

Exercises 1 Algebra 3

① Find 3 examples of categories not in the lecture.

② - Recall a preorder $(X, \leq)$ is a set X with a binary rel $\leq$ satisfying $(x \leq x) \& (x \leq y \& y \leq z \Rightarrow x \leq z)$.
A preorder gives rise to a small category X^* with objects the elements of X & a (unique) morphism $x \rightarrow y \Leftrightarrow x \leq y$.
- Can you construct a preorder from a small category?

③ Recall that a monoid M gives rise to a 1-object category ΣM .
Can you construct monoid(s) from a small category?

④ Prove in detail that the free monoid construction $F: \text{Set} \rightarrow \text{Mon}$ described in the lecture satisfies the axioms for a functor.