CVIČENÍ 3

3.1

$f(x) = x^3 - 8$

1. $D(f) = \mathbb{R}$

2. $H(f)$: $y = x^3 - 8$
$x^3 = -8 - y$
$x = \sqrt[3]{-8-y}$
$x = -\sqrt[3]{8+y} \rightarrow H(f) = \mathbb{R}$

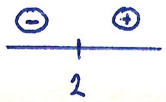
3. $f(-x) = -x^3 - 8$
$-f(x) = -x^3 + 8$
$f(x) = x^3 - 8$

$f(x) \neq f(-x) \rightarrow$ není sudá
$f(-x) \neq -f(x) \rightarrow$ není lichá

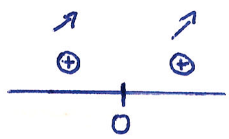
4. není periodická, neskládá se z periodické funkce

5. BN nemá

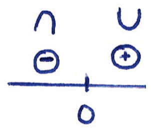
6. $x^3 - 8 = 0$
$x^3 = 8$
$x = \sqrt[3]{8} = 2$



7. $(x^3 - 8)' = 0$
$3x^2 = 0$
$x^2 = 0$
$x = 0$



8. $(x^3 - 8)'' = 0$
$(3x^2)' = 0$
$6x = 0$



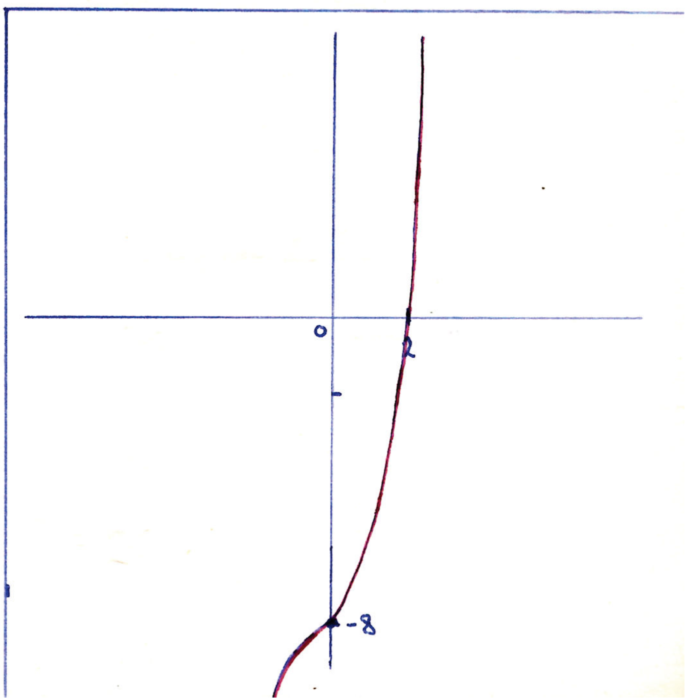
9. ABS: nemá BN $\rightarrow$ nemá ABS

ASS: $\lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} \frac{x^3 - 8}{x} =$

$= \lim\limits_{x \to \infty} x^2 + \lim\limits_{x \to \infty} -\frac{8}{x} = \infty \Rightarrow$ ASS v ∞ není

$\lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} x^2 + \lim\limits_{x \to -\infty} -\frac{8}{x} = \infty \rightarrow$ ASS v $-\infty$ není

10. $f(0) = -8$
$f(2) = 0$



3.2

$f(x) = 2x^2 - 6x + 4$

1. $D(f) = \mathbb{R}$

2. $H(f)$: $2x^2 - 6x + 4 = y$

$2x^2 - 6x + 4 - y = 0$

$D = b^2 - 4ac$
$= 36 - 4 \cdot 2(4-y)$
$= 36 - 32 + 8y = 4 + 8y$

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{4+8y}}{4}$
$= \frac{6 \pm \sqrt{4(1+2y)}}{4}$
$= \frac{3 \pm \sqrt{1+2y}}{4}$

$1 + 2y \geq 0$
$1 \geq -2y$
$-\frac{1}{2} \leq y$
$\Downarrow$
$H(f) = \langle -\frac{1}{2}; \infty)$

3. $f(-x) = 2(-x)^2 - 6(-x) + 4$
$= 2x^2 + 6x + 4$

$-f(x) = -2x^2 + 6x - 4$

$f(x) \neq f(-x) \rightarrow$ není sudá
$f(-x) \neq -f(x) \rightarrow$ není lichá

4. není periodická, neskládá se z periodických fcí

5. BN nemá

6. $2x^2 - 6x + 4 = 0$

$D = b^2 - 4ac = 36 - 4 \cdot 2 \cdot 4 = 36 - 32 = 4$

$\frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm 2}{4} \begin{cases} 2 \\ 1 \end{cases}$

⊕ 1 ⊖ 2 ⊕

7. $(2x^2 - 6x + 4)' = 0$
$4x - 6 = 0$
$4x = 6$
$x = \frac{6}{4} = \frac{3}{2}$

↘ ⊖ $\frac{3}{2}$ ⊕ ↗

8. $(2x^2 - 6x + 4)'' = 0$
$(4x - 6)' = 0$
$4 = 0 \ldots \rightarrow$ ⊕ na celém $D(f) \Rightarrow$

∪
⊕
$-\infty$ ∞

9. ABS: nemá BN → nemá ABS

ASS: $\lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{2x^2 - 6x + 4}{x} =$

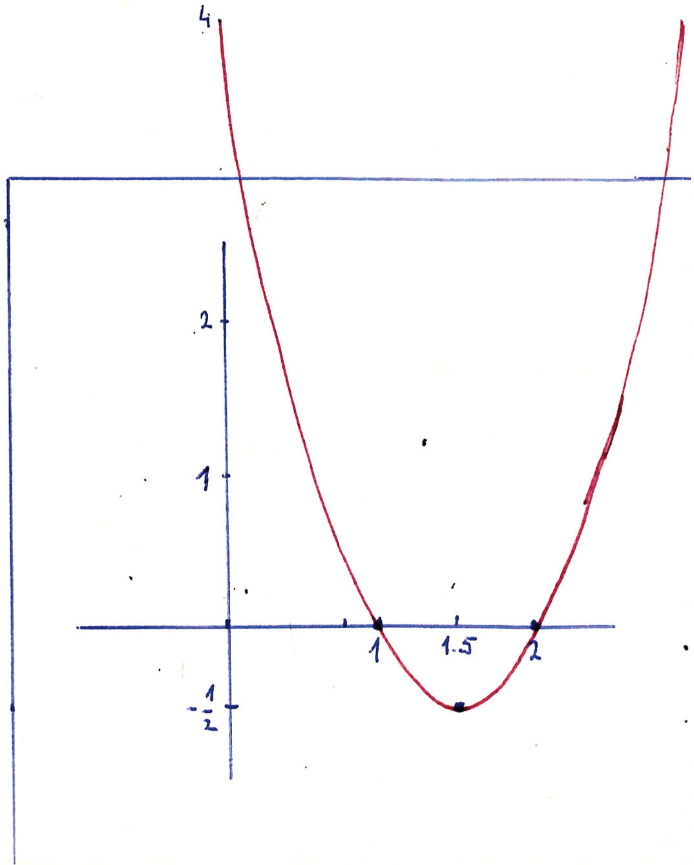
$= \lim_{x \to \infty} 2x - 6 + \frac{4}{x} = 2\infty - 6 + \frac{4}{\infty} = \infty$

$\Rightarrow$ ASS v ∞ není

$\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} 2x - 6 + \frac{4}{x} = -2\infty - 6 - \frac{4}{\infty} =$
$= -\infty$

$\Rightarrow$ ASS v $-\infty$ není

10. $f(1) = 0$
$f(2) = 0$
$f(\frac{3}{2}) = 2 \cdot \frac{9}{4} - \frac{18}{2} + 4 = \frac{9}{2} - 9 + 4 = 8\frac{1}{2} - 9 = -\frac{1}{2}$
$f(0) = 4$



3.3

$f(x) = -\frac{9}{2x}$

1. $D(f) = \mathbb{R} \setminus \{0\}$
2. $H(f)$: $-\frac{9}{2x} = y$

$-9 = 2xy$

$\frac{-9}{2y} = x$

$H(f) = \mathbb{R} \setminus \{0\}$

Zk: $0 = -\frac{9}{2x}$

$0 = -9 \ldots$ spor $\Rightarrow y \neq 0$

3. $f(-x) = -\frac{9}{2(-x)} = +\frac{9}{2x}$

$-f(x) = \frac{9}{2x}$

$f(x) \neq f(-x) \Rightarrow$ není sudá

$f(-x) = -f(x) \Rightarrow$ je lichá

4. není periodická, neobsahuje period. fci

5. BN: $x = 0$

6. $-\frac{9}{2x} = 0$

$-9 = 0 \ldots$

$\downarrow$ zlom pouze v BN.

$\oplus \quad | \quad \ominus$, bod 0

7. $\left(-\frac{9}{2x}\right)' = 0$

$\frac{9 \cdot 2}{(2x)^2} = 0$

$\frac{18}{4x^2} = 0$

$\frac{9}{2x^2} = 0$

$\nearrow \oplus \quad | \quad \nearrow \oplus$, bod 0

8. $\left(-\frac{9}{2x}\right)'' = 0$

$\left(\frac{9}{2x^2}\right)' = 0$

$\left(\frac{9}{2} \cdot x^{-2}\right)' = 0$

$\frac{9}{2} \cdot (-2) x^{-3} = 0$

$-\frac{9}{x^3} = 0$

$\cup \oplus \quad | \quad \cap \ominus$, bod 0

9. ABS: $\lim_{x \to 0^+} -\frac{9}{2x} = -\infty$

$\lim_{x \to 0^-} -\frac{9}{2x} = \infty$

ASS $\lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} -\frac{9}{2x^2} = 0 \Rightarrow a = 0$

$\lim_{x \to \infty} (f(x) - ax) = \lim_{x \to \infty} -\frac{9}{2x} = 0 \Rightarrow b = 0$

$\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} -\frac{9}{2x^2} = 0 \Rightarrow a = 0$

$\lim_{x \to -\infty} (f(x) - ax) = 0 \Rightarrow b = 0$

ASS: $y = 0$

10. $f(0) = $ BN

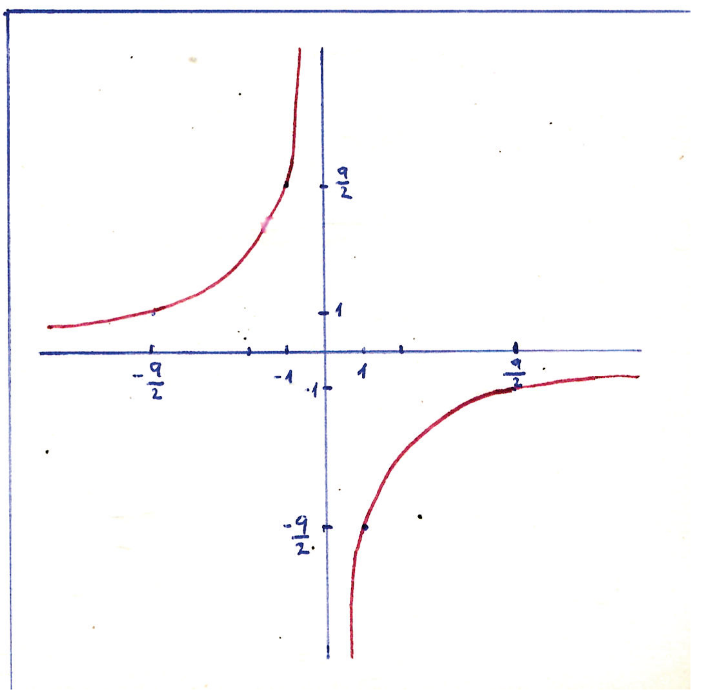
$f(1) = -\frac{9}{2}$

$f(-1) = \frac{9}{2}$

$-\frac{9}{2x} = 1$

$-9 = 2x$

$x = -\frac{9}{2}$



3.4

$f(x) = -\frac{x}{2x+4}$

1. $D(f)$: $2x+4 \neq 0$
   $2x \neq -4$
   $x \neq -2 \rightarrow D(f) = \mathbb{R} \setminus \{-2\}$

2. $H(f)$: $\frac{-x}{2x+4} = y$
   $-x = y(2x+4)$
   $-x = 2xy + 4y$
   $-x - 2xy = 4y$
   $+x(1+2y) = -4y$
   $x = \frac{-4y}{1+2y}$

   $1+2y \neq 0$
   $2y \neq -1$
   $y \neq -\frac{1}{2} \rightarrow H(f) = \mathbb{R} \setminus \{-\frac{1}{2}\}$

   Zk: $\frac{-x}{2x+4} = -\frac{1}{2}$
   $-x = -\frac{1}{2}(2x+4)$
   $-x = -x - 2$
   $0 = -2 \rightarrow$ spor $\rightarrow y \neq -\frac{1}{2}$

3. $f(-x) = \frac{-(-x)}{2(-x)+4} = \frac{x}{-2x+4} = \frac{-x}{2x-4}$ $f(-x) \neq f(x)$... není sudá

   $-f(x) = \frac{x}{2x+4}$ $f(-x) \neq -f(x)$... není lichá

4. není periodická, neobsahuje periodickou fci
5. BN: $x = -2$
6. $\frac{-x}{2x+4} = 0$
   $-x = 0$

   $\ominus \quad -2 \quad \oplus \quad 0 \quad \ominus$

7. $\left(\frac{-x}{2x+4}\right)' = 0$
   $\frac{-2x-4+x2}{(2x+4)^2} = 0$
   $\frac{-4}{(2x+4)^2} = 0$ ... $x \neq -2$

   $\searrow \ominus \quad -2 \quad \searrow \ominus$

8. $\left(-\frac{x}{2x+4}\right)'' = 0$
   $\left(\frac{-4}{(2x+4)^2}\right)' = 0$
   $\left(-4(2x+4)^{-2}\right)' = 0$
   $-4\cdot(-2)(2x+4)^{-3}\cdot 2 = 0$
   $\frac{16}{(2x+4)^3} = 0$ $x \neq -2$

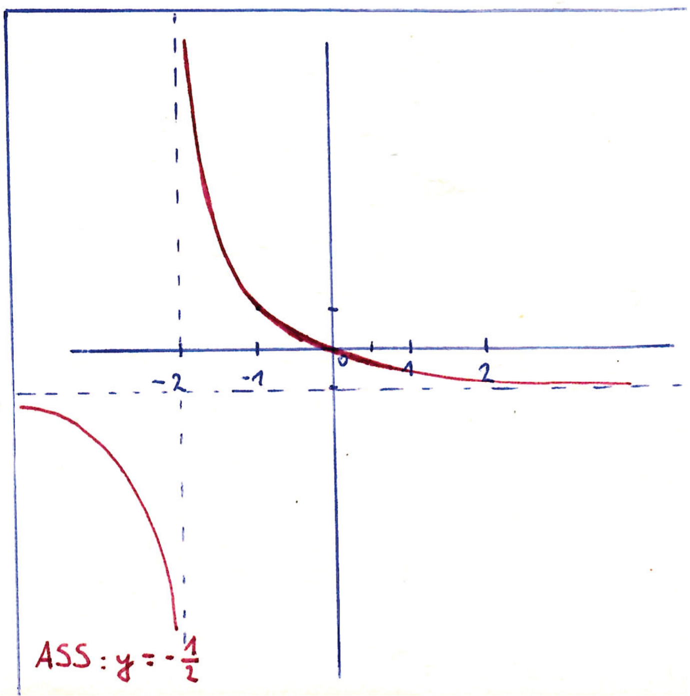
   $\cap \ominus \quad -2 \quad \cup \oplus$

9. ABS: $\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{-x}{2x+4} = -\infty$
   $\lim_{x\to -2^-} \frac{-x}{2x+4} = \infty$

   ASS: $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{-1}{2x+4} = 0 \rightarrow a = 0$
   $\lim_{x\to\infty} (f(x) - ax) = \lim_{x\to\infty} \frac{-x}{2x+4} = \lim_{x\to\infty} \frac{-x}{x(2+\frac{4}{x})} =$
   $\lim_{x\to\infty} \frac{-1}{2-\frac{4}{x}} = -\frac{1}{2} \rightarrow b = -\frac{1}{2}$

   $\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{-1}{2x+4} = 0 \rightarrow a = 0$
   $\lim_{x\to-\infty} f(x) - ax = \lim_{x\to-\infty} \frac{-1}{2-\frac{4}{x}} = -\frac{1}{2} \rightarrow b = -\frac{1}{2}$

10. $f(0) = 0$
    $f(-2) =$ neex.
    $f(-1) = \frac{1}{2}$



ASS: $y = -\frac{1}{2}$

3.5

$f(x) = \frac{x^2-1}{3x}$

1. $D(f) = \mathbb{R} \setminus \{0\}$

2. $H(f)$: $\frac{x^2-1}{3x} = y$ $\quad D = b^2-4ac = 9y^2+4$ $\quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{3y \pm \sqrt{9y^2+4}}{2}$ vždy > 0

$x^2-1 = 3xy$

$x^2-3xy-1=0$

$H(f) = \mathbb{R}$

3. $f(-x) = \frac{(-x)^2-1}{3(-x)} = \frac{x^2-1}{-3x} = -\frac{x^2-1}{3x}$ $\quad f(x) \neq f(-x) \to$ není sudá

$-f(x) = -\frac{x^2-1}{3x}$ $\quad -f(x) = f(-x) \to$ lichá

4. není periodická, neobsahuje periodickou fci

5. BN: $x=0$

6. $\frac{x^2-1}{3x} = 0$ $\quad$ $\ominus$ -1 $\oplus$ 0 $\ominus$ 1 $\oplus$

$x^2-1=0$

$x^2=1$

$x = \pm 1$

7. $\left(\frac{x^2-1}{3x}\right)' = 0$ $\quad$ $\oplus \nearrow$ 0 $\oplus \nearrow$

$\frac{2x \cdot 3x - (x^2-1)3}{9x^2} = 0$

$\frac{6x^2-3x^2+3}{9x^2} = 0$

$\frac{(x^2+1)}{3x^2} = 0$ … vždycky kladné

8. $\left(\frac{x^2-1}{3x}\right)'' = 0$

$\left(\frac{(x^2+1)}{3x^2}\right)' = 0$

$\left(\frac{1}{3}(x^2+1)\,x^{-2}\right)' = 0$

$\frac{1}{3}\, 2x \cdot x^{-2} + \frac{1}{3}(x^2+1)(-2)\cdot x^{-3} = 0$

$\frac{2}{3x} - \frac{2}{3}\frac{x^2+1}{x^3} = 0$ $\quad$ $\cup$ $\oplus$ 0 $\cap$ $\ominus$

$\frac{2}{3}\,\frac{x^2-x^2-1}{x^3} = 0$

$\frac{-2}{3x^3} = 0$

9. ABS: $\lim_{x\to 0^+} f(x) = \lim_{x \to 0^+} \frac{x^2-1}{3x} = -\infty$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x^2-1}{3x} = \infty$

ASS: $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2-1}{3x^2} = \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{3} = \frac{1}{3}$ … $a = \frac{1}{3}$

$\lim_{x\to\infty} f(x) - ax = \lim_{x\to\infty} \frac{x^2-1}{3x} - \frac{x}{3} = \lim_{x\to\infty} \frac{x^2-1-x^2}{3x} =$

$\lim_{x\to\infty} \frac{-1}{3x} = 0 \to b = 0$

$\lim_{x\to-\infty} \frac{x^2-1}{3x^2} = \lim_{x\to-\infty} \frac{1-\frac{1}{x^2}}{3} = \frac{1}{3}$ … $a = \frac{1}{3}$

$\lim_{x\to-\infty} \frac{x^2-1}{3x} - \frac{x}{3} = \lim_{x\to-\infty} \frac{-1}{3x} = 0$

ASS: $y = \frac{1}{3}x$

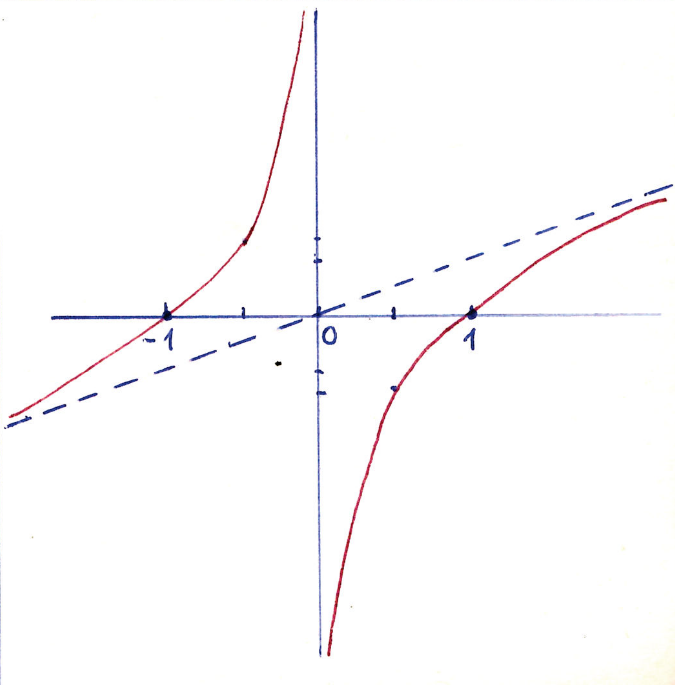
10. $f(-1) = 0$

$f(1) = 0$

$f(0)$ neex.

$f\left(\frac{1}{2}\right) = \frac{\frac{1}{4}-1}{\frac{3}{2}} = -\frac{3}{4}\cdot\frac{2}{3} = -\frac{1}{2}$

$f\left(-\frac{1}{2}\right) = \frac{\frac{1}{4}-1}{-\frac{3}{2}} = +\frac{3}{4}\cdot\frac{2}{3} = \frac{1}{2}$



3.6

$f(x) = -\frac{1}{x^2-2}$

1. $D(f) = \mathbb{R} \setminus \{-\sqrt{2}; \sqrt{2}\}$

2. $H(f)$: $-\frac{1}{x^2-2} = y$

$-1 = yx^2 - 2y$

$yx^2 = 2y - 1$

$x^2 = \frac{2y-1}{y}$

$x = \pm\sqrt{\frac{2y-1}{y}}$

$\frac{2y-1}{y} > 0$

1. $\frac{\oplus}{\oplus} > 0$

$2y - 1 \geq 0$

$2y \geq 1$

$y \geq \frac{1}{2} \wedge y > 0$

$0 \quad \frac{1}{2}$

$y \in \langle \frac{1}{2}; \infty)$

2. $\frac{\ominus}{\ominus} > 0$

$2y - 1 \leq 0$

$2y \leq 1$

$y \leq \frac{1}{2} \wedge y < 0$

$0 \quad \frac{1}{2}$

$y \in (-\infty; 0)$

$H(f) = (-\infty; 0) \cup \langle \frac{1}{2}; \infty) = \mathbb{R} \setminus \langle 0; \frac{1}{2})$

3. $f(-x) = -\frac{1}{(-x)^2-2} = \frac{-1}{x^2-2}$

$-f(x) = \frac{1}{x^2-2}$

$f(-x) = f(x) \to$ sudá

$f(-x) \neq -f(x) \to$ není lichá

4. není periodická, neobsahuje periodickou fci

5. BN: $x = -\sqrt{2}$ $\quad x = \sqrt{2}$

6. $-\frac{1}{x^2-2} = 0$

$-1 = 0$

$\ominus \quad -\sqrt{2} \quad \oplus \quad \sqrt{2} \quad \ominus$

7. $\left(-\frac{1}{x^2-2}\right)' = 0$

$\frac{2x}{(x^2-2)^2} = 0$

$x = \sqrt{2} \quad x = -\sqrt{2} \quad x = 0$

$\ominus \quad -\sqrt{2} \quad \ominus \quad 0 \quad \oplus \quad \sqrt{2} \quad \oplus$

8. $\left(-\frac{1}{x^2-2}\right)'' = 0$

$\left(\frac{2x}{(x^2-2)^2}\right)' = 0$

$(2x(x^2-2)^{-2})' = 0$

$2(x^2-2)^{-2} + 2x(-2)(x^2-2)^{-3} \cdot 2x = 0$ vždy > 0

$\frac{2(x^2-2)}{(x^2-2)^3} - \frac{8x^2}{(x^2-2)^3} = -\frac{6x^2+4}{(x^2-2)^3}$ $\quad x = \pm\sqrt{2}$

$\cap \quad \cup \quad \cap$

$\ominus \quad -\sqrt{2} \quad \oplus \quad \sqrt{2} \quad \ominus$

9. ABS: $\lim_{x \to \sqrt{2}^+} -\frac{1}{x^2-2} = -\infty$

$\lim_{x \to \sqrt{2}^-} -\frac{1}{x^2-2} = +\infty$

$\lim_{x \to -\sqrt{2}^+} -\frac{1}{x^2-1} = \infty$

$\lim_{x \to -\sqrt{2}^-} -\frac{1}{x^2-1} = -\infty$

ASS: $\lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} -\frac{1}{x^3-2x} = 0 \to a = 0$

$\lim_{x \to \infty} f(x) - ax = \lim_{x \to \infty} -\frac{1}{x^2-2} = 0 \to b = 0$

$\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} -\frac{1}{x^3-2x} = 0 \quad a = 0$

$\lim_{x \to -\infty} f(x) - ax = \lim_{x \to -\infty} -\frac{1}{x^2-2} = 0 \quad b = 0$

ASS: $y = 0$

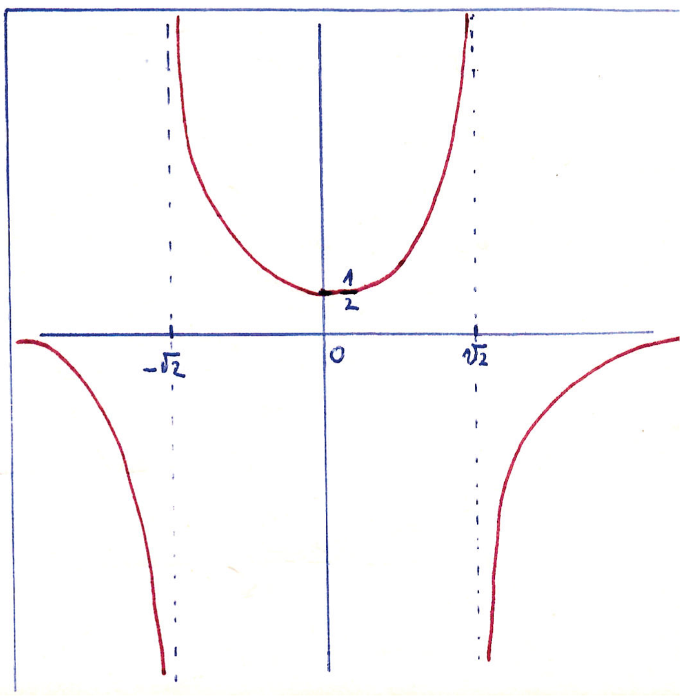
10. $f(\sqrt{2})$ neex.

$f(-\sqrt{2})$ neex.

$f(0) = \frac{1}{2}$

$f(1) = 1$

$f(-1) = 1$



CVIČENÍ 4

4.1

1. $\frac{\partial}{\partial x}(xy^2 - e^x + \cos y) = y^2 - e^x$

$\frac{\partial}{\partial y}(xy^2 - e^x + \cos y) = 2xy - \sin y$

2. $\frac{\partial}{\partial x}(x^2 \cos y^2) = 2x \cos y^2$

$\frac{\partial}{\partial y}(x^2 \cos y^2) = x^2 \sin y^2 (-1) 2y = -2x^2 y \sin y^2$

3. $\frac{\partial}{\partial x} \ln \frac{x}{y} = \frac{1}{\frac{x}{y}} \cdot \frac{1}{y} = \frac{\cancel{y}}{x} \cdot \frac{1}{\cancel{y}} = \frac{1}{x}$

$\frac{\partial}{\partial y} \ln \frac{x}{y} = \frac{1}{\frac{x}{y}} \cdot \frac{x}{y^2}(-1) = -\frac{x}{y^2} \cdot \frac{y}{x} = -\frac{1}{y}$

4. $\frac{\partial}{\partial x} y^2 e^{xy} = y^2 e^{xy} \cdot y = y^3 e^{xy}$

$\frac{\partial}{\partial y} y^2 e^{xy} = 2y e^{xy} + y^2 e^{xy} x = y e^{xy}(2 + xy)$

5. $\frac{\partial}{\partial x} \frac{y-2}{x+1} = \frac{-y+2}{(x+1)^2} = \frac{2-y}{(x+1)^2}$

$\frac{\partial}{\partial y} \frac{y-2}{x+1} = \frac{1}{x+1}$

6. $\frac{\partial}{\partial x} \sin(x^2+y^2) = \cos(x^2+y^2) \cdot 2x = 2x \cos(x^2+y^2)$

$\frac{\partial}{\partial y} \sin(x^2+y^2) = \cos(x^2+y^2) \cdot 2y = 2y \cos(x^2+y^2)$

7. $\frac{\partial}{\partial x} x^2 \ln y^2 = 2x \ln y^2$

$\frac{\partial}{\partial y} x^2 \ln y^2 = x^2 \frac{1}{y^2} \cdot 2y = x^2 \frac{2}{y} = \frac{2x^2}{y}$

4.2

1. $$\frac{\partial^2}{\partial x^2} x^2y^2 - e^x + \cos y = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} x^2y^2 - e^x + \cos y\right)$$
$$= \frac{\partial}{\partial x}\left(y^2 - e^x\right) = -e^x$$

$$\frac{\partial^2}{\partial y^2} x^2y^2 - e^x + \cos y = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y} x^2y^2 - e^x + \cos y\right)$$
$$= \frac{\partial}{\partial y}\left(2xy - \sin y\right) = 2x - \cos y$$

$$\frac{\partial^2}{\partial xy} x^2y^2 - e^x + \cos y = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} x^2y^2 - e^x + \cos y\right)$$
$$= \frac{\partial}{\partial y}\left(y^2 - e^x\right) = 2y$$

$$\frac{\partial^2}{\partial yx} x^2y^2 - e^x + \cos y = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} x^2y^2 - e^x + \cos y\right) =$$
$$= \frac{\partial}{\partial x}\left(2xy - \sin y\right) = 2y$$

2. $f(x,y) = x^2\cos y^2 \qquad \frac{\partial}{\partial x} = 2x\cos y^2 \qquad \frac{\partial}{\partial y} = -2x^2 y \sin y^2$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(2x\cos y^2\right) = 2\cos y^2$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{\partial y}\left(-2x^2y\sin y^2\right) = -2x^2\sin y^2 - 2x^2y\cos y^2\, 2y = -2x^2\sin y^2 - 4x^2y^2\cos y^2 = -2x^2(\sin y^2 + 2y^2\cos y^2)$$

$$\frac{\partial^2}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(2x\cos y^2\right) = 2x\sin y^2(-1)2y = -4xy\sin y^2$$

$$\frac{\partial^2}{\partial yx} f(x,y) = \frac{\partial}{\partial x}\left(-2x^2y\sin y^2\right) = -4xy\sin y^2$$

3. $f(x,y) = \ln\frac{x}{y} \qquad \frac{\partial}{\partial x} = \frac{1}{x} \qquad \frac{\partial}{\partial y} = -\frac{1}{y}$

$$\frac{\partial^2}{\partial x^2} f(x,y) = \frac{\partial}{\partial x}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$\frac{\partial^2}{\partial y^2} f(x,y) = \frac{\partial}{y}\left(-\frac{1}{y}\right) = \frac{1}{y^2}$$

$$\frac{\partial^2}{\partial xy} f(x,y) = \frac{\partial}{\partial y}\left(\frac{1}{x}\right) = 0$$

$$\frac{\partial^2}{\partial yx} f(x,y) = \frac{\partial}{\partial x}\left(-\frac{1}{y}\right) = 0$$

4. $f(x,y)=y^2e^{xy}$ $\quad \frac{\partial}{\partial x}=y^3e^{xy}$ $\quad \frac{\partial}{\partial y}=ye^{xy}(2+xy)$

$$\frac{\partial^2}{\partial x^2}f(x,y)=\frac{\partial}{\partial x}\left(y^3e^{xy}\right)=y^3e^{xy}y=y^4e^{xy}$$

$$\frac{\partial^2}{\partial y^2}f(x,y)=\frac{\partial}{\partial y}\left(ye^{xy}(2+xy)\right)=\frac{\partial}{\partial y}\left(2ye^{xy}+xy^2e^{xy}\right)=2e^{xy}+2ye^{xy}x+x2ye^{xy}+xy^2e^{xy}x$$
$$=2e^{xy}+2xye^{xy}+2xye^{xy}+x^2y^2e^{xy}=e^{xy}(2+4xy+x^2y^2)$$

$$\frac{\partial^2}{\partial xy}f(x,y)=\frac{\partial}{\partial y}\left(y^3e^{xy}\right)=3y^2e^{xy}+y^3e^{xy}x=3y^2e^{xy}+xy^3e^{xy}=y^2e^{xy}(3+xy)$$

$$\frac{\partial^2}{\partial yx}f(x,y)=\frac{\partial}{\partial x}\left(ye^{xy}(2+xy)\right)=\frac{\partial}{\partial x}\left(2ye^{xy}+xy^2e^{xy}\right)=2ye^{xy}y+y^2e^{xy}+xy^2e^{xy}y$$
$$=2y^2e^{xy}+y^2e^{xy}+xy^3e^{xy}=y^2e^{xy}(3+xy)$$

5. $f(x,y)=\frac{y-2}{x+1}$ $\quad \frac{\partial}{\partial x}=\frac{-y+2}{(x+1)^2}$ $\quad \frac{\partial}{\partial y}=\frac{1}{x+1}$

$$\frac{\partial^2}{\partial x^2}f(x,y)=\frac{\partial}{\partial x}\left(\frac{-y+2}{(x+1)^2}\right)=\frac{(y-2)2(x+1)}{(x+1)^4}=\frac{2(y-2)}{(x+1)^3}$$

$$\frac{\partial^2}{\partial y^2}f(x,y)=\frac{\partial}{\partial y}\left(\frac{1}{x+1}\right)=0$$

$$\frac{\partial^2}{\partial xy}f(x,y)=\frac{\partial}{\partial y}\left(\frac{-y+2}{(x+1)^2}\right)=\frac{-1}{(x+1)^2}$$

$$\frac{\partial^2}{\partial yx}f(x,y)=\frac{\partial}{\partial x}\left(\frac{1}{x+1}\right)=\frac{-1}{(x+1)^2}$$

6. $f(x,y)=\sin(x^2+y^2)$ $\quad \frac{\partial}{\partial x}=2x\cos(x^2+y^2)$ $\quad \frac{\partial}{\partial y}=2y\cos(x^2+y^2)$

$$\frac{\partial}{\partial x^2}f(x,y)=\frac{\partial}{\partial x}\left(2x\cos(x^2+y^2)\right)=2\cos(x^2+y^2)+2x\sin(x^2+y^2)(-1)2x=2\cos(x^2+y^2)-4x^2\sin(x^2+y^2)$$

$$\frac{\partial}{\partial y}f(x,y)=\frac{\partial}{\partial y}\left(2y\cos(x^2+y^2)\right)=2\cos(x^2+y^2)+2y\sin(x^2+y^2)(-1)2y=2\cos(x^2+y^2)-4y^2\cos(x^2+y^2))$$

$$\frac{\partial}{\partial xy}f(x,y)=\frac{\partial}{\partial y}\left(2x\cos(x^2+y^2)\right)=2x\sin(x^2+y^2)(-1)2y=-4xy\sin(x^2+y^2)$$

$$\frac{\partial}{\partial yx}f(x,y)=\frac{\partial}{\partial x}\left(2y\cos(x^2+y^2)\right)=2y\sin(x^2+y^2)(-1)2x=-4xy\sin(x^2+y^2)$$

7. $f(x,y) = x^2 \ln y^2$ $\quad \frac{\partial}{\partial x} = 2x \ln y^2$ $\quad \frac{\partial}{\partial y} \frac{2x^2}{y}$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}(2x \ln y^2) = 2 \ln y^2$$

$$\frac{\partial^2}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{2x^2}{y}\right) = -\frac{2x^2}{y^2}$$

$$\frac{\partial^2}{\partial xy} = \frac{\partial}{\partial y}(2x \ln y^2) = 2x \frac{1}{y^2} 2y = \frac{4x}{y}$$

$$\frac{\partial^2}{\partial yx} = \frac{\partial}{\partial x}\left(\frac{2x^2}{y}\right) = \frac{4x}{y}$$

4.3

1. $f(x,y) = x^2 + y^2 - xy - 2x + y$

$\frac{\partial}{\partial x} f(x,y) = 2x - y - 2$ $\qquad \frac{\partial}{\partial y} f(x,y) = 2y - x + 1 = 0$

$2(2y+1) - y - 2 = 0$ $\qquad x = 2y + 1$

$4y + 2 - y - 2 = 0$ $\qquad x = 2 \cdot 0 + 1$

$3y = 0$ $\qquad x = 1$ $\qquad \Rightarrow SB = [1,0]$

$y = 0$

$\frac{\partial^2}{\partial x^2} = 2$ $\quad \frac{\partial^2}{\partial y^2} = 2$ $\quad \frac{\partial^2}{\partial xy} = -1$ $\quad \frac{\partial^2}{\partial yx} = -1$ $\quad \Rightarrow$ Hessova matice : $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

$\left|\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\right| = 4 - 1 = 3 \Rightarrow H = 3 > 0 \Rightarrow [1,0]$ je lokální extrém

Hessián

$\oplus \rightarrow \cup$

$\frac{\partial^2}{\partial x^2} f(x,y)_{|[1,0]} = 2 > 0 \Rightarrow$ konvexní tvar $f(x,y) \Rightarrow m\,[1,0]$

2. $f(x,y) = y^2 x + 3xy - 6y$

$\frac{\partial}{\partial x} f(x,y) = y^2 + 3y = 0$ $\qquad \frac{\partial}{\partial y} f(x,y) = 2xy + 3x - 6 = 0$

$y(y+3) = 0$ $\qquad 3x - 6 = 0$

$y = 0 \quad y = -3$ $\qquad 3x = 6$

$x = 2 \Rightarrow SB = [2,0]$

$-6x + 3x - 6 = 0$

$-3x - 6 = 0$

$3x = -6$

$x = -2 \Rightarrow SB = [-2,-3]$

$\frac{\partial^2}{\partial x^2} f(x,y) = 0 \qquad \frac{\partial^2}{\partial y^2} f(x,y) = 2x \qquad \frac{\partial^2}{\partial xy} = 2y+3 \qquad \frac{\partial^2}{\partial yx} = 2y+3$

Hessova matice: $\begin{pmatrix} 0 & 2y+3 \\ 2y+3 & 2x \end{pmatrix}$

$[2,0]$

$\left|\begin{pmatrix} 0 & 3 \\ 3 & 4 \end{pmatrix}\right| = 0-9 = -9 \Rightarrow H = -9 < 0 \Rightarrow S[2,0]$

$[-2,-3]$

$\left|\begin{pmatrix} 0 & -3 \\ -3 & -4 \end{pmatrix}\right| = 0-9 = -9 \Rightarrow H = -9 < 0 \Rightarrow S[-2,-3]$

3. $f(x,y) = 4(x-y) - x^2 - y^2$

$\frac{\partial}{\partial x} f(x,y) = 4-2x = 0$
$2x = 4$
$x = 2$

$\frac{\partial}{\partial y} f(x,y) = -4-2y = 0$
$2y = -4$
$y = -2 \quad \Rightarrow SB = [2,-2]$

$\frac{\partial^2}{\partial x^2} = -2 \qquad \frac{\partial^2}{\partial y^2} = -2 \qquad \frac{\partial^2}{\partial xy} = 0 \qquad \frac{\partial^2}{\partial yx} = 0 \Rightarrow$ Hessova matice: $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$

$[2,-2]$

$\left|\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\right| = 4-0 = 4 \Rightarrow H = 4 > 0 \Rightarrow [2,-2]$ je lokální extrém

$\ominus \rightarrow \cap$

$\frac{\partial^2}{\partial x^2} f(x,y)\big|_{[2,-2]} = -2 < 0 \Rightarrow$ konkávní tvar $f(x,y) \rightarrow M[2,-2]$

4. $f(x,y) = (2x^2-3)(y+1)$

$\frac{\partial}{\partial x} f(x,y) = 4x(y+1) = 0$
$4\sqrt{\frac{3}{2}}(y+1) = 0$
$y = -1 \rightarrow SB[\sqrt{\frac{3}{2}},-1]$
$-4\sqrt{\frac{3}{2}}(y+1) = 0$
$y = -1 \rightarrow SB[-\sqrt{\frac{3}{2}},-1]$

$\frac{\partial}{\partial y} f(x,y) = 2x^2-3 = 0$
$2x^2 = 3$
$x^2 = \frac{3}{2}$
$x = \pm\sqrt{\frac{3}{2}}$

$\frac{\partial^2}{\partial x^2} = 4(y+1) \qquad \frac{\partial^2}{\partial y^2} = 0 \qquad \frac{\partial^2}{\partial xy} = 4x \qquad \frac{\partial^2}{\partial yx} = 4x \Rightarrow$ Hessova matice: $\begin{pmatrix} 4(y+1) & 4x \\ 4x & 0 \end{pmatrix}$

$[\sqrt{\frac{3}{2}},-1]$

$\left|\begin{pmatrix} 0 & 4\sqrt{\frac{3}{2}} \\ 4\sqrt{\frac{3}{2}} & 0 \end{pmatrix}\right| = 0 - 16\frac{3}{2} = 0-24 = -24 \Rightarrow H = -24 < 0 \Rightarrow S[\sqrt{\frac{3}{2}},-1]$

$[-\sqrt{\frac{3}{2}},-1]$

$\left|\begin{pmatrix} 0 & -4\sqrt{\frac{3}{2}} \\ -4\sqrt{\frac{3}{2}} & 0 \end{pmatrix}\right| = 0 - 16\sqrt{\frac{3}{2}} = -24 \Rightarrow H = -24 < 0 \Rightarrow S[-\sqrt{\frac{3}{2}},-1]$

$5.\ f(x,y) = 2x^2 - 6xy + 5y^2 - x + 3y + 2$

$$\frac{\partial}{\partial x} f(x,y) = 4x - 6y - 1 = 0$$

$$4x - \frac{36x}{10} + \frac{18}{10} - 1 = 0$$

$$\frac{40x - 36x + 18 - 10}{10} = 0$$

$$4x + 8 = 0$$

$$4x = -8$$

$$x = -2$$

$$\frac{\partial}{\partial y} f(x,y) = -6x + 10y + 3$$

$$10y = 6x - 3$$

$$y = \frac{6x}{10} - \frac{3}{10}$$

$$y = -\frac{12}{10} - \frac{3}{10} = -\frac{15}{10}$$

$$y = -\frac{3}{2} \Rightarrow SB = [-2, -\tfrac{3}{2}]$$

$$\frac{\partial^2}{\partial x^2} = 4 \quad \frac{\partial^2}{\partial y^2} = 10 \quad \frac{\partial^2}{\partial xy} = -6 \quad \frac{\partial^2}{\partial yx} = -6 \Rightarrow \text{Hessova matice: } \begin{pmatrix} 4 & -6 \\ -6 & 10 \end{pmatrix}$$

$[-2, -\frac{3}{2}]$

$$\left|\begin{pmatrix} 4 & -6 \\ -6 & 10 \end{pmatrix}\right| = 40 - 36 = 4 \Rightarrow H = 4 > 0 \rightarrow [-2, -\tfrac{3}{2}] \text{ je lokální extrém}$$

$\oplus \rightarrow \cup$

$$\frac{\partial^2}{\partial x^2} f(x,y)\Big|_{[-2,-\frac{3}{2}]} = 4 > 0 \Rightarrow \text{konvexní tvar } f(x,y) \Rightarrow m[-2, -\tfrac{3}{2}]$$

$6.\ f(x,y) = xy(4-x-y)$

$$\frac{\partial}{\partial x} f(x,y) = y(4-x-y) - xy = 0$$

$$4y - xy - y^2 - xy = 0$$

$$4(2-\tfrac{x}{2}) - 2x(2-\tfrac{x}{2}) - (2-\tfrac{x}{2})^2 = 0$$

$$8 - 2x - 4x + x^2 - (4 - 2x + \tfrac{x^2}{4}) = 0$$

$$8 - 6x + x^2 - 4 + 2x - \tfrac{x^2}{4} = 0$$

$$4 - 4x + \tfrac{3x^2}{4} = 0$$

$$16 - 16x + 3x^2 = 0$$

$$3x^2 - 16x - 16 = 0$$

$$D: b^2 - 4ac = 16^2 - 4 \cdot 16 \cdot 3$$

$$16(16-12)$$

$$16 \cdot 4 = 64$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{16 \pm 8}{6} = \begin{cases} 4 \\ \frac{4}{3} \end{cases}$$

$$\frac{\partial}{\partial y} f(x,y) = x(4-x-y) - xy = 0$$

$$4x - x^2 - xy - xy = 0$$

$$4x - x^2 - 2xy = 0$$

$$2xy = 4x - x^2 \rightarrow x = 0$$

$$2y = 4 - x$$

$$y = 2 - \frac{x}{2}$$

$$y = 2 - \frac{4}{2} = 0 \qquad SB = [4, 0]$$

$$y = 2 - \frac{2}{3} = \frac{4}{3} \qquad SB = [\tfrac{4}{3}, \tfrac{4}{3}]$$

$$4y - y^2 = 0$$

$$y(4-y) = 0$$

$$y = 0 \qquad y = 4 \Rightarrow SB = [0,0]$$

$$SB = [0,4]$$

$\frac{\partial}{\partial x^2} = -2y \qquad \frac{\partial}{\partial y^2} = -2x \qquad \frac{\partial}{\partial xy} = 4-2x-2y \qquad \frac{\partial}{\partial yx} = 4-2x-2y$

Hessova matice: $\begin{pmatrix} -2y & 4-2x-2y \\ 4-2x-2y & -2x \end{pmatrix}$

$[0,0]$

$\left|\begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}\right| = 0-16 = -16 \Rightarrow H = -16 < 0 \Rightarrow S[0,0]$

$[0,4]$

$\left|\begin{pmatrix} -8 & -4 \\ -4 & 0 \end{pmatrix}\right| = 0-16 = -16 \Rightarrow H = -16 < 0 \Rightarrow S[0,4]$

$[4,0]$

$\left|\begin{pmatrix} 0 & -4 \\ -4 & -8 \end{pmatrix}\right| = 0-16 = -16 \Rightarrow H = -16 < 0 \Rightarrow S[4,0]$

$[\frac{4}{3}, \frac{4}{3}]$

$\left|\begin{pmatrix} -2\cdot\frac{4}{3} & 4-\frac{16}{3} \\ 4-4\frac{4}{3} & -\frac{8}{3} \end{pmatrix}\right| = \left(\frac{8}{3}\right)^2 - \left(-\frac{4}{3}\right)^2 = \frac{64}{9} - \frac{16}{9} = \frac{48}{9} \Rightarrow H = \frac{48}{9} > 0 \rightarrow [\frac{4}{3}, \frac{4}{3}]$ je lokální extrém

$\ominus \rightarrow \cap$

$\frac{\partial^2}{\partial x^2} f(x,y)\Big|_{[\frac{4}{3},\frac{4}{3}]} = -\frac{8}{3} < 0 \Rightarrow$ konkávní tvar $f(x,y) \rightarrow M[\frac{4}{3}, \frac{4}{3}]$