

Exercises 4

① Consider the Functor cat $[A, B]$,
whose objects are functors and
arrows are natural transformations.
Prove that $\eta: F \rightarrow G \in [A, B]$
is an isomorphism \iff
 $\eta_x: Fx \rightarrow Gx$ is an iso in B for all
 $x \in A$.

② Let $PX = \{U: U \subseteq X\}$ be the set of subsets
of X .
Extend P to a functor $\text{Set} \xrightarrow{P} \text{Set}$
& describe a natural transformation
 $1 \implies P$.

③ Consider the adjoint functors
 $\text{Vect} \xrightleftharpoons[\perp]{F} \text{Set}$
Describe a natural transformation
 $\epsilon: F U \implies \text{Id}$.

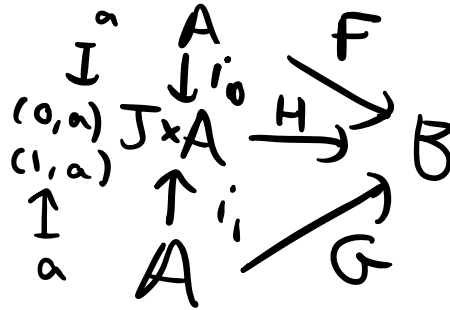
(4) - Let A, B be categories. The product category $A \times B$ has objects (a, b) where $a \in A, b \in B$ & morphisms $(a, b) \xrightarrow{(f, g)} (c, d)$ where $f: a \rightarrow c$ & $g: b \rightarrow d$.

- Let $F, G: A \Rightarrow B$ be functors, and $J = \{0 \rightarrow 1\}$.

Show that natural transformations $F \Rightarrow G$ are the same thing as functors

$$H: J \times A \rightarrow B$$

such that the diagram



commutes.