Hvězdná noc, 1889

Struktura a kinematika galaxií

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IV. Poissonova rovnice
V. Rotační křivka diskových galaxií
VI. Logaritmický potenciál
VII. Pohyb hvězd kolmo na galaktickou rovinu

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DRÁHY HVĚZD V GALAXIÍCH

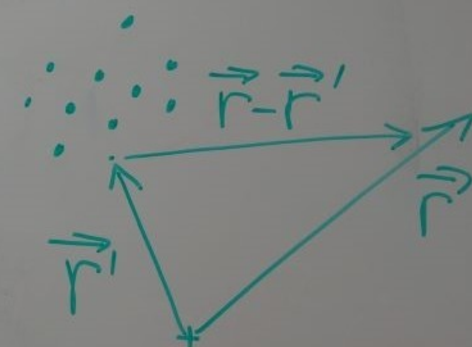
$$\nabla^2 \Phi = 4\pi G \rho \quad \text{(Poissonova rovnice)} \Rightarrow$$

ρ – hustota hmoty

Φ – gravitační potenciál

FORMÁLNÍ ŘEŠENÍ:

$$\Phi(\vec{r}) = -G \int \frac{\rho(\vec{r}\,')\, d^3\vec{r}\,'}{|\vec{r} - \vec{r}\,'|}$$



$$\vec{F} = -\nabla \Phi$$

$\vec{F}$ – gr. síla na jednotku hmoty (= intenzita gr. pole)

$$\ddot{\vec{r}} = -\nabla \Phi$$

(pohybová rovnice hvězdy v inerciální soustavě)

Spherical systems

Newton's first theorem

A body that is inside a spherical shell of matter experiences no net gravitational force from that shell.

Newton's second theorem

The gravitational force on a body that lies outside a spherical shell of matter is the same as it would be if all the shell's matter were concentrated into a point at its center.

Zdroj: Binney & Tremaine, Galactic Dynamics

Pro sférickou symetrii:

$$\Phi(r) = -4\pi G \left[\frac{1}{r}\int_0^r \rho(r')r'^2 dr' + \int_r^\infty \rho(r')r'dr'\right]$$

$$\mathbf{F}(r) = -\frac{d\Phi}{dr}\hat{\mathbf{e}}_r = -\frac{GM(r)}{r^2}\hat{\mathbf{e}}_r,$$

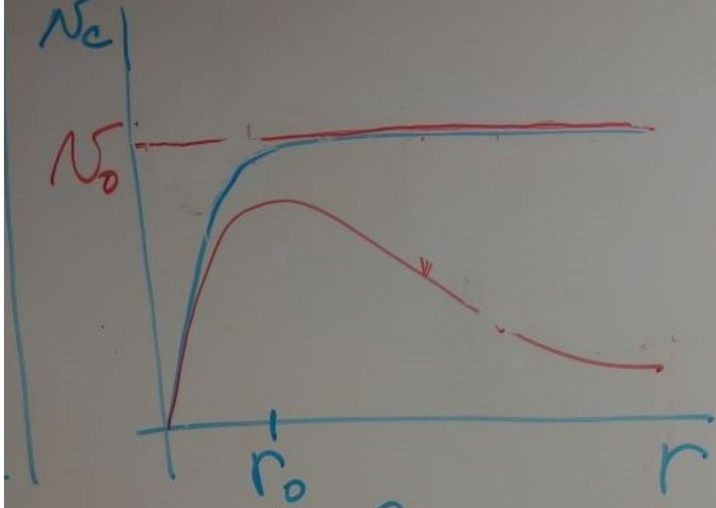
$$M(r) = 4\pi \int_0^r \rho(r')r'^2 dr'.$$

$$v_c^2 = r\frac{d\Phi}{dr} = r|\mathbf{F}| = \frac{GM(r)}{r}.$$

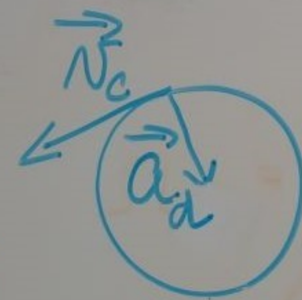
$$v_e(r) = \sqrt{2|\Phi(r)|}.$$

Zdroj: Binney & Tremaine, Galactic Dynamics

ROTAČNÍ KŘIVKA GALAXIÍ



v_c – KRUHOVÁ RYCHLOST



$a_d = \frac{v_c^2}{r}$, $a_d = |F_r|$

RADIÁLNÍ SLOŽKA GR. SÍLY

$$\Rightarrow v_c^2 = r \cdot |F_r| = r \cdot \left|\frac{\partial \Phi}{\partial r}\right| = r \cdot \frac{\partial \Phi}{\partial r}$$

$$\Phi(r) = \int \frac{v_c^2(r)}{r}\, dr$$

APROXIMACE: $v_c = v_0 = \text{Const.}$

$$\boxed{\Phi = v_0^2 \ln r} + C \Rightarrow F_r = -\frac{d\Phi}{dr} = -\frac{v_0^2}{r}$$

LOGARITMICKÝ POTENCIÁL

MODIFIKACE:

$$\Phi = \frac{v_0^2}{2} \ln(r^2 + r_0^2) \Rightarrow$$

DOMÁCÍ ÚKOL:

$F_r(r) = ?$, $v_c(r) = ?$, $\rho(r) = ?$

Rotation curves of spiral galaxies

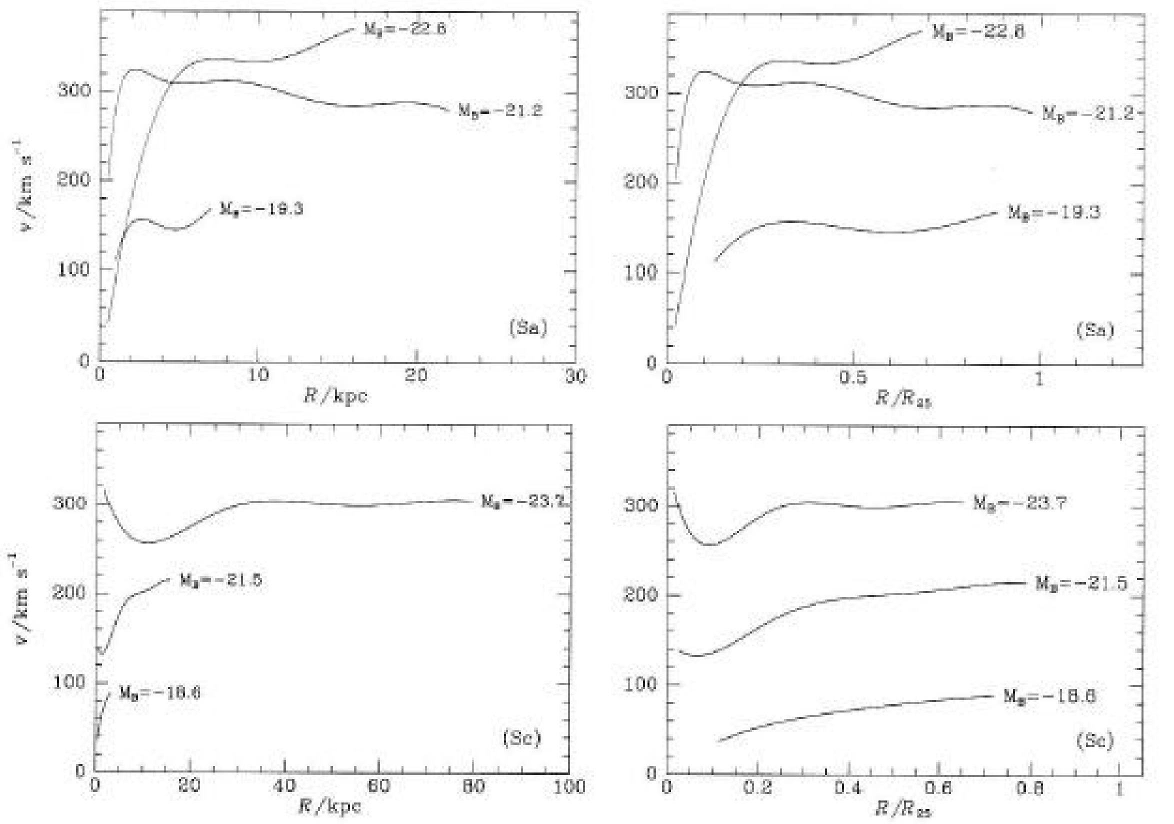


Figure 8.33 The upper panels show the rotation curves of three Sa galaxies of very different luminosities from the sample of Rubin *et al.* (1985) plotted both on the same linear scale (left) and rescaled by their optical radii, R_{25} (right). The lower panels show similar plots for three Sc galaxies from the sample of Burstein *et al.* (1982).

Zdroj: Binney & Tremaine, Galactic Dynamics

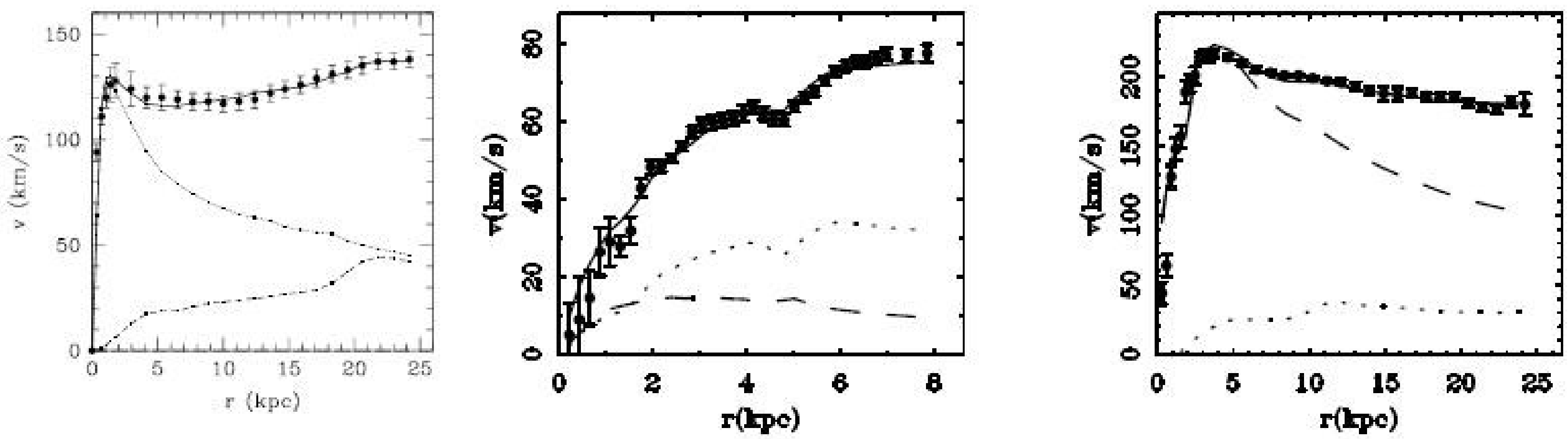


Fig. 2. The observed and MOND rotation curves (in solid lines) for NGC 3657 (left), NGC 1560 (cnter), and NGC 2903 (right). The first from Sanders (2006), the last two from Sanders and McGaugh (2002). Points are data, dashed and dotted lines for the last two galaxies are the Newtonian curves calculated for the stars and gas alone; the reverse for the first (they add in quadrature to give the full Newtonian curve).

Milgrom, 2009

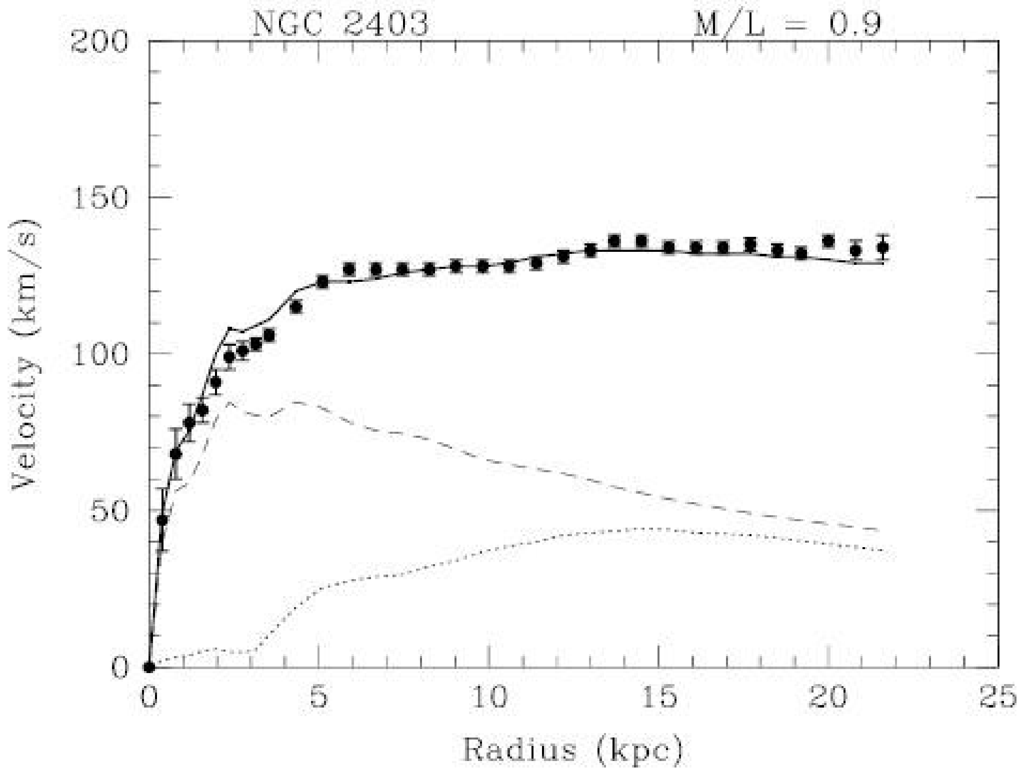


Figure 1: The points show the rotation curve of NGC 2403 as deduced from 21 cm line observations [6]. The dashed curve is the Newtonian rotation curve of the stellar component as deduced from the observed surface brightness distribution with M/L=0.9, and the dotted curve is the Newtonian rotation curve deduced from the observed HI surface density distribution. The solid curve is that calculated from Milgrom's formula. Here $a_0 = 10^{-8}$ cms^{-2}.

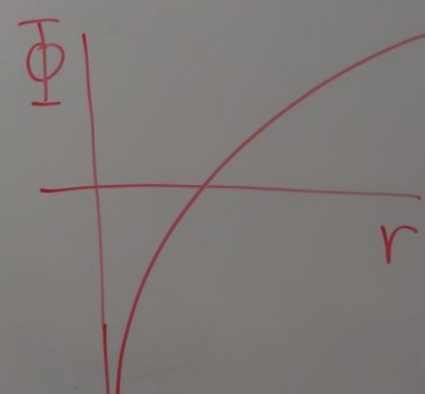
Sanders 2008

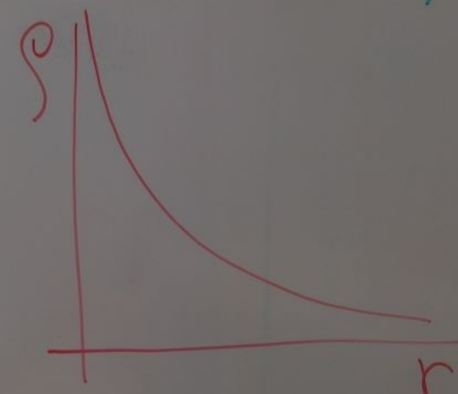
Výpočet hustoty pro $\Phi = v_0^2 \ln r$

$$\nabla^2 \Phi = 4\pi G \rho \,, \quad \frac{d\Phi}{dr} = \frac{v_0^2}{r}$$

pro sférickou symetrii (tj. $\rho(\vec{r}) = \rho(|\vec{r}|)$, $\Phi(\vec{r}) = \Phi(|\vec{r}|)$):

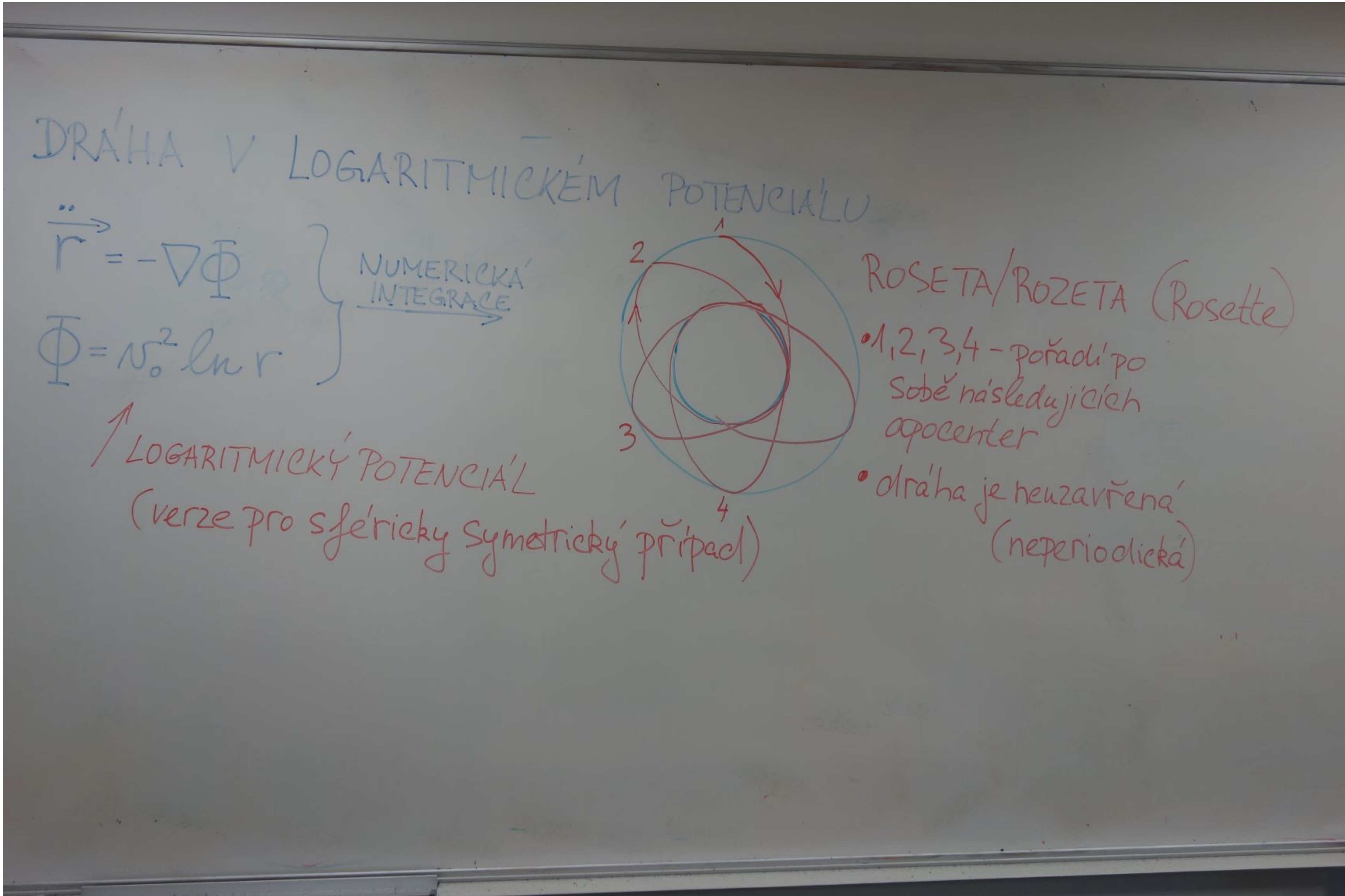
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right) = 4\pi G \rho$$





$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{v_0^2}{r}\right) = 4\pi G \rho$$

$$\frac{v_0^2}{r^2} = 4\pi G \rho \Longrightarrow \boxed{\rho(r) = \frac{v_0^2}{4\pi G r^2}}$$


DRÁHA V LOGARITMICKÉM POTENCIÁLU
$\ddot{\vec{r}} = -\nabla\Phi$
$\Phi = v_0^2 \ln r$
NUMERICKÁ INTEGRACE
LOGARITMICKÝ POTENCIÁL
(verze pro sféricky symetrický případ)
1
2
3
4
ROSETA/ROZETA (Rosette)
• 1,2,3,4 - pořadí po sobě následujících apocenter
• dráha je neuzavřená (neperiodická)

Logarithmic potential

$$\Phi_L = \frac{1}{2} v_0^2 \ln \left(R_c^2 + R^2 + \frac{z^2}{q_\Phi^2} \right) + \text{constant}$$

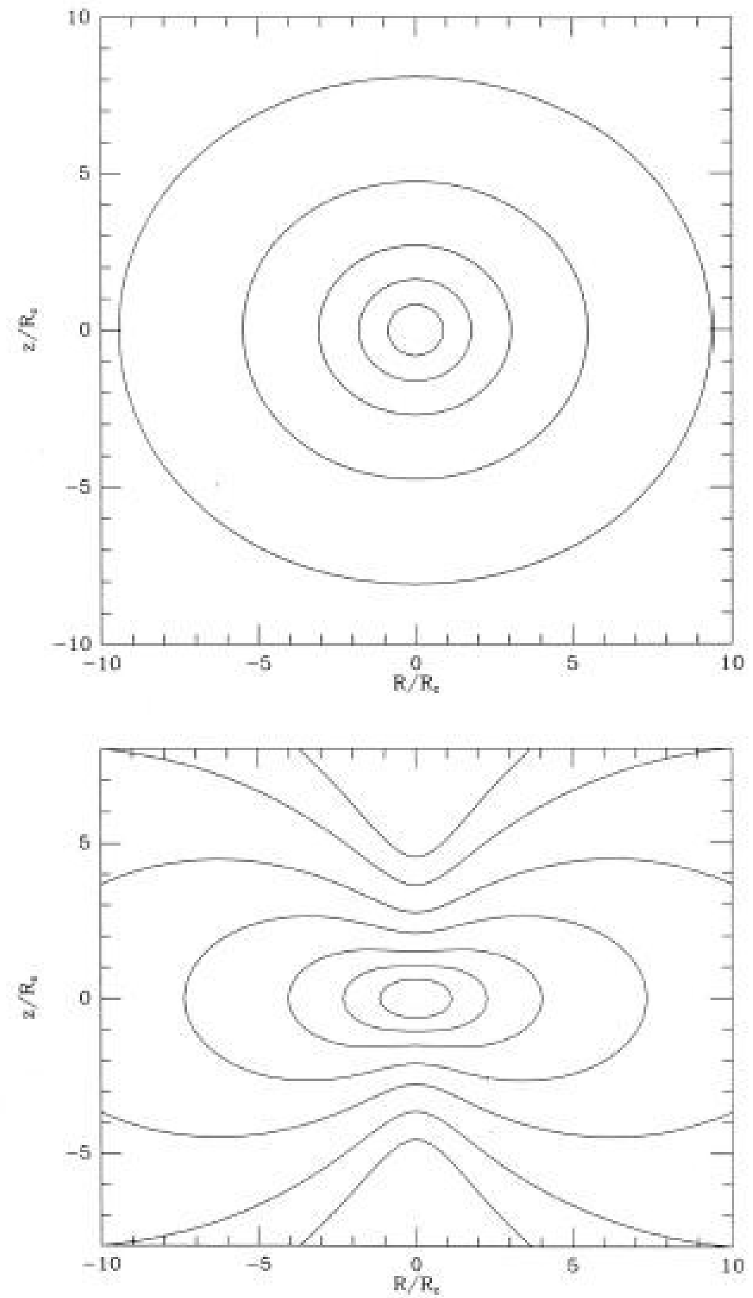


Figure 2-8. Contours of equal density in the (R, z) plane for ρ_L [eq. (2-54b)] when: $q_\Phi = 0.95$ (top); $q_\Phi = 0.7$ (bottom). In each case the contour levels are $0.1 v_0^2/(GR_c^2) \times (1, 0.3, 0.1, \ldots)$. When $q_\Phi = 0.7$ the density is negative near the z-axis for $|z| \gtrsim 7R_c$.

Zdroj: Binney & Tremaine, Galactic Dynamics

POHYB HVĚZD KOLMO NA GALAKTICKOU ROVINU

$\ddot{z} = -\frac{\partial \Phi}{\partial z}$, $\frac{\partial \Phi}{\partial z}$ odhadneme z Poissonovy rovnice:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \rho$$

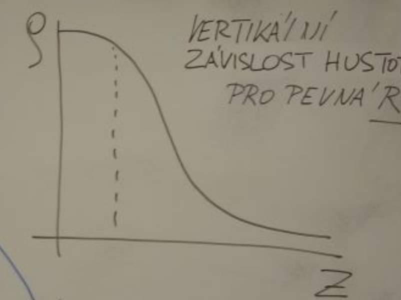
První člen: $= v_c^2$, ϕ (plochá rotační křivka)

Druhý člen: ϕ (osová symetrie)

$$\frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \rho(R,z)$$

$$\frac{\partial^2 \Phi}{\partial z^2} \doteq 4\pi G \rho_0(R)$$

ŘEŠÍME ZVLÁŠŤ PRO KAŽDÉ R



Aproximace:
– pro malé z je $\rho(R,z) \doteq \rho(R, z=0)$

$$\Rightarrow \frac{\partial \Phi}{\partial z} = 4\pi G \rho_0 z$$

$$\Rightarrow \ddot{z} = -4\pi G \rho_0 z = -\nu_z^2 z$$

$$z = Z \sin(\nu_z t + \varphi_z)$$

VERTIKÁLNÍ HARMONICKÉ KMITY (PRO MALÉ VERTIKÁLNÍ AMPLITUDY)

VERTIKÁLNÍ FREKVENCE $\nu_z(R) = \sqrt{4\pi G \rho_0(R)} = \sqrt{\left.\frac{\partial^2 \Phi}{\partial z^2}\right|_R}$

3D rozety ve zploštělém osově symetrickém potenciálu – pohled v rotující meridionální rovině (R, z)

Rovinná (2D) rozeta ve sféricky symetrickém potenciálu

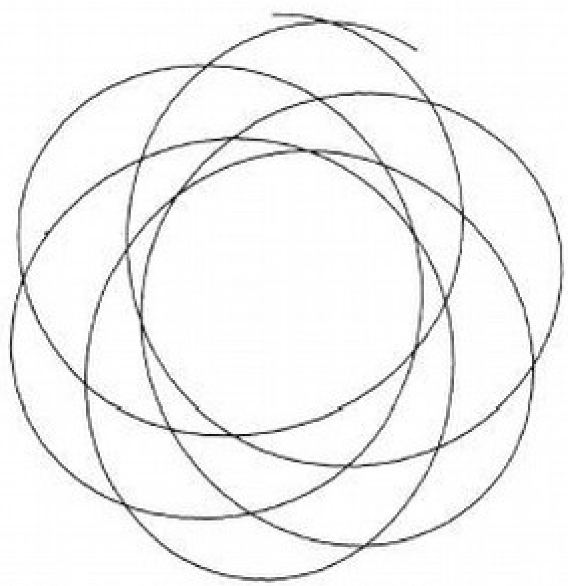


Figure 3-1. A typical orbit in a spherical potential forms a rosette.

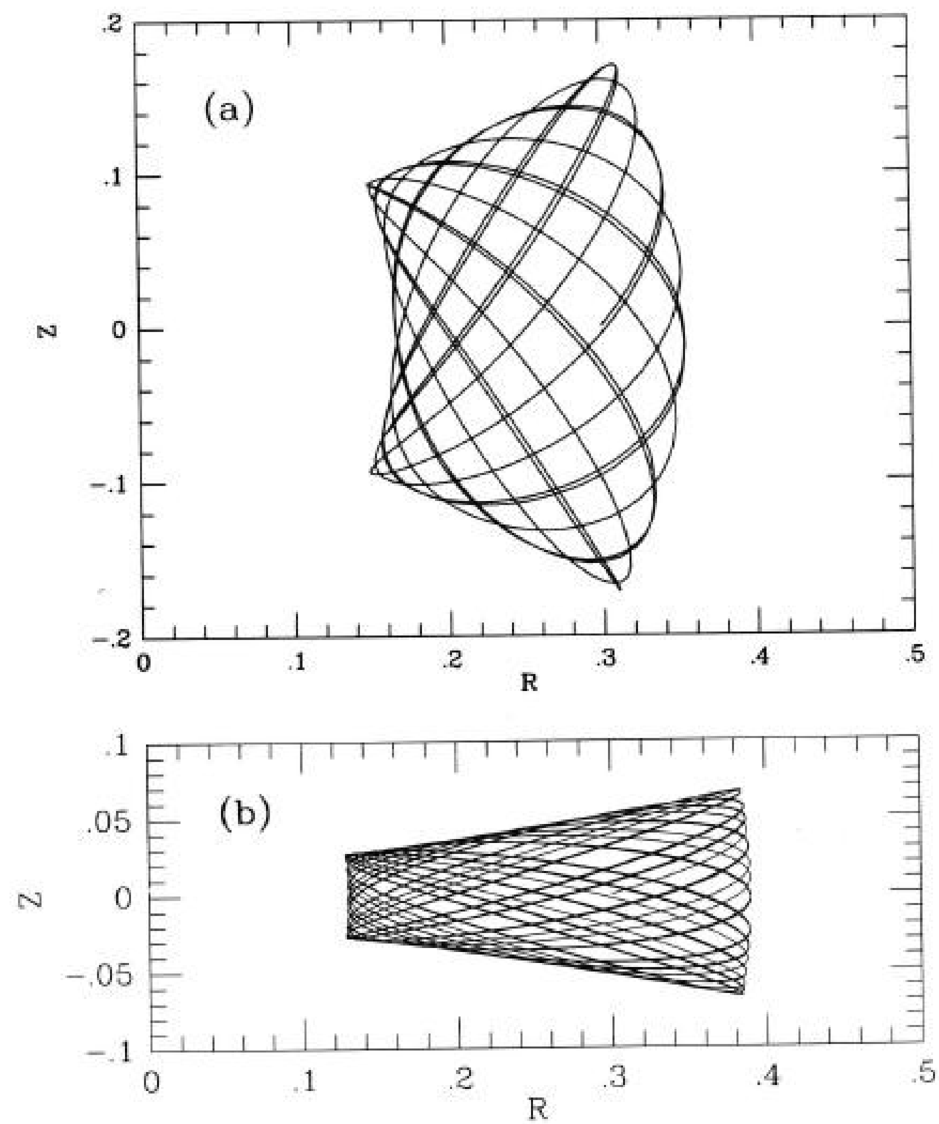
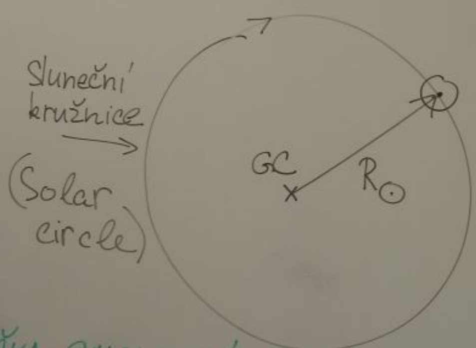


Figure 3-3. Two orbits in the potential of equation (3-50) with $q = 0.9$. Both orbits are at energy $E = -0.8$ and angular momentum $L_z = 0.2$, and we assume $v_0 = 1$.

Zdroj: Binney & Tremaine, Galactic Dynamics

OKOLÍ SLUNCE



ODBOČKA - GALAKTICKÉ JEDNOTKY:

$[d] = \mathrm{kpc}$, $[v] = \mathrm{km/s}$, $G \equiv 1$

$\Longrightarrow$

$[\Omega] = \mathrm{km/s/kpc}$, $[M] = 2{,}32 \cdot 10^5 \, M_\odot$,

$[t] = 0{,}978 \cdot 10^9 \, \mathrm{yr} \doteq 10^9 \, \mathrm{yr}$ (1 Gyr)

$$R_\odot = 8{,}2 \pm 0{,}2 \ \mathrm{kpc}$$

$$v_c(R_\odot) = 240 \pm 20 \ \mathrm{km/s}$$

$$\Omega(R_\odot) = \frac{v_c(R_\odot)}{R_\odot} \doteq 29 \ \mathrm{km/s/kpc} \quad (\pm 3 \ \mathrm{km/s/kpc})$$

$$\kappa(R_\odot) \doteq \sqrt{2}\, \Omega(R_\odot) \doteq 40 \ \mathrm{km/s/kpc}$$

$$\rho(R_\odot, z=0) \doteq 0{,}1 \ M_\odot/\mathrm{pc}^3$$

$$\Longrightarrow \nu_z(R_\odot) = \sqrt{4\pi G \rho(R_\odot, z=0)} \doteq 73 \ \mathrm{km/s/kpc}$$

$$\boxed{\nu_z > \kappa > \Omega}$$

Periody oběhu a radiálních/vertikálních kmitů:

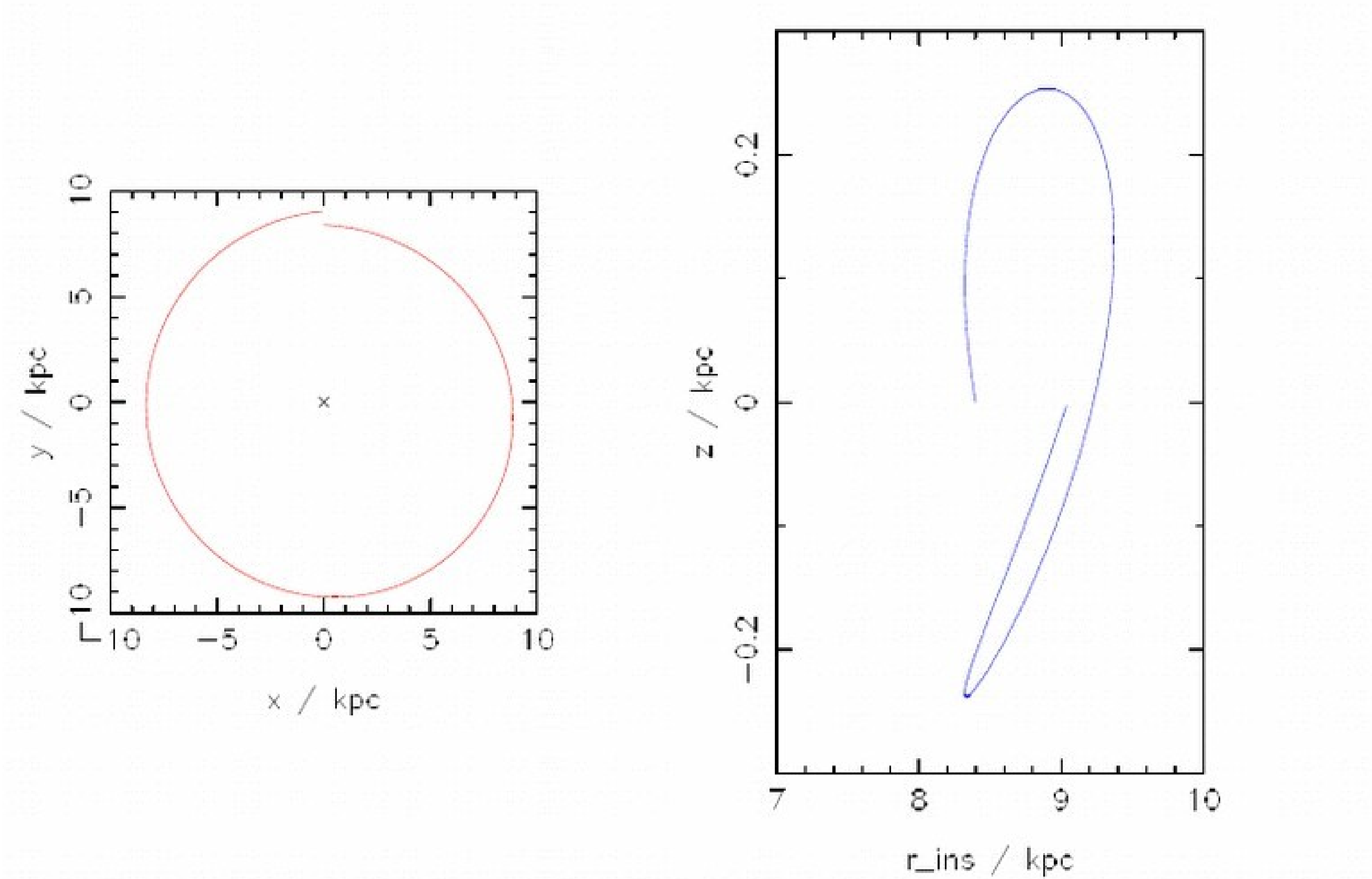
$$T(R_\odot) = \frac{2\pi}{\Omega(R_\odot)} \doteq 210 \cdot 10^6 \, \mathrm{yr} \ (210 \ \mathrm{Myr})$$

$$T_r(R_\odot) = \frac{2\pi}{\kappa(R_\odot)} \doteq 160 \cdot 10^6 \, \mathrm{yr} \ (160 \ \mathrm{Myr})$$

$$T_{\nu_z}(R_\odot) = \frac{2\pi}{\nu_z(R_\odot)} \doteq 84 \cdot 10^6 \, \mathrm{yr} \ (84 \ \mathrm{Myr})$$

Orbit of the Sun in our Galaxy (1 turn = 220 million years)

speed = 240 km/s
distance from the Galactic center: R = 8.4 kpc (27,000 l.y.)
(U,V,W = 11.1, 12.2, 7.2 km/s)



Orbit of the Sun: 21 turns in 4.57 billion years