Přechod do rotačních souřadnic

Paraxiální rovnice v komplexních souřadnicích

$$w'' + \frac{\gamma\Phi'}{2\Phi^*} w' + \frac{\gamma\Phi''}{4\Phi^*} w - i\frac{\eta B}{\sqrt{\Phi^*}} w' - i\frac{\eta B'}{2\sqrt{\Phi^*}} w = 0$$

$$w(z) = e^{i\theta(z)} u(z)$$

$$w' = i\theta' e^{i\theta} u + e^{i\theta} u' = e^{i\theta}(i\theta' u + u')$$

$$w'' = i\theta' e^{i\theta}(i\theta' u + u') + e^{i\theta}(i\theta'' u + i\theta' u' + u'')$$

$$= e^{i\theta}(u'' + 2i\theta' u' + i\theta'' u - \theta'^2 u)$$

$$u'' + 2i\theta' u' + i\theta'' u - \theta'^2 u + \frac{\gamma\Phi'}{2\Phi^*}(u' + i\theta' u) + \frac{\gamma\Phi''}{4\Phi^*} u -$$

$$- i\frac{\eta B}{\sqrt{\Phi^*}}(u' + i\theta' u) - i\frac{\eta B'}{2\sqrt{\Phi^*}} u = 0$$

$$u'' + \left(\frac{\gamma\Phi'}{2\Phi^*} + 2i\theta' - i\frac{\eta B}{\sqrt{\Phi^*}}\right) u' + \left(\frac{\gamma\Phi''}{4\Phi^*} - \theta'^2 + i\theta'' + \frac{\eta B}{\sqrt{\Phi^*}}\theta' - i\frac{\eta B'}{2\sqrt{\Phi^*}}\right) u =$$

pokud zvolíme $\theta' = \frac{\eta B}{2\sqrt{\Phi^*}}$ dostaneme

$$u'' + \frac{\gamma\Phi'}{2\Phi^*} u' + \left(\frac{\gamma\Phi'}{4\Phi^*} + \frac{\eta^2 B^2}{4\Phi^*}\right) u = 0$$