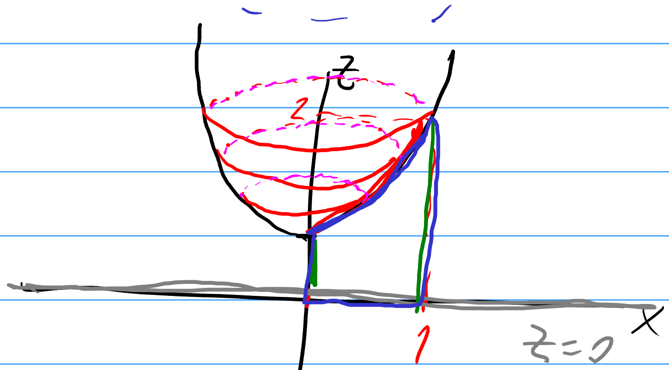
$z=0: \ 0 = x^2+y^2+1$ NEMÁ ŘEŠ.

$z=1$

$z=2$

$z=5$

$z=\frac{3}{2}$



$z = x^2+y^2+1$

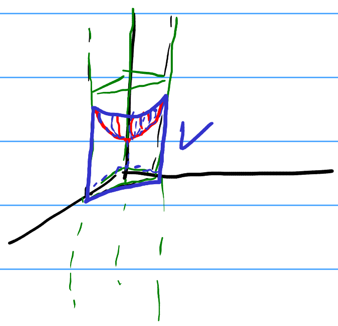
$y=0 \Downarrow$

$z = x^2+1$

$0 \le x \le 1$

$0 \le y \le 1-x$

$0 \le z \le x^2+y^2+1$



$$\int_0^1\int_0^{1-x}\int_0^{x^2+y^2+1} xy\,dz\,dy\,dx = \int_0^1\int_0^{1-x} xy(1+x^2+y^2)\,dy\,dx =$$

$$= \int_0^1 \left[x\frac{y^2}{2} + x^3\frac{y^2}{2} + x\frac{y^4}{4} \right]_0^{1-x} dx = \int_0^1 \underline{\frac{x(1-x)^2}{2}} + x^3\frac{(1-x)^2}{2} + \underline{x\frac{(1-x)^4}{4}}\, dx$$

$1-2x+x^2$

$t = 1-x$

$dt = -dx$

$$= +\int_0^1 \frac{(1-t)t^2}{2} + \frac{(1-t)t^4}{4}\, dt + \frac{1}{2}\int_0^1 x^3 - 2x^4 + x^5\, dx =$$

$$= \frac{1}{2}\left[\frac{t^3}{3} - \frac{t^4}{4}\right]_0^1 + \frac{1}{4}\left[\frac{t^5}{5} - \frac{t^6}{6}\right]_0^1 + \frac{1}{2}\left[\frac{x^4}{4} - 2\frac{x^5}{5} + \frac{x^6}{6}\right]_0^1 =$$

$$= \frac{1}{2}\left(\frac{4-3}{12}\right) + \frac{1}{4}\,\frac{6-5}{30} + \frac{1}{2}\left(\frac{5-8}{20} + \frac{1}{6}\right) =$$

$$= \frac{1}{24} + \frac{1}{4}\cdot\frac{1}{30} + \frac{1}{2}\left(\frac{-9+10}{60}\right) = \frac{1}{4}\left(\underbrace{\frac{5+1}{30}}_{\frac{1}{5}}\right) + \frac{1}{2}\cdot\frac{1}{60} =$$

$$= \frac{1}{4}\left(\frac{1}{5} + \frac{1}{30}\right) = \frac{1}{4}\,\frac{7}{30} = \underline{\underline{\frac{7}{120}}}$$

$z^2 = y^2 - x^2$

$\Leftrightarrow z = \pm\sqrt{y^2 - x^2}$

④ $\iiint\limits_V y \, dx\, dy\, dz \qquad V: y = 1,\ y = \sqrt{x^2 + z^2}$

$z = 0$

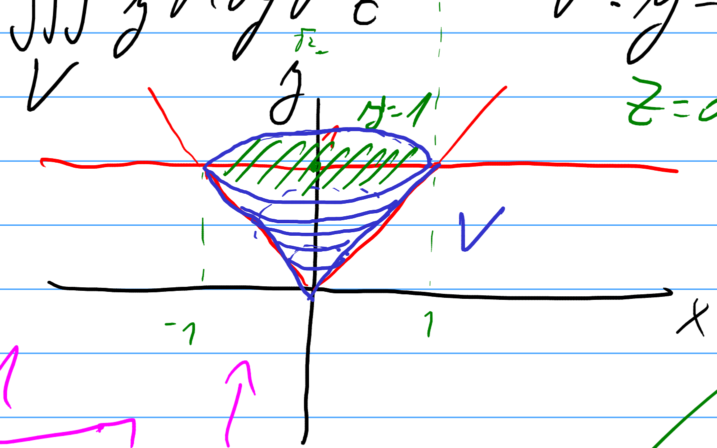
$z = 0: y = \sqrt{x^2} = |x|$

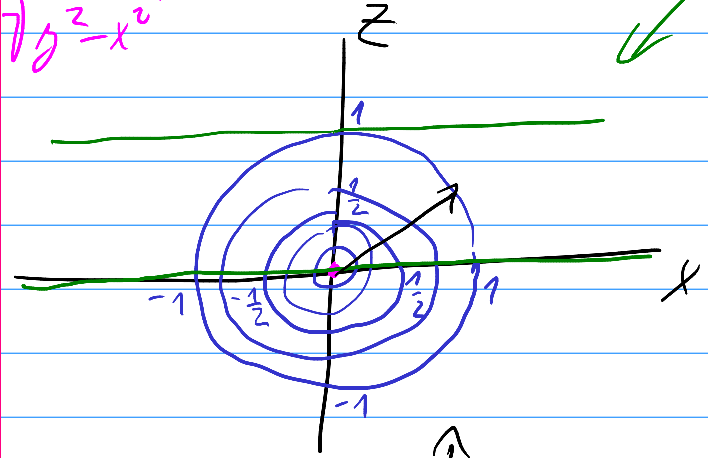
$z = 1: y = \sqrt{x^2+1}$

$-1 \le x \le 1$

$|x| \le y \le 1$

$-\sqrt{y^2-x^2} \le z \le \sqrt{y^2 - x^2}$





$y = \sqrt{x^2 + z^2}$

$y = 0: \quad 0 = \sqrt{x^2 + z^2}$

$\qquad\qquad 0 = x^2 + z^2$

$y = 1 \quad 1 = x^2 + y^2$

$y = 2 \quad z^2 = x^2 + y^2$

$x^2 + z^2 = 1$

1) $-1 \le x \le 1$

2) $-\sqrt{1-x^2} \le z \le \sqrt{1-x^2}$

3) $\sqrt{x^2+z^2} \le y \le 1$

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+z^2}}^{1} y \, dy\, dz\, dx = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left[\frac{y^2}{2}\right]_{\sqrt{x^2+z^2}}^{1} dz\, dx$$

$$= \frac{1}{2} \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1 - x^2 - z^2 \, dz\, dx =$$

$$= \frac{1}{2}\int_{-1}^{1} 2(1-x^2)\sqrt{1-x^2} - 2\frac{\sqrt{(1-x^2)^3}}{3}\,dx =$$

$$= \frac{2}{3}\int_{-1}^{1}(1-x^2)^{\frac{3}{2}}dx = \left| \begin{array}{l} x = \sin t \\ dx = \cos t\,dt \\ 1 \to \frac{\pi}{2} \\ -1 \to -\frac{\pi}{2} \end{array} \right| =$$

$\cos^2 t + \sin^2 t = 1$

$$= \frac{2}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\,(\underbrace{1-\sin^2 t}_{\cos^2 t})^{\frac{3}{2}}dt = \frac{2}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t \cos^3 t\,dt =$$

$$= \frac{2}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos^2 t)^2 dt = \frac{2}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\cos 2t}{2}\right)^2 = \frac{2}{12}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1+2\cos 2t+\cos^2 2t\,dt$$

$$\cos^2 t = \frac{1+\cos 2t}{2}$$

$$\cos 2t = \cos^2 t - \sin^2 t = 2\cos^2 t - 1$$

$$\cos^2 t = \frac{\cos 2t + 1}{2}$$

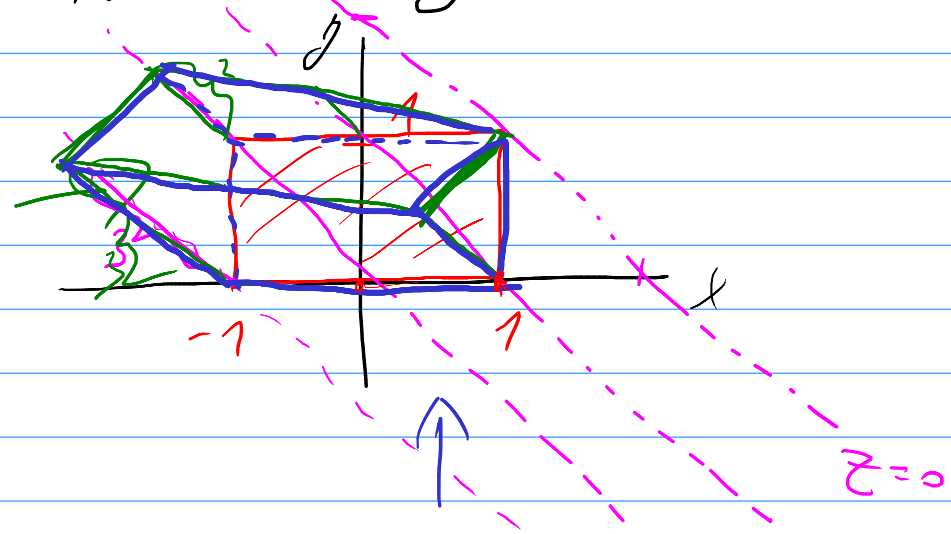
$$= \frac{1}{6}\left[t + 2\frac{\sin 2t}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{1}{6}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 4t}{2}\,dt$$

$$= \frac{\pi}{6} + \frac{1}{12}\pi + \frac{1}{12}\left[\frac{\sin 4t}{4}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{3}{12}\pi = \frac{1}{4}\pi$$

5) ZAMĚŇTE POŘADÍ INT. PROM

$$\int_{-1}^{1}\int_{0}^{1}\int_{0}^{2-x-y} B(x,y,z)\,dz\,dy\,dx = \int_{0}^{1}\int_{-1}^{1}\int_{0}^{2-x-y} B(x,y,z)\,dz\,dy\,dx$$

$$-1 \le x \le 1$$
$$0 \le y \le 1$$
$$0 \le z \le 2-x-y$$



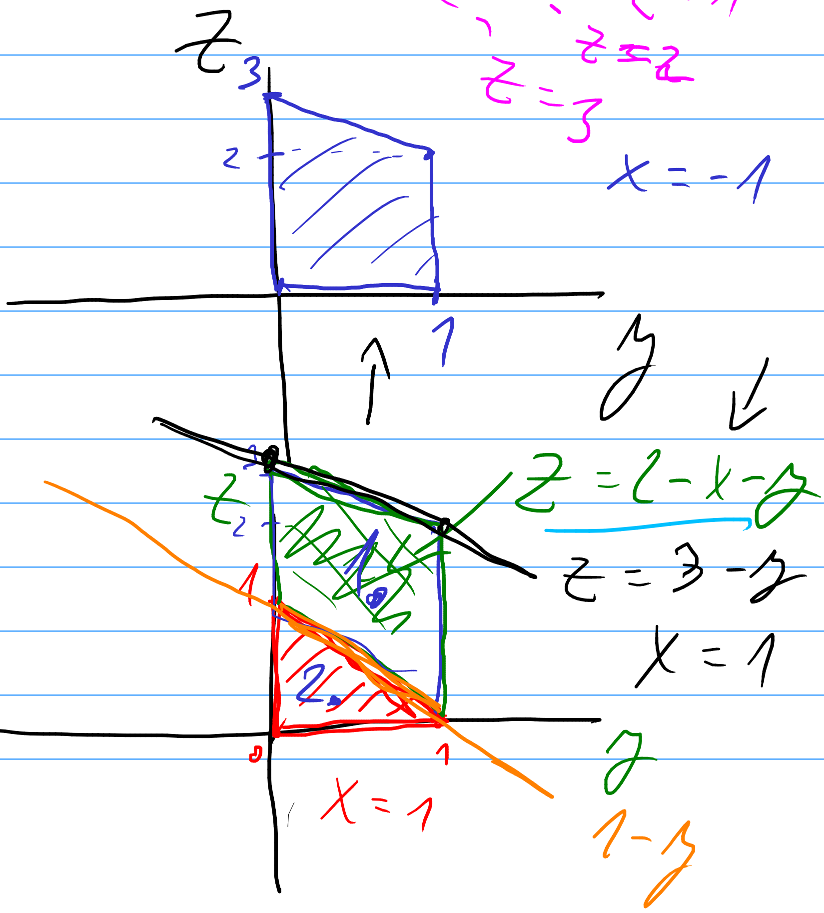
$z = 2 - x - y$

$z = 0 \rightarrow y = 2 - x$

$z = 1 \rightsquigarrow y = 1 - x$

$z = 2 \quad y = -x$

$z = 3 \quad y = -1 - x$



$0 \le y \le 1$

$\le z \le$

$-1 \le x \le$

1) $0 \le y \le 1$

$1-y \le z \le 3-y$

$-1 \le x \le 2-z-y$

2) $0 \le y \le 1$

$0 \le z \le 1-y$

$-1 \le x \le 1$

$$I = \int_0^1 \int_{1-y}^{3-y} \int_{-1}^{2-z-y} B \, dx \, dz \, dy \; +$$

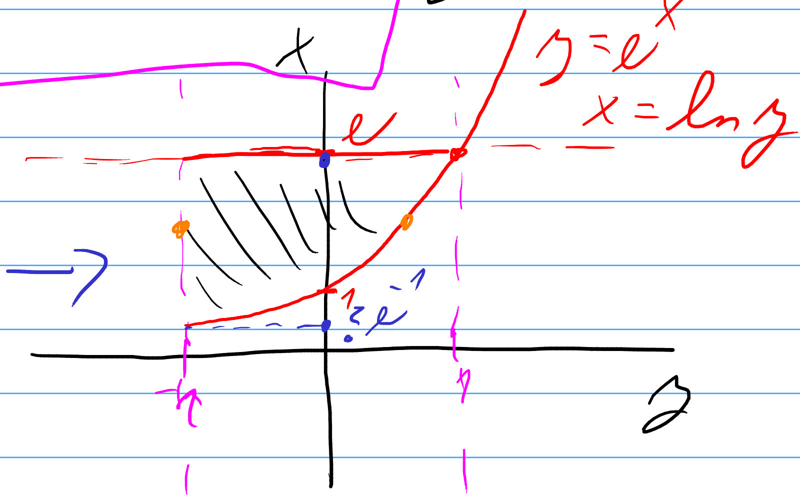
$$+ \int_0^1 \int_0^{1-y} \int_{-1}^{1} B \, dx \, dz \, dy$$

(6) $$\int_{-1}^{1}\int_{e^{y}}^{e}\int_{0}^{3-y} G(x,y,z)\,dz\,dx\,dy$$

$-1 \le y \le 1$

$e^{y} \le x \le e$

$0 \le z \le 3-y$



$e^{-1} \le x \le e$

$-1 \le y \le \ln y$

$$I = \int_{e^{-1}}^{e}\int_{-1}^{\ln y}\int_{0}^{3-y} G\,dz\,dy\,dx$$

$$I = \int_{-1}^{1}\int_{0}^{3-y}\int_{e^{y}}^{e} G\,dx\,dz\,dy$$