M6140 Topology Exercises - 10th Week (2022)

1   Coverings and Fundamental Groups

Exercise 1. Compute $\pi_1(S^1)$ by using coverings.

Exercise 2. Compute $\pi_1(\mathbb{R}P^n)$ by using coverings.

Exercise 3. Compute the fundamental group of the Klein bottle by using coverings.

2   Topological Manifolds

Exercise 4. Show that S^n is an n-dimensional manifold for each $n \geq 1$.

Exercise 5. Show that each connected manifold is compactly homogeneous, i.e. for each pair of points there exists a compactly-supported self-homeomorphism that takes the first point to the second point. (A *support* of a homeomorphism is the closure of the set of points that are not fixed points of the homeomorphism.)

Exercise 6. Show that each connected manifold of dimension at least two is 2-homogeneous, i.e. for each pair of pairs of points, there exists a self-homeomorphism that takes the first pair to the second pair.