

Marked assignment 2

- Due date 20/12/2023

① A morphism $f: A \rightarrow B$ is said to be regular epi if there is a coequaliser diagram:

$$X \begin{array}{c} \xrightarrow{p} \\ \xrightarrow{q} \end{array} A \xrightarrow{f} B.$$

a) Prove that each regular epi is epi. (2 pts)

b) In $(\Omega, E)\text{-Alg}$, using results from the course prove that $f: A \rightarrow B$ is regular epi $\Leftrightarrow f$ is a surjective algebra homomorphism. (3 pts)

② let Ω be a signature. Prove that the forgetful functor $U: \Omega\text{-Alg} \rightarrow \text{Set}$ preserves coproducts $\iff \Omega$ is unary (ie. all its operation symbols are unary)

(4 pts)

③ let R be a ring. Show that submodules of R (viewed as a left R -module) are the same as left ideals of R .

(1 pt)