Crystalline State

Basic Structural Chemistry

Structure Types

Degree of Crystallinity



Amorphous – no order, random



Degree of Crystallinity

A crystalline solid: HRTEM image of strontium titanate $SrTiO_3$ Brighter atoms are Sr and darker are Ti

A TEM image of amorphous interlayer at the Ti/(001)Si interface in an as-deposited sample



Single-Crystal X-ray Diffraction

Structure Analysis



a four circle X-ray diffractometer

Single-Crystal X-ray Diffraction Structure Analysis



Crystals

- Crystal = a periodic arrangement of structural motifs = building blocks
- Building block is called a basis: an atom, a molecule, or a group of atoms or molecules
- A periodic arrangement must have translational symmetry such that if you move a building block by a distance:

$$\overline{T} = n_1 \overline{a} + n_2 \overline{b} + n_3 \overline{c}$$

where n_1, n_2 , and n_3 are integers, and $\overline{a}, \overline{b}, \overline{c}$ are vectors.

then it falls on another identical building block with the same orientation

 If we remove the building blocks and replace them with points, then we have a point lattice or Bravais lattice

Crystal Structure

Crystal Structure = Lattice + Motifs



(POINT) LATTICE

the geometrical pattern repeating periodically in space (2D or 3D) formed by points that have identical environment representing the locations of basis or motifs

MOTIF (BASIS)

the repeating unit of a pattern (an atom, a group of atoms, a molecule etc.) inside the unit cell

UNIT CELL

the smallest repetitive volume of the crystal, which when stacked together with replication ⁸ reproduces the whole crystal

Unit Cell

An "imaginary" parallel sided region (parallelepiped) of a structure from which the entire crystal can be constructed by purely translational displacements

Contains one unit of the translationally repeating pattern

Content of a unit cell represents its chemical composition

The unit cells that are commonly formed by joining neighbouring lattice points by straight lines, are called primitive unit cells



Crystal = Periodic Arrays of Atoms





Primitive Cell:

- Smallest building block for the crystal lattice
- Repetition of the primitive cell gives a crystal lattice

Five Planar Lattices



Name	Nu Brava	nber of is lattices	Conditions
Square	а	1	$a_1 = a_2$, $\alpha = 90^{\circ}$
Rectangular	b,c	2	$a_1 \neq a_2$, $\alpha = 90^\circ$
Hexagonal	d	1	$a_1 = a_2$, $\alpha = 120^{\circ}$
Oblique	e	1	a ₁ ≠a ₂ ,α≠120°,α≠90°

graphene

Lattice points of spherical symmetry 11

Ten Planar Point Groups

Symmetry

preservation of form and configuration across a point, a line, or a plane symmetry elements

Symmetry Element a geometric entity (line, point, plane) about which a symmetry operation takes place

Symmetry Operation a permutation of atoms such that an object (molecule or crystal) is transformed into a state indistinguishable from the starting state



The Ten Planar Point Groups

Point Group the collection of symmetry elements of an isolated shape, does not consider translation

Lattice points occupied by motifs of nonspherical symmetry

17 Plane Space Groups - Wallpaper

A space group = a complete set of all symmetry elements and translations



Seven Crystal Systems in 3D



Fourteen Bravais Lattices in 3D





Conventional Unit Cell ≠ Primitive Cell

3D Lattices and Space Groups



Primitive Cell

- The smallest cell that can be translated throughout space to completely recreate the entire lattice
- Volume of space translated through all the vectors in a lattice that just fills all of space without overlapping or leaving voids
- Contains just one Bravais lattice point (Z = 1)
- There is not one unique primitive cell, many (∞) possible shapes
- The primitive cell for the simple cubic lattice is equal to the simple cubic unit cell (they are identical)



Body-Centered Cubic (I)

Unit Cell



Primitive Cell

A primitive cell of the lattice may be constructed in 2 ways:

The primitive cell may have the lattice point confined at its CENTER = the WIGNER-SEITZ cell

The primitive cell may be formed by constructing lines BETWEEN lattice points, the lattice points lie at the VERTICES of the cell



Nonprimitive Unit Cell vs. Primitive Cell



The primitive cell is smaller or equal in size to the unit cell The unit cells possesses the highest symmetry present in the **lattice (for example Cubic)** 20

Cubic (F)

Unit Cell

Index System for Points

- 1) Coordinates within a unit cell
- 2) Express the coordinates u, v, w as fractions of unit cell vectors (lattice parameters) a, b, and c: (h, k, l)
- 3) Do not clear fractions
- 4) Entire lattice can be referenced by one unit cell



Index System for Directions (Miller Indices)



A = [100] B = [111] $C = [1^{-}2^{-}2]$ 1) Determine coordinates of two points in direction of interest (simplified – origin):

u1 v1 w1 and u2 v2 w2

2) Subtract coordinates of the second point from those of the first point:

u' = u1 - u2, v' = v1 - v2, w' = w1 - w2

3) Clear fractions from the differences to give indices in lowest integer values

4) Write indices in [] brackets - [uvw]

5) Negative = a bar over the integer

Index System for Directions (Miller Indices)



In the cubic system directions having the same indices regardless of order or sign are equivalent

For cubic crystals, the directions are all equivalent by symmetry:

 $[1 \ 0 \ 0], [1^{-} \ 0 \ 0], [0 \ 1 \ 0], [0 \ 1^{-} \ 0], [0 \ 0 \ 1], [0 \ 0 \ 1^{-}]$

Families of crystallographic directions e.g. <1 0 0>

Angled brackets denote a family of crystallographic directions

- 1. If the plane passes through the origin, select an equivalent plane or move the origin
- Find the intercepts on the axes in terms of the lattice constants a, b,
 The axes may be those of a primitive or nonprimitive unit cell
- 3. Take the reciprocals of these numbers and then reduce to three integers having the same ratio, usually the smallest three integers
- 4. $(1/\infty = 0)$
- 5. The result enclosed in parenthesis (*hkl*), is called the index of the plane



-

					f
Exa	ample	а	b	С	
1.	Intercepts	1	1	∞	9
2.	Reciprocals	1/1	1/1	1/∞	
	·	1	1	0	
3.	Reduction	1	1	0	
Λ	Millor Indicos	(110)			a D
4.	Willer Mulces	(110)			x
Example		а	b	C	Z t
1.	Intercepts	1/2	~	~	
2	Reciprocals	1/1/2	1/∞	1/∞	
		2	0	0	
3.	Reduction	2	Õ	0	
4.	Miller Indices	(100)			a y b
					X

Cubic system - planes having the same indices regardless of order or sign are equivalent - braces {hkl}

(111), (11⁻1), (111⁻) belong to {111} family (100)(001)(100), (1⁻00), (010), and (001) belong to {100} family



(010)

(110)

(101)





The Miller indices (hkl) is the same vector as the normal to the plane [hkl]







Atomic planes influence

- Optical properties
- Reactivity
- Surface tension 28
- Dislocations

Quasiperiodic Crystals

Quasiperiodic crystal = a structure that is ordered but not periodic continuously fills all available space, but it lacks translational symmetry



Only 2, 3, 4, 6fold symmetry allowed to fill 2D plane completely

Penrose - a plane filled in a nonperiodic fashion using two different types of tiles



Crystals and Crystal Bonding

- metallic (Cu, Fe, Au, Ba, alloys) metallic bonding, electron delocalization
- ionic (NaCl, CsCl, CaF₂, ...)

ionic bonds, cations and anions, electrostatic interactions, ions pack into extremely regular crystalline structures, in an arrangement that minimizes the lattice energy (maximizing attractions and minimizing repulsions). The lattice energy is the summation of the interaction of all sites with all other sites.

- covalent network solid (diamond, graphite, SiO₂, AIN,...) atoms, covalent bonding, a chemical compound (or element) in which the atoms are bonded by covalent bonds in a continuous network extending throughout the material, there are no individual molecules, the entire crystal or amorphous solid may be considered a macromolecule
- molecular (Ar, C₆₀, HF, H₂O, organics, proteins)
 molecules, van der Waals and hydrogen bonding



Covalent Network Solids



Three Cubic Cells



SC or Primitive (P) BCC (I)



Coordination number

Z = number of lattice points per unit cell

Cube



a = edge

d = face diagonl ($d^2 = a^2 + a^2 = 2a^2$)

$$d = \sqrt{2} \cdot a$$

$$D = \sqrt{3} \cdot a$$

Simple Cubic SC = Polonium



Space filling 52%

BCC = W, Tungsten




FCC = Copper, Cu = CCP







Space filling 74% CN 12

Close Packing in Plane 2D



(a) An "open" packing

(b) Close packing



Close Packing in Space 3D





Mg, Be, Zn, Ni, Li, Os, Sc, Ti, Co, Y, Ru, solid He



Cu, γ -Fe (austenite), Ca, Sr, Ag, Au, Ni, Rh, solid Ne-Xe, F₂, C₆₀, opal (300 nm)

CCP = FCC



Close packed layers of CCP are oriented perpendicularly to the body diagonal of the cubic cell of FCC

Structures with Larger Motifs



 C_{60} - FCC = CCP



Structures with Larger Motifs



TEM images of superlattices composed of 11.3 nm Ni nanoparticles

Structures with Larger Motifs



Coordination Polyhedrons

Which is HCP and which is CCP?



Space Filling

a = lattice parameter	Atom Radius, r	Number of Atoms (lattice points), Z	Space filling
SC	a/2	1	52%
BCC	√3a/4	2	68%
FCC	√2a/4	4	74%
Diamond	√3a/8	8	34%

Periodic Table of Metal Structures



Two Types of Voids (Holes) in Close-Packed Structures (CCP and HCP)









Tetrahedral Holes T+

Octahedral Holes

Tetrahedral Holes T-

N cp atoms in a lattice cell

N Octahedral Holes 2N Tetrahedral Holes



Two Types of Voids (Holes)

Octahedral Holes (O)



Z = 4

number of atoms in the CCP cell (N)

O = 4 number of octahedral holes (N) **Tetrahedral Holes (T)**



Z = 4

number of atoms in the CCP cell (N)

T = 8 number of tetrahedral holes (2N) 53

Two Types of Voids (Holes)

HCP

N hcp atoms in a lattice cell

N Octahedral Holes (OC)

2N Tetrahedral Holes (TE)



Characteristic Structures of Solids = Structure Types

Rock salt NaCI: LiCI, KBr, AgCI, MgO, TiO, FeO, SnAs, UC, TiN, ...

Fluorite CaF₂: BaCl₂, K₂O, PbO₂ ...

Lithium bismutide Li₃Bi: Fe₃AI, M₃C₆₀

ICSD 3555 NaCl 3438 MgAl2O4 2628 GdFeO3

Sphalerite (zinc blende) ZnS: CuCl, HgS, GaAs ...

Nickel arsenide NiAs: FeS, PtSn, CoS ...

Wurtzite ZnS: ZnO, MnS, SiC

Structure Types Derived from CCP = FCC



Fluorite structure = a face-centered cubic array (FCC) of cations = cubic close packing (CCP) of cations with all tetrahedral holes filled by anions = a simple cubic (SC) array of anions

Antifluorite structure = a face-centred cubic (FCC) array of anions = cubic close packing (CCP) of anions, with cations in all of the tetrahedral holes (the reverse of the fluorite structure)







Oxides: Na₂O, K₂O, UO₂, ZrO₂, ThO₂

alkali metal sulfides, selenides and tellurides

 $K_2[PtCI_6]$, $(NH_4)_2[PtCI_6]$, $Cs_2[SiF_6]$, $[Fe(NH_3)_6][TaF_6]_2$

CaF₂, SrF₂, SrCl₂, BaF₂, BaCl₂, CdF₂, HgF₂, EuF₂, β -PbF₂, PbO₂ Li₂O, Li₂S, Li₂Se, Li₂Te, Na₂O, Na₂S, Na₂Se, Na₂Te, K₂O, K₂S ₅₉



Fluorite A-cell

Plan view

FCa₄ Tetrahedra



Pyrochlores = Disordered Fluorite

$CaF_2 (Ca_4F_8)$ Pyrochlore $A_2B_2O_7$



(Na,Ca)₂Nb₂O₆(OH,F) Y₂Ti₂O₇, La₂Zr₂O₇

Sphalerite (zincblende, ZnS)







Cubic close packing of anions with 1/2 tetrahedral holes filled by cations

Sphalerite (zincblende, ZnS)



13-15 compounds: BP, BAs, AIP, AIAs, GaAs, GaP, GaSb, AISb, InP, InAs, InSb

12-16 compounds: BeS, BeSe, BeTe, b-MnS (red), β -MnSe, β -CdS, CdSe, CdTe, HgS, HgSe, HgTe, ZnSe, ZnTe

Halogenides: Agl, CuF, CuCl, CuBr, Cul, NH₄F

Borides: PB, AsB Carbides: β-SiC Nitrides: BN

Cubic Diamond







Ice-hexagonal ⁶⁵ Replace C-C with O-H····O

Diamond Structure

Elements of Group 14: C, Si, Ge, grey-Sn



- Add 4 atoms to FCC/CCP to 1/2 of tetrahedral holes
- Tetrahedral bond arrangement of all atoms
- Each atom has 4 nearest neighbors and 12 next nearest neighbors

Cuprite Cu₂O - Cubic Diamond Lattices





Two interpenetrating diamond lattices

Replace C-C with O-Cu-O

Wurzite, ZnS



O = Zn O = S

Hexagonal close packing of anions with 1/2 tetrahedral holes filled by cations

> ZnO, ZnS, ZnSe, ZnTe, BeO, CdS, CdSe, MnS, Agl, AIN, GaN Lonsdaleite

Zincite, ZnO



Semiconductors of 13-15 and 12-16 type



Rock Salt, NaCl



Rock Salt, NaCl



Anion and cation sublattices = same FCC/CCP

Rock salt (NaCl) = Anti-rock salt (CINa)
Rock Salt Structures (NaCI)



Hydrides: LiH, NaH, KH, NH₄BH₄ – H₂ storage material Pd(H)

Borides: ZrB, HfB

Carbides: TiC, ZrC, VC, UC

Nitrides: ScN, TiN, UN, CrN, VN, ZrN

Oxides: MgO, CaO, SrO, BaO, TiO, VO, MnO, FeO, CoO, NiO Chalcogenides: MgS, CaS, SrS, BaS, a-MnS, MgSe, CaSe, SrSe, BaSe, CaTe



Halides: LiF, LiCl, LiBr, Lil, NaF, NaBr, Nal, KF, KCl, KBr, Kl, RbF, RbCl, RbBr, Rbl, CsF, AgCl, AgF, AgBr

Intermetallics: SnAs Other FeS₂ (pyrite), CaC₂, NaO₂

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Rock Salt Structures (NaCI)

Palladium-Hydrogen system





fcc structure

The spaces occupied by hydrogen are the interstitial octahedral (O) sites of palladium.



PdH(D)x (x:hydrogen concentration H(D)/Pd)

Rock Salt Structures (NaCI)

FeS₂ (pyrite), CaC₂, NaO₂







SiO₂ pyrite - high pressure polymorph, in Uranus and Neptune core

Nickel Arsenide, NiAs



Hexagonal close packing of anions (As) with all octahedral holes filled by cations (Ni)

NiS, NiAs, NiSb, NiSe, NiSn, NiTe, FeS, FeSe, FeTe, FeSb, PtSn, CoS, CoSe, CoTe, CoSb, CrSe, CrTe, CoSb, PtB (anti-NiAs structure)



ReB₂ - type



Hexagonal close packing of anions with all tetrahedral holes filled by cations

Li₃Bi (anti BiF₃)



 Fe_3AI [Cr(NH₃)₆]Cl₃ K₃[Fe(CN)₆] M₃C₆₀



Cubic close packing of anions with all tetrahedral and octahedral holes filled by cations





Primitive cubic packing of anions with all cubic holes filled by cations



CsCl

Primitive cubic packing of CsCl₈ cubes sharing all faces



CsBr, CsI, CsCN, NH₄CI, NH₄Br, TICI, TIBr, TII, CuZn, CuPd, LiHg

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Both sublattices form independent diamond structures. The atoms sit on the sites of a bcc lattice with $a_{bcc} = \frac{1}{2} a$

Niggli – 230 space groups – restrictions on arrangement of atoms: There are only 4 possible AB cubic structures: NaCI, ZnS-sphalerite, CsCI, and NaTI







What type of unit cell? sc, bcc, fcc

SC of ReO₆ octahedra

NaCl structure with 3/4 of cations removed and 1/4 of anions removed

Cubic-WO₃, UO₃, MoF₃, NbF₃, TaF₃, AIF₃, Cu₃N

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Perovskite Structure CaTiO₃



1839 G. Rose named mineral after

C. A. Perovski

TiO₆ – octahedra

CaO₁₂ – cuboctahedra

(Ca²⁺ and O^{2–} form a cubic close packing)

preferred structure of piezoelectric, ferroelectric and superconducting materials

Goldschmidt's tolerance factor

Perovskite, CaTiO₃

Two equvivalent views of the unit cell of perovskite



Perovskite, **ABX**₃



MgSiO₃, CaSiO₃

KNbO₃, KTaO₃, KIO₃, NaNbO₃, NaWO₃, LaCoO₃, LaCrO₃, LaFeO₃, LaGaO₃, LaVO₃, SrTiO₃, SrZrO₃, SrFeO₃

ThTaN₃, BaTaO₂N

Goldschmidt's tolerance factor

$$t = \frac{r_{A-X}}{\sqrt{2} r_{B-X}} = \frac{r_{A} + r_{X}}{\sqrt{2} \left(r_{B} + r_{X}\right)}$$

0.8 < t < 0.9 orthorhombic/monoclinic 0.9 < t < 0.97 cubic 0.97 < t < 1.02 tetragonal

Perovskite - Ferroelectric BaTiO₃



Perovskite, BaTiO₃



Perovskite - Ferroelectric BaTiO₃



Perovskite Structure of YBCO



Perovskite Structure of CH₃NH₃Pbl₃



Three Polymorphs of TiO₂



anatase (a), rutile (b) and brookite (c)



Rutile, TiO₂





 $CN - stoichiometry Rule A_x B_y$

CN(A) / CN(B) = y / x

Distorted hexagonal close packing of anions with 1/2 octahedral holes filled by cations (giving a tetragonal lattice)

Rutile, TiO₂

Rutile Crystal Structure





 TiO_6 – octahedra OTi₃ – trigonal planar (alternative to CaF₂ for highly charged smaller cations)

GeO₂, CrO₂, IrO₂, MoO₂, WO₂, NbO₂, β-MnO₂, OsO₂, VO₂ (>340 K), RuO₂, CoF₂, FeF₂, MgF₂, MnF₂

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The Spinel Structure: MgAl₂O₄



FCC array of O²⁻ ions, A²⁺ occupies 1/8 of the tetrahedral and B³⁺ 1/2 of the octahedral holes

 \rightarrow normal spinel: AB₂O₄

 \rightarrow inverse spinel: B[AB]O₄

 $Fe_{3}O_{4} = Fe^{3+}[Fe^{2+}Fe^{3+}]O_{4}$

→ basis structure for several magnetic materials

Magnetite (Fe_3O_4) and Maghemite (γ - Fe_2O_3)

Cubic inverse spinel O²⁻ atoms are arranged in closepacked FCC lattice Fe²⁺ occupy ¹/₂ of OCT sites Fe³⁺ are split evenly across the remaining OCT and TET sites Fully oxidized form of magnetite Inverse spinel with cation deficiency

One of every six octahedral sites in magnetite is vacant Stoichiometry Fe^{tet}(Fe_{5/3}□_{1/3})^{oct}O₄





Spinels, AB₂X₄

AB₂X₄ Spinel normal: Cubic close packing of anions with 1/2 octahedral holes filled by B cations and 1/8 tetrahedral holes by A cations

MgAl₂O₄, CoAl₂O₄, MgTi₂O₄, Fe₂GeO₄, NiAl₂O₄, MnCr₂O₄

AB₂X₄ Spinel inverse: As for spinel but A cations and 1/2 of B cations interchanged

$$\label{eq:mgFe2O4} \begin{split} \mathsf{MgFe_2O_4, NiFe_2O_4, MgIn_2O_4, MgIn_2S_4, Mg_2TiO_4, Zn_2TiO_4, Zn_2SnO_4, } \\ \mathsf{FeCo_2O_4.} \end{split}$$

δ = the inversion parameter (A_δB_{1-δ})_A[A_{1-δ}B_{1+δ}]_BO₄

Values from $\delta = 1$ (normal) to $\delta = 0$ (inverse) May depend on synthesis conditions

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Corundum, Al₂O₃



Al₂O₃ lattice consists of HCP array of O²⁻ ions

Al³⁺ ions fill of all octahedral holes

The AI centres are surrounded by oxides

Oxides are coordinated by Al³⁺ ions

Garnets

Naturally occuring garnets $A_3B_2Si_3O_{12} = A_3B_2(SiO_4)_3$

A₃ = divalent cation (Mg, Fe, Mn or Ca) dodecahedral

B₂ = trivalent (AI, Fe³⁺, Ti, or Cr) octahedral

Si₃ = tetravalent, tetrahedral



Since Ca is much larger in radius than the other divalent cations, there are two series of garnets: one with calcium and one without:

pyralspite contain AI (pyrope, almandine, spessartine) ugrandite contain Ca (uvarovite, grossular, andradite)

Synthetic garnets A₃B₅O₁₂

 $\begin{array}{l} \mathsf{A}_3 = \text{trivalent cations, large size (Y, La,...)} \\ \mathsf{B}_5 = \text{trivalent (AI, Fe}^{3+}, \text{Ti, or Cr}) \ 2\mathsf{B} \ \mathsf{octahedral}, 3\mathsf{B} \ \mathsf{tetrahedral} \\ \mathsf{Y}_3\mathsf{AI}_5\mathsf{O}_{12} \\ \mathsf{Y}_3\mathsf{Fe}_5\mathsf{O}_{12} \end{array}$

Synthetic Garnets A₃B₅O₁₂

YAG Garnet Y₃Al₅O₁₂

Y₃ = red - dodecahedral trivalent cations, large size

Al₅ = blue 2 octahedral 3 tetrahedral

O₁₂



Fullerides

M₁C₆₀ all the octahedral (O) sites (dark blue) are occupied (NaCI)

M₂C₆₀ all the tetrahedral (T) sites (light blue) are occupied (CaF₂)

M₃C₆₀ both the O and the T sites are occupied (BiF₃)

M₄C₆₀ rearranged to a body-centered tetragonal (bct) cell and both the O and the T sites of the bct lattice are occupied

M₆C₆₀ a bcc lattice and all its T sites are occupied



Layered Structures

Cdl₂ Hexagonal close packing of anions with 1/2 octahedral holes filled by cations

Col₂, Fel₂, Mgl₂, Mnl₂, Pbl₂, Thl₂, Til₂, Tml₂, Vl₂, Ybl₂, Znl₂, VBr₂, TiBr₂, MnBr₂, FeBr₂, CoBr₂, TiCl₂, TiS₂, TaS₂ Mg(OH)₂ - brucite

CdCl₂ Cubic close packing of anions with 1/2 octahedral holes filled by cations

CdCl₂, CdBr₂, CoCl₂, FeCl₂, MgCl₂, MnCl₂, NiCl₂, Nil₂, ZnBr₂, Znl₂, Cs₂O* (anti-CdCl₂ structure)

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Cdl₂ Hexagonal Close Packing



HCP of anions with 1/2 octahedral holes filled by cations

Fully occupied and completely empty planes alternate



CdCl₂ Cubic Close Packing

CCP of anions with 1/2 octahedral holes filled by cations, fully occupied and completely empty planes alternate





Vocabulary of terms

Parallelepiped = rovnoběžnostěn