Efficiency of the European labour markets: The Case of Czech Republic (A Stochastic frontier model approach)

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Abstract. This contribution aims to provide a consistent methodology to evaluate the performance of the European labour markets in the last 20 years and to reveal the most important factors influencing the efficiency of these labour markets. In particular, it focuses on the Czech labour market. Labour markets and their dynamics may be described by the standard Cobb-Douglas matching functions. Successful matches are thus treated as an output of a production process where unemployed are paired with available vacancies. Unemployment outflows are determined by the efficiency of corresponding matching process. Using stochastic frontier model approach, we estimate the efficiency of matching functions of Czech labour market, reveal the dynamics of quantified efficiency indicators and evaluate the differences among the regions. The stochastic frontier is estimated using the regional panel data set for the period 1997-2013. The model specification includes fixed effect term where individual effect terms and inefficiency terms are estimated jointly.

Keywords: matching efficiency matching function, European labour markets, Czech Republic, stochastic frontier model, panel data.

JEL Classification: J41, C23, E24
AMS Classification: 62P20

1 Introduction

Labour market dynamics is influenced by many economic and institutional factors. The institutional factors may be connected with labour market efficiency. Most of the concepts dealing with the "efficiency" approach are based on the matching function framework. In this framework, successful labour market matches are treated as an outcome of interactions between unemployed job seeker and vacancies. Quantifying the efficiency of the European labour markets belongs to the highly relevant question in the last decade. Unfortunately, using the aggregate labour market statistics for European countries does not allow us to estimate directly the corresponding efficiencies. We are able to obtain the time series of vacancies and unemployed but the successful matches are missing and cannot be recovered in an easy way. This kind of statistics must be obtained from the regional labour market statistics for each country. Panel data structure is thus rich enough to use stochastic frontiers model approach separately for country specific regional labour markets.

This contribution aims to provide a consistent methodology to evaluate and quantify the effectiveness of the Czech labour market from the view of regional labour markets. We are using the stochastic frontier panel data model approach with quarterly regional data and explicitly treated fixed effects term in the matching function model equation. On the one hand, this approach extends the previous investigations of the efficiency of the Czech labour market carried out by Němec [5], [6] or by Tvrdoň and Verner [7]. Their results have been based on the aggregate labour market statistics. On the other hand, using the data from quarterly regional labour market statistics and stochastic frontier panel data model methodology, it offers a new insight into the outcomes of the Czech labour market in the last 15 years and extends the detailed analysis of Galuščák and Münich [2] in a specific way, i.e. by dealing with efficiency issues. Stochastic frontier model approach has been used by Ilmakunnas and Pesola [4] in their study of regional labour markets in Finland. They used annual data and did not take into account explicitly possible individual fixed effects of examined regions. Gorter et al. [3] investigated the efficiency in the Dutch labour market in Netherland along the same lines. They have observed that the estimated labour market efficiency increases during the recession and recovery period while it decreases during the economic booms. This interesting feature is considered in this contribution as well.

2 Stochastic frontier model with panel data

Stochastic frontier model approach is a modern econometric tool that allows us to measure the performance of production units. Production technology is described by the production function where inputs are transformed

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into outputs of any kind. This parametric approach to measure technical inefficiency is very flexible and may be used in many applications. As for the labour market framework, the production technology of a labour market is usually described by the matching function.

2.1 Matching function and matching efficiency

Dynamic relationship between the flows of unemployed and the flows of unfilled job vacancies can be described simply by a standard production function with two inputs: the unemployed and the vacancies. New matches are thus an outcome of this matching process. In this contribution, the regional labour markets are represented by a standard Cobb-Douglas matching function in log-linear form:

$$\ln M_{it} = \alpha_i + \beta_{\log(u)} \ln U_{it} + \beta_{\log(v)} \ln V_{it} + \varepsilon_{it}\,, \tag{1}$$

where $i = 1, \ldots, N$ denotes the regions and $t = 1, \ldots, T$ the time periods. The α_i terms are fixed regional effects and ε_{it} represents stochastic factors which will be discussed below. This basic form of matching function may be extended and modified in many ways. Ilmakunnas and Pesola [4] implemented regional and labour force characteristics directly into the matching function by means of other explanatory variables. Resulting efficiency was thus a linear function of regional fixed effects and various regional characteristics. In their view, the term ε_{it} was treated purely as white noise process. Similar approach may be found in the work of Gorter et al. [3]. Galuščák and Münich [2] enhanced the basic matching function form by the flow factors (i.e. unemployment and vacancy inflows realized during the time period). Stochastic frontier model approach tries to model the stochastic term ε_{it} as consisting of combination of random variations in the matching process and the region specific inefficiency term. Regional and labour force characteristics are then implemented directly into this inefficiency term. This approach was used by Ilmakunnas and Pesola [4]. But they did not include the fixed (or random) region effects. In this contribution, we try to estimate the inefficiency of the Czech regional labour market using fixed effect panel stochastic model. This model approach is thus able to capture region specific individual effects, basic matching function characteristics and time-varying regional inefficiency terms at once.

2.2 Fixed effect panel stochastic model

To estimate matching function parameters and the inefficiency of the matching process we use the approach proposed by Wang and Ho [8]. Their specification of a stochastic frontier model is as follows:

$$y_{it} = \alpha_i + \mathbf{x_{it}\beta} + \varepsilon_{it}\,, \tag{2}$$

$$\varepsilon_{it} = v_{it} - u_{it}\,, \tag{3}$$

$$v_{it} \sim N\left(0, \sigma_v^2\right), \tag{4}$$

$$u_{it} = h_{it} \cdot u_i^*\,, \tag{5}$$

$$h_{it} = f\left(\mathbf{z_{it}\delta}\right), \tag{6}$$

$$u_i^* \sim N^+\left(\mu, \sigma_u^2\right), \qquad i = 1, \ldots, N \qquad t = 1, \ldots, T\,. \tag{7}$$

In this model framework, α_i is individual fixed effect for the unit i, $\mathbf{x_{it}}$ is a $1 \times K$ vector of explanatory variables, v_{it} is a random error with zero mean, u_{it} is a stochastic variable measuring inefficiency, and h_{it} is a positive function of a $1 \times L$ vector of non-stochastic determinants of inefficiency ($\mathbf{z_{it}}$). Constant term is excluded from explanatory variables and inefficiency determinants. It should be clear that the notation N^+ means that the realized values of the variable u_i^* are positive. In case of $\mu = 0$ the variable u_i^* follows a half-normal distribution. Wang and Ho [8] showed how to remove the fixed individual effect from the model. This procedure allows us to estimate all the model parameters. Of course, the individual effects may be recovered from the final parameter estimates. There are two possible approaches to model transformation: first-differencing and within-transformation. Both methods are equivalent (see Wang and Ho [8]). Stochastic frontier model of the Czech regional labour markets has been identified using the first-difference transformation. The main points of this methodology may be described as follows (for detailed discussion see Wang and Ho [8]).

It is necessary to define first difference of corresponding variables as $\Delta w_{it} = w_{it} - w_{it-1}$ and the stacked vector of Δw_{it} for a given i and $t = 2, \ldots, T$ is denoted as $\Delta \widetilde{w}_i = \left(\Delta w_{i2}, \Delta w_{i3}, \ldots, \Delta w_{iT}\right)'$. Assuming that the function h_{it} is not constant, i.e. the vector z_{it} contains at least one time-varying variable, the model in its first-difference form may be expressed as:

$$\Delta\tilde{y}_i = \Delta\tilde{\mathbf{x}}_\mathbf{i}\boldsymbol{\beta} + \Delta\tilde{\varepsilon}_i, \tag{8}$$

$$\Delta\tilde{\varepsilon}_i = \Delta\tilde{v}_i - \Delta\tilde{u}_i, \tag{9}$$

$$\Delta\tilde{v}_i \sim MN(0, \Sigma), \tag{10}$$

$$\Delta\tilde{u}_i = \Delta\tilde{h}_i u_i^*, \tag{11}$$

$$u_i^* \sim N^+\left(\mu, \sigma_u^2\right), \qquad i = 1, \ldots, N \tag{12}$$

It is obvious from panel data models that first-difference introduces correlations of Δv_{it} within the ith panel. The covariance matrix of the multivariate distribution of $\Delta\tilde{v}_i$ is

$$\Sigma = \begin{bmatrix} 2\sigma_v^2 & -\sigma_v^2 & 0 & \cdots & 0 \\ -\sigma_v^2 & 2\sigma_v^2 & -\sigma_v^2 & \cdots & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -\sigma_v^2 \\ 0 & 0 & \cdots & -\sigma_v^2 & 2\sigma_v^2 \end{bmatrix}. \tag{13}$$

The covariance matrix Σ has elements $2\sigma_v^2$ on the diagonal and $-\sigma_v^2$ on the off-diagonal. The key point revealed by Wang and Ho [8] is that the distribution of the term u_i^* is unaffected by the transformation. This fact helps to derive marginal log-likelihood function for each panel unit:

$$\ln L_i = -\frac{1}{2}(T-1)\ln(2\pi) - \frac{1}{2}(T-1)\ln\left(\sigma_v^2\right) - \frac{1}{2}\Delta\tilde{\varepsilon}_i'\Sigma^{-1}\Delta\tilde{\varepsilon}_i + \frac{1}{2}\left(\frac{\mu_*^2}{\sigma_*^2} - \frac{\mu^2}{\sigma_u^2}\right) + \ln\left(\sigma_*\Phi\left(\frac{\mu_*}{\sigma_*}\right)\right) - \ln\left(\sigma_u\Phi\left(\frac{\mu}{\sigma_u}\right)\right) \tag{14}$$

where

$$\mu_* = \frac{\mu/\sigma_u^2 - \Delta\tilde{\varepsilon}_i'\Sigma^{-1}\Delta\tilde{h}_i}{\Delta\tilde{h}_i'\Sigma^{-1}\Delta\tilde{h}_i + 1/\sigma_u^2} \qquad \sigma_*^2 = \frac{1}{\Delta\tilde{h}_i'\Sigma^{-1}\Delta\tilde{h}_i + 1/\sigma_u^2} \qquad \Delta\tilde{\varepsilon}_i = \Delta\tilde{y}_i - \Delta\tilde{\mathbf{x}}_\mathbf{i}\boldsymbol{\beta}. \tag{15}$$

In this expression, Φ is the cumulative density function of a standard normal distribution. Log-likelihood function of the model is obtained by summing the above function over al panel units.

Wang and Ho [8] approximated the observation specific technical inefficiency as conditional expectation

$$E\left(u_{it}\middle|\Delta\tilde{\varepsilon}_i\right) = h_{it}\left[\mu_* + \frac{\phi(\mu_*/\sigma_*)\sigma_*}{\Phi(\mu_*/\sigma_*)}\right] \tag{16}$$

evaluated at estimated values of term $\Delta\tilde{\varepsilon}_i$. This is a modified estimator of inefficiency terms which uses $\Delta\tilde{\varepsilon}_i$ instead of $\tilde{\varepsilon}_i$ as the conditional term. The main advantage is that the vector $\Delta\tilde{\varepsilon}_i$ contains all the information of individual unit in the sample and does not depend on individual effect term α_i that has the variance of higher order in case of small time dimension of the sample (variance of order $1/T$ in comparison to the variance of $1/((N-1)T)$ for the estimator $\hat{\beta}$). Technical efficiency may be obtained in accordance with other studies (see Battese and Coelli [1]) as $\exp(-u_{it})$. For derivation of individual fixed effects terms see Wang and Ho [8].

2.3 Data and model specification

The model for the Czech regional labour markets is estimated using the quarterly data set covering a sample of 14 regions from the 2nd quarter 1997 to the 2nd quarter 2013. The original data come from database of the Ministry of Labour and Social Affairs which cover the monthly data from regional Employment offices (77 districts). I used the following variables: the number of registered successful matches, M_{it}, in the corresponding month, the number of unemployed at the start of the month, U_{it}, and the number of vacancies at the start of the month, V_{it}. All the data are seasonally unadjusted. The quarterly data were transformed as monthly averages and summed up across the districts corresponding to the specific region.

Panel data set consists of 14 regions and 65 quarters. The estimated model has the form defined by the equations (2)-(7), where $y_{it} = \ln M_{it}$, $\mathbf{x_{it}} = (\ln U_{it}, \ln V_{it})$, $\boldsymbol{\beta} = \left(\beta_{\log(u)}, \beta_{\log(v)}\right)'$, and

$$h_{it} = \left|\delta_{quarter}q_t + \delta_{quarter^2}q_t^2 + \delta_{quarter^3}q_t^3 + \delta_{\Delta\log GDP}\Delta\log GDP_t + \delta_{\Delta\log INV}\Delta\log INV_t\right|.$$

The variable q represents the time trend in a year, i.e. $q = \{1,2,3,4\}$. Variables $\Delta \log GDP_t$ and $\Delta \log INV_t$ are the quarter-on-quarter growths of gross domestic product and gross fixed capital formation in the Czech Republic respectively. These statistics are obtained from quarterly national accounts provided by the Czech statistical office. We have assumed $\mu = 0$ and we have thus a half-normal representation of the model. Inefficiency terms capture time trend (which is usual in many applications; see e.g. Battese and Coelli [1]). For computational purposes, the variance parameters were parameterised as $c_v = \log \sigma_v^2$ and $c_u = \log \sigma_u^2$ respectively.

3 Efficiency estimates

The model parameters were estimated by numerically maximizing the sum of marginal log-likelihood functions (14). All the estimation procedures were performed using Matlab version 2013b and its implemented function for unconstrained maximization.

$\beta_{\log(u)}$	$\beta_{\log(v)}$	$\delta_{quarter}$	$\delta_{quarter^2}$	$\delta_{quarter^3}$	$\delta_{\Delta \log GDP}$	$\delta_{\Delta \log INV}$	$\log \sigma_v^2$	$\log \sigma_u^2$
0.470	0.058	-0.551	0.353	-0.048	3.019	-0.3478	-3.666	-0.463
(0.001)	(0.000)	(0.002)	(0.001)	(0.000)	(0.536)	(0.054)	(0.002)	(0.095)

Table 1 Parameter estimates (full sample 1997-2013, standard errors in parenthesis)

Table 1 presents the estimates of the parameters. Estimated coefficients of the Cobb-Douglas matching function, $\beta_{\log(u)}$ and $\beta_{\log(v)}$, do not confirm the empirical findings that with regional data it may be more likely to find increasing returns in matching (see Ilmakunnas and Pesola [4]). The Czech regional labour market proves the diminishing returns in matching. The elasticity of matches to vacancies is extremely low. That means that the vacancy creation is not a sufficient condition to diminish the unemployment. As the results suggests, new vacancies do not correspond with the structure of the unemployed (e.g. qualification of the unemployed). Elasticity of matching to unemployed, $\beta_{\log(u)}$, shows that approximately half of the new unemployed is able to find a new job immediately. Of course, it is only an approximation.

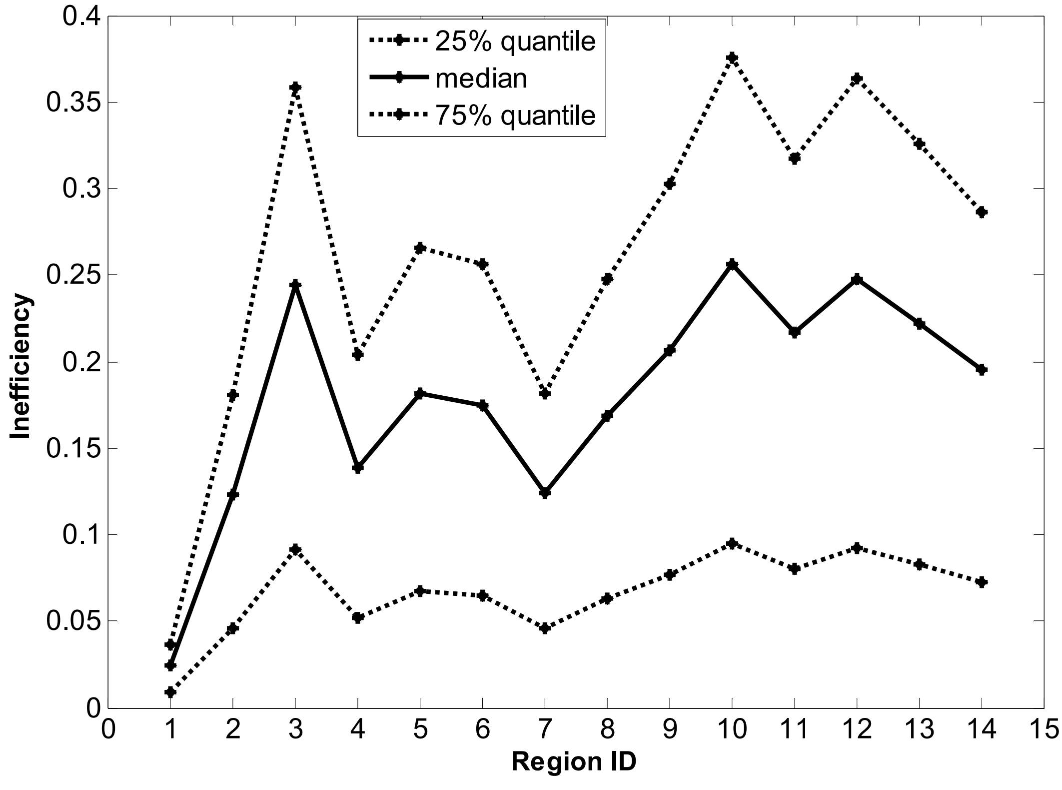

Figure 1 Inefficiency range (full sample 1997-2013)

As for the time trend parameters describing the development of the inefficiency during the period of one year, we can see that the marginal effect of the time on inefficiency is positive but decreasing. Surprisingly, the

GDP growth tends to lower matching efficiency. This may be due to fact, that the positive economic development supports the vacancies creation. But, these vacancies cannot be filled immediately or they are filled by unregistered unemployed. The reason may be in the inappropriate structure of unemployed registered at the employment office. On the other hand, the investment activities lead to the more effective matching. Investment is connected with new vacancies creation and the firms seems to search for the new workers at the employment offices. Moreover, this kind of matching may be supported by the government subsidies. Higher variability, σ_u^2, of the inefficiency term in comparison to the white noise process variability, σ_v^2, contributes to the satisfying identification of the stochastic frontier model (as stated by Wang and Ho [8]).

Figure 1 shows the interquartile range of inefficiency terms distributions for all 14 regions. The identification numbers correspond with those included in the **Table 2**. Minimum inefficiency values are almost zero for all investigated labour markets. It is this clear that all the regions are able to match the unemployed with the vacancies at the full rate. It is caused mostly by the seasonal factors. **Table 2** suggests that there are some regions with exceptionally good performance (Praha or Liberecký kraj) or bad efficiency performance (e.g. Jihočeský kraj or Vysočina). It may be surprising that some discussed regions suffering from high unemployment (e.g. Ústecký kraj) have not the less efficient labour markets. But, it should be stressed that low inefficiency does not automatically means low unemployment. It expresses the potential for new created matches which can be constituted by the interaction between unemployed and available vacancies.

ID	Region	Minimum	25% quantile	50% quantile	75% quantile	Maximum
1	Praha	0.001	0.009	0.025	0.037	0.047
2	Středočeský kraj	0.007	0.046	0.123	0.181	0.230
3	Jihočeský kraj	0.013	0.091	0.245	0.359	0.457
4	Plzeňský kraj	0.007	0.052	0.139	0.204	0.259
5	Karlovarský kraj	0.010	0.068	0.186	0.266	0.339
6	Ústecký kraj	0.010	0.065	0.175	0.256	0.326
7	Liberecký kraj	0.007	0.046	0.124	0.182	0.231
8	Královehradecký kraj	0.009	0.063	0.169	0.247	0.314
9	Pardubický kraj	0.011	0.077	0.207	0.303	0.386
10	Vysočina	0.014	0.095	0.256	0.375	0.478
11	Jihomoravský kraj	0.011	0.081	0.217	0.317	0.404
12	Olomoucký kraj	0.013	0.092	0.248	0.364	0.463
13	Zlínský kraj	0.012	0.083	0.222	0.326	0.415
14	Moravskoslezský kraj	0.010	0.073	0.195	0.286	0.364

Table 2 Regional labour markets inefficiency pattern (full sample 1997-2013)

From this point of view, the results imply that the potential of some problematic labour markets is utilized quite well. There may be an appropriate structure of unemployed and vacancies, unobserved characteristics of the unemployed support their willingness to active job search and finally, the surrounding regions may offer other possibilities for employing unemployed job applicants (this spatial dependency is not implemented in estimated models so far). The unfavourable efficiency outcomes of the regions Vysočina or Olomoucký kraj may be thus explained in a similar way.

Table 2 depicts the distribution of inefficiency terms across the Czech regions during the period from 1998 to 2012. This figure summarise the aggregate regional inefficiency changes in a straightforward way. We can observe relative stable distributions during the whole examined period. The differences across the regions diminished in 2008. These results do not indicate that the estimated labour market inefficiency may rise during the recession and recovery period while it decreases during the economic booms (see Gorter et al. [3]). Although looking at the medians of inefficiency in 2006 and 2009 may suggest some indications of this kind of behaviour.

4 Conclusion

Obtained results shows, that the stochastic frontier model approach is able to capture some interesting patterns of these labour markets controlling individual fixed effects of examined regions and possible time-varying changes in the inefficiency terms. The model estimates using the full sample display disparities in the labour market inefficiency among the regions although the distributions remain stable through the whole examined period. The low inefficiency does not necessary mean the low unemployment in the investigated regions. It will be of great importance in further research to focus on the model outcomes with region specific variables. Moreover, the spatial properties of the labour markets dynamics should be investigated, i.e. the efficiency terms should incorporate the influence of neighbouring regions.

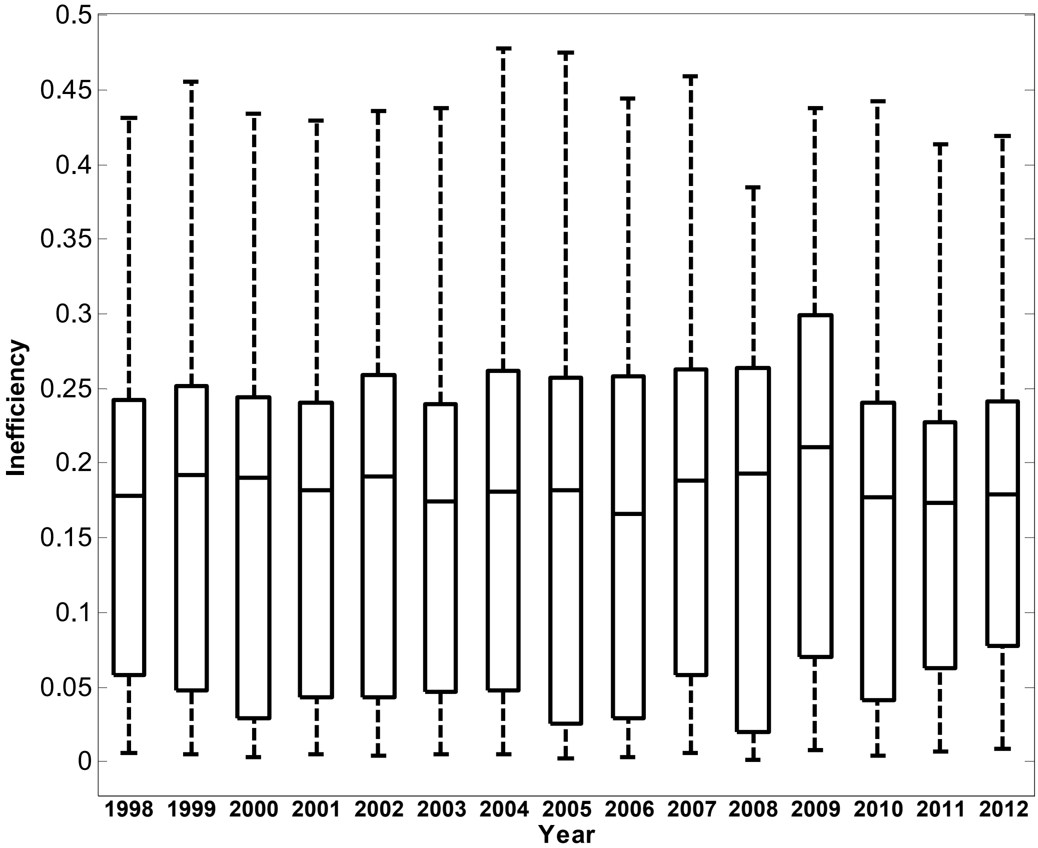

Figure 2 Inefficiency distributions among years (1998-2012)

Acknowledgements

This work is supported by funding of specific research at Faculty of Economics and Administration, project MUNI/A/0808/2013.

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