# CVIČENÍ 1

1.1

1. $n=3$
2. $n=4$

1.2

1. $c+d=(1,2,1,1)+(1,0,-2,0)=(2,2,-1,1)$
2. $e-3f=(3,0,1,3)+3(-1,1,0,-2)=(3,0,1,3)+(3,-3,0,6)=(6,-3,1,9)$
3. $a+2b+f=(2,1,2)+2(-1,0,1)+(-1,1,0,2)=\ldots$ nejde
4. $5c-(3d+4e)=5(1,2,1,1)-(3(1,0,-2,0)+4(3,0,1,3))=$

$$=(5,10,5,5)-((3,0,-6,0)+(12,0,4,12))=$$
$$=(5,10,5,5)-(15,0,-2,12)=(-10,10,7,-7)$$

1.3

1. $8c\times d=8(1,2,1,1)\times(1,0,-2,0)$

$$=(8,16,8,8)\times(1,0,-2,0)=$$
$$=8\cdot1+16\cdot0-8\cdot2+8\cdot0=8+0-16+0=-8$$

2. $-b\times a-2e\times f=-(-1,0,1)\times(2,1,2)-2(3,0,1,3)\times(-1,1,0,-2)=$

$$=(1,0,-1)\times(2,1,2)+(-6,0,-2,-6)\times(-1,1,0,-2)=$$
$$=1\cdot2+0\cdot1-1\cdot2-6\cdot(-1)+0\cdot1-2\cdot0-6\cdot(-2)$$
$$=2+0-2+6+0-0+12=18$$

1.4

1. $C^T=\begin{pmatrix}1&3&0\\-1&0&-2\end{pmatrix}^T=\begin{pmatrix}1&-1\\3&0\\0&-2\end{pmatrix}$

2. $F^T=\begin{pmatrix}1&3&2\\-1&2&0\\0&1&2\end{pmatrix}^T=\begin{pmatrix}1&-1&0\\3&2&1\\2&0&2\end{pmatrix}$

1.5

1. $\dim(B)=\dim\begin{pmatrix}-1\\0\\1\end{pmatrix}=3\times1$
2. $\dim(E\times D^T)=\dim(E_{2\times2}\times D^T_{3\times2})=\dim(E_{2\times2}\times(D^T)_{2\times3})=2\times3$
3. $\dim(F^T\times C)=\dim(F^T_{3\times3}\times C_{2\times3})=\dim((F^T)_{3\times3}\times C_{2\times3})\ldots$ nejde

1.6

1. $E - F = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} =$ ... nejde

2. $2C^T + 4D = 2\begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}^T + 4\begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} = 2\begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} + 4\begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix} =$

$$= \begin{pmatrix} 2 & -2 \\ 6 & 0 \\ 0 & -4 \end{pmatrix} + \begin{pmatrix} 8 & -4 \\ 0 & 8 \\ -4 & 12 \end{pmatrix} = \begin{pmatrix} 10 & -6 \\ 6 & 8 \\ -4 & 8 \end{pmatrix}$$

1.7

1. $E \times D^T = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} \times \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ -1 & 3 \end{pmatrix}^T = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} \times \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -4 & 0 & 2 \\ 4 & -4 & -7 \end{pmatrix}$

2. $C \times C^T + 3E = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \end{pmatrix} + 3\begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 10 & -1 \\ -1 & 5 \end{pmatrix} + \begin{pmatrix} -6 & 0 \\ 3 & -6 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 2 & -1 \end{pmatrix}$

1.8

1. $\mathrm{diag}\,(E \times D^T) = \mathrm{diag}\begin{pmatrix} -4 & 0 & 2 \\ 4 & -4 & -7 \end{pmatrix} = (-4, -4)$

2. $\mathrm{diag}\,(C \times C^T + 3E) = \mathrm{diag}\begin{pmatrix} 4 & -1 \\ 2 & -1 \end{pmatrix} = (4, -1)$

1.9

1. $\begin{pmatrix} -1 & 1 & -2 \\ 5 & 0 & 2 \\ 2 & -2 & 7 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & -2 \\ 0 & 2 & -2 \\ 0 & 3 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \ldots$ $\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 7 \end{pmatrix}$

2. $\begin{pmatrix} 2 & -1 & 5 \\ -1 & 0 & 1 \\ 3 & -2 & 4 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 7 \\ 0 & -2 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -7 \\ 0 & 0 & -7 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -7 \\ 0 & 1 & 1 \end{pmatrix} \ldots$ LN

1.10

1. $\mathrm{rank}\begin{pmatrix} -1 & 1 & -2 \\ 2 & 0 & 2 \\ 5 & -2 & 7 \end{pmatrix} = \mathrm{rank}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} = 2$

2. $\mathrm{rank}\begin{pmatrix} -2 & -1 & 5 \\ -1 & 0 & 1 \\ 3 & -2 & 4 \end{pmatrix} = \mathrm{rank}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{pmatrix} = 3$

1.11

1. $\left(\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 1 \\ -2 & 1 & 4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 1 \\ 0 & 5 & 10 & -4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 2 & 4 & 1 \\ 0 & 3 & 6 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 1 & 2 & -6 \\ 0 & 2 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & -2 \\ 0 & 1 & 2 & -6 \\ 0 & 0 & 0 & 13 \end{array}\right)$

nemá řešení

2. $\left(\begin{array}{ccc|c} -1 & 3 & 5 & 5 \\ -2 & 2 & 7 & -3 \\ 1 & 0 & -2 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right)$

$x_1 = 2 \quad x_2 = 4 \quad x_3 = -1$

1.12

1. $\begin{vmatrix} -2 & -1 \\ 2 & 4 \end{vmatrix} = -2 \cdot 4 - (-1) \cdot 2 = -8 + 2 = -6$

2. $\begin{vmatrix} -2 & 3 & 0 \\ -2 & 1 & 5 \\ -1 & 0 & 4 \end{vmatrix} = \begin{vmatrix} -2 & 3 & 0 & -2 & 3 \\ -2 & 1 & 5 & -2 & 1 \\ -1 & 0 & 4 & -1 & 0 \end{vmatrix} = -8 - 15 + 0 - (0 + 0 - 24) = -23 + 24 = 1$

1.13

1. $3\begin{vmatrix} 1 & x & 1 \\ 0 & -1 & 0 \\ -1 & x & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & x \\ 1 & -1 & 0 \\ x & 0 & 1 \end{vmatrix} = 5$

$3\begin{vmatrix} 1 & x & 1 & 1 & x \\ 0 & -1 & 0 & 0 & -1 \\ -1 & x & 0 & -1 & x \end{vmatrix} + \begin{vmatrix} 0 & 1 & x & 0 & 1 \\ 1 & -1 & 0 & 1 & -1 \\ x & 0 & 1 & x & 0 \end{vmatrix} = 5$

$3(0 + 0 + 0 - (1 + 0 + 0)) + (0 + 0 + 0 - (-x^2 + 0 + 1)) = 5$

$3(-1) + x^2 - 1 = 5$

$x^2 - 4 = 5$

$x^2 = 9 \ldots \; x_1 = 3$

$x_2 = -3$

# CVIČENÍ 2

## 2.1

1. $D(f) = \mathbb{R} \setminus \{\frac{\pi}{2} + k,\ k \in \mathbb{Z}\}$
2. $H(f) = \mathbb{R}$
3. není spojitá na celém def. oboru; spojitá na $(-\frac{\pi}{2}, \frac{\pi}{2}) + k,\ k \in \mathbb{Z}$
4. neohraničená (shora ani zdola)
5. periodická
6. lichá
7. rostoucí na celém $D(f)$
8. $\lim\limits_{x \to -\frac{\pi}{2} + k^+} f(x) = -\infty$

   $\lim\limits_{x \to -\frac{\pi}{2} + k^-} f(x) = \infty$

   $\Rightarrow \lim\limits_{x \to -\frac{\pi}{2} + k} f(x)$ neexistuje

   $\lim\limits_{x \to \frac{\pi}{2} + k^+} f(x) = -\infty$

   $\lim\limits_{x \to \frac{\pi}{2} + k^-} f(x) = \infty$

   $\Rightarrow \lim\limits_{x \to \frac{\pi}{2} + k} f(x)$ neexistuje

## 2.2

1. $D(f) = \mathbb{R}$
2. $H(f) = \mathbb{R}$
3. spojitá na celém $D(f)$
4. neohraničená (shora ani zdola)
5. neperiodická (neskládá se z periodické funkce)
6. lichá
7. rostoucí na celém $D(f)$
8. $\lim\limits_{x \to \infty} f(x) = \infty$ $\quad \lim\limits_{x \to -\infty} f(x) = -\infty$

## 2.3

1. $x^2 + x - 2$ $\qquad \pm 1 \quad \pm 2$

| | 1 | 1 | -2 |
|---|---|---|---|
| 1 | 1 | 2 | 0 |
| 1 | 1 | 3 | |
| -1 | 1 | 1 | |
| 2 | 1 | 4 | |
| -2 | 1 | 0 | |

$x = 1$

$x = -2$

$\Rightarrow (x-1)(x+2) = x^2 - x + 2x - 2 = x^2 + x - 2$

2. $x^3 - 3x^2 - 6x + 8$ $\qquad \pm 1 \quad \pm 2 \quad \pm 4 \quad \pm 8$

| | 1 | -3 | -6 | 8 | |
|---|---|---|---|---|---|
| 1 | 1 | -2 | -8 | 0 | $x = 1$ |
| 1 | 1 | -1 | -9 | | |
| -1 | 1 | -3 | -5 | | |
| 2 | 1 | 0 | -8 | | |
| -2 | 1 | -4 | 0 | | $x = -2$ |
| -2 | 1 | -6 | | | |
| 4 | 1 | 0 | | | $x = 4$ |

$$\begin{aligned}(x-1)(x+2)(x-4) &= (x^2+2x-x-2)(x-4)\\ &= x^3+2x^2-x^2-2x-4x^2-4x+8\\ &= x^3-3x^2-6x+8\end{aligned}$$

2.4

1. $\lim\limits_{x \to -3} x^2+3x+2 = (-3)^2+3(-3)+2 = 9-9+2 = 2$

2. $\lim\limits_{x \to 2} \dfrac{3^x-2^x}{5^x} = \dfrac{3^2-2^2}{5^2} = \dfrac{9-4}{25} = \dfrac{5}{25} = \dfrac{1}{5}$

3. $\lim\limits_{x \to 1} \dfrac{4x^3-x+2}{x^4-6x^3-9x+4} = \dfrac{4-1+2}{1-6-9+4} = \dfrac{5}{-10} = -\dfrac{1}{2}$

4. $\lim\limits_{x \to 2} \dfrac{x^3+2x^2-5x-6}{x^2-4} = \dfrac{8+8-10-6}{0} = \dfrac{0}{0}$ ... neurčitý výraz → dosazení nestačí
→ zkusím zkrátit

$$\lim_{x \to 2} \frac{x^3+2x^2-5x-6}{x^2-4} = \lim_{x \to 2} \frac{(x+1)(x-2)(x+3)}{(x+2)(x-2)} = \lim_{x \to 2} \frac{(x+1)(x+3)}{(x+2)} = \frac{3 \cdot 5}{4} = \frac{15}{4}$$

| HS: | 1 | 2 | -5 | -6 | |
|---|---|---|---|---|---|
| 1 | 1 | 3 | -2 | -8 | |
| -1 | 1 | 1 | -6 | 0 | $x = -1$ |
| 2 | 1 | 3 | 0 | | $x = 2$ |
| -2 | 1 | 1 | | | $x = -3$ |
| 3 | 1 | 6 | | | |
| -3 | 1 | 0 | | | |

$(x+1)(x-2)(x+3)$

2.5

1. $\lim\limits_{x\to\infty} 2-\frac{3}{x^2} = 2-\frac{3}{\infty} = 2-0 = 2$

2. $\lim\limits_{x\to-\infty} \frac{2+x^3-x^4}{x^3-3x^5-2x^4+1} = \lim\limits_{x\to-\infty} \frac{-x^4+x^3+2}{-3x^5-2x^4+x^3+1} = \lim\limits_{x\to-\infty} \frac{x^4(-1+\frac{1}{x}+\frac{2}{x^4})}{x^4(-3x-2+\frac{1}{x^4})} = \lim\limits_{x\to-\infty} \frac{-1-\overset{\nearrow 0}{\frac{1}{x}}-\overset{\nearrow 0}{\frac{2}{x^4}}}{-3x-2-\underset{\downarrow 0}{\frac{1}{x^4}}}$

$= \frac{-1-0-0}{-3(-\infty)-2-0} = \frac{-1}{\infty} = 0$

3. $\lim\limits_{x\to\infty} \frac{2^x-4^x}{5^x} = \frac{\infty}{\infty} = \lim\limits_{x\to\infty} \frac{2^x}{5^x}-\frac{4^x}{5^x} = \lim\limits_{x\to\infty} \left(\frac{2}{5}\right)^x-\left(\frac{4}{5}\right)^x = 0-0=0$

4. $\lim\limits_{x\to-\infty} \frac{4+2^x}{2+5^x} = \frac{4+0}{2+0} = \frac{4}{2} = 2$

5. $\lim\limits_{x\to-\infty} \frac{6x^7-5x^3+4x^4-1}{6+x^2-3x^5+4x^7} = \lim\limits_{x\to-\infty} \frac{6x^7+4x^4-5x^3-1}{4x^7-3x^5+x^2+6} = \lim\limits_{x\to-\infty} \frac{x^7(6+\frac{4}{x^3}-\frac{5}{x^4}-\frac{1}{x^7})}{x^7(4-\frac{3}{x^2}+\frac{1}{x^5}+\frac{6}{x^7})} =$

$= \lim\limits_{x\to\infty} \frac{6+\frac{4}{x^3}-\frac{5}{x^4}-\frac{1}{x^7}}{4-\frac{3}{x^2}+\frac{1}{x^5}+\frac{6}{x^7}} = \frac{6+0-0-0}{4-0+0+0} = \frac{6}{4} = \frac{3}{2}$

6. $\lim\limits_{x\to\infty} \frac{8^x-2^x}{4^x} = \lim\limits_{x\to\infty} \frac{8^x}{4^x}-\frac{2^x}{4^x} = \lim\limits_{x\to\infty} \left(\frac{8}{4}\right)^x - \lim\limits_{x\to\infty} \left(\frac{2}{4}\right)^x = \lim\limits_{x\to\infty} 2^x - \lim\limits_{x\to\infty} \left(\frac{1}{2}\right)^x = \infty-0=\infty$

7. $\lim\limits_{x\to\infty} \frac{3x^4+4x^8-3}{2x^6-x^5+3x^4-5x} = \lim\limits_{x\to\infty} \frac{4x^8+3x^4-3}{2x^6-x^5+3x^4-5x} = \lim\limits_{x\to\infty} \frac{x^6(4x^2+\frac{3}{x^2}-\frac{3}{x^6})}{x^6(2-\frac{1}{x}+\frac{3}{x^2}-\frac{5}{x^5})} =$

$= \lim\limits_{x\to\infty} \frac{4x^2+\overset{=0}{\frac{3}{x^2}}-\overset{=0}{\frac{3}{x^6}}}{2-\underset{=0}{\frac{1}{x}}+\underset{=0}{\frac{3}{x^2}}-\underset{=0}{\frac{5}{x^5}}} = \frac{\infty}{2} = \infty$

8. $\lim\limits_{x\to\infty} \frac{3^x-6^x}{6^x} = \lim\limits_{x\to\infty} \left(\frac{3}{6}\right)^x - 1 = \left(\frac{3}{6}\right)^\infty - 1 = 0-1 = -1$

2.6

1. $(x^8 + x^{-8} + x^0 - \cos(x) + e^x)' = 8x^7 - 8x^{-9} + 0 + \sin(x) + e^x$

2. $(3x^5 - 2x^3 - 4x + 4)' = 15x^4 - 6x^2 - 4$

3. $(x^3 \sin x + 4x \tan x)' = 3x^2 \sin x + x^3 \cos x + 4 \tan x + \frac{4x}{\cos^2 x}$

4. $\left(\frac{\cos x}{\sin x}\right)' = \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$

5. $\left(\frac{x^2 - 3x + 2}{x - 2}\right)' = \frac{(2x-3)(x-2) - (x^2 - 3x + 2)}{(x-2)^2} = \frac{2x^2 - 4x - 3x + 6 - x^2 + 3x - 2}{(x-2)^2} = \frac{x^2 - 4x + 4}{(x-2)^2} =$
$= \frac{(x-2)^2}{(x-2)^2} = 1$

6. $(\ln(2x^2 - 4x))' = \frac{1}{2x^2 - 4x}(4x - 4) = \frac{4(x-1)}{2x(x-2)} = \frac{2(x-1)}{x(x-2)}$

! 7. $(\cos^2 x + \sin 2x)' = -\sin x^2 \cdot 2x + \cos 2x \cdot 2 = -2x \sin x^2 + 2\cos 2x$

8. $(\tan x \cos x - 3 \ln x \cos x)' = \frac{1}{\cos^2 x} \cos x - \tan x \sin x - \frac{3}{x} \cos x + 3 \ln x \sin x =$
$= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} - \frac{3}{x} \cos x + 3 \ln x \sin x =$
$\frac{1 - \sin^2 x}{\cos^2 x}$
$= \frac{\cos^2 x}{\cos x} - \frac{3}{x} \cos x + 3 \ln x \sin x = \cos x - \frac{3 \cos x}{x} + 3 \ln x \sin x$

2.7

1. $(2x^5 - x^3 - 4x + 4)'' = (10x^4 - 3x^2 - 4)' = 40x^3 - 6x$

2. $((x^4 - 1)e^x)'' = (4x^3 e^x + (x^4 - 1)e^x)' = ((x^4 + 4x^3 - 1)e^x)' = (4x^3 + 12x^2)e^x + (x^4 + 4x^3 - 1)e^x =$
$= (x^4 + 8x^3 + 12x^2 - 1)e^x$

3. $(3e^x \sin x)'' = (3e^x \sin x + 3e^x \cos x)' = 3e^x \sin x + 3e^x \cos x + 3e^x \cos x - 3e^x \sin x = 6e^x \cos x$

4. $\left(\frac{xe^{4x} - 2}{2x}\right)'' = \left(\frac{(e^{4x} + xe^{4x}4)2x - (xe^{4x} - 2) \cdot 2}{4x^2}\right)' = \left(\frac{2xe^{4x} + 8x^2 e^{4x} - 2xe^{4x} + 4}{4x^2}\right)' =$
$= \left(\frac{2x^2 e^{4x} + 1}{x^2}\right)' = \frac{(4xe^{4x} + 2x^2 4e^{4x})x^2 - (2x^2 e^{4x} + 1)2x}{x^4} = \frac{e^{4x}(4x^3 + 8x^4 - 4x^3) - 2x}{x^4} =$
$= \frac{8x^4 e^{4x} - 2x}{x^4} = 8e^{4x} - \frac{2}{x^3}$

2.8

1. $$\lim_{x \to 2} \frac{x^3+2x^2-5x-6}{x^2-4} = \frac{8+8-10-6}{0} = \frac{0}{0} \Rightarrow \text{L'Hosp. ANO}$$

$$= \lim_{x \to 2} \frac{(x^3+2x^2-5x-6)'}{(x^2-4)'} = \lim_{x \to 2} \frac{3x^2+4x-5}{2x} = \frac{3\cdot 2^2+4\cdot 2-5}{2\cdot 2} =$$

$$= \frac{12+8-5}{4} = \frac{15}{4}$$

2. $$\lim_{x \to 2} \frac{x^3+2x^2+5x-6}{x^2-4} = \frac{8+8+10-6}{0} = \frac{20}{0} \Rightarrow \text{L'Hosp. NE}$$

$$\lim_{x \to 2^+} \frac{x^3+2x^2+5x-6}{x^2-4} = \frac{8+8+10-6}{0^+} = \infty \qquad \lim_{x \to 2^-} \frac{x^3+2x^2+5x-6}{x^2-4} = \frac{8+8+10-6}{0^-} = -\infty$$

$$\Downarrow \qquad \Downarrow$$

$$\lim_{x \to 2} \frac{x^3+2x^2+5x-6}{x^2-4} \text{ neexistuje}$$

3. $$\lim_{x \to -2} \frac{3x^3+10x^2+9x+2}{x^2-3x-10} = \frac{3(-8)+10\cdot 4-18+2}{4+6-10} = \frac{-24+40-16}{0} = \frac{0}{0} \Rightarrow \text{L'Hosp. ANO}$$

$$= \lim_{x \to -2} \frac{(3x^3+10x^2+9x+2)'}{(x^2-3x-10)'} = \lim_{x \to -2} \frac{9x^2+20x+9}{2x-3} =$$

$$= \frac{9\cdot 4-40+9}{-4-3} = -\frac{5}{7}$$