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Output gap in the Czech economy: DSGE approach

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Abstract: This paper presents three measures of the output gap estimated by a dynamic stochastic general equilibrium model of the Czech economy. We argue that the most plausible description of the business cycle provides the output gap defined as a deviation from a flexible price level of output, which is generated solely by permanent growth shocks. Our model shows that 2006-2008 overheating of the economy and the following 2008-2009 slump can be largely attributed to development in a world economy and export and import sectors, while the 2012-2013 recession was caused mainly by a combination of adverse domestic demand and cost shocks.

Keywords: DSGE model, labour market, output gap, small open economy

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1. Introduction

The Czech economy has recently gone through tumultuous changes. Impacts of the 2008-2009 global financial crisis were followed by the longest recession in the Czech history during 2012-2013. The subsequent recovery has been then characterised by the historically lowest unemployment rates, which have reached 2% at the end of 2018. In this paper, we use data that embody all these events to quantify one of the key economic indicators – an output gap. The concept of the output gap is defined as a deviation of actual output from its potential level. It can be used to identify the position of an economy in the business cycle, during macroeconomic forecasting, or for assessment of the monetary policy. The estimates of the output gap are therefore calculated regularly by policymaking institutions; see e.g., Inflation Reports of the Czech National Bank.

There are various methods on how to estimate the potential output and the output gap. The simplest way is to use some univariate time series filter such as widely used Hodrick and Prescott (1997) filter and to define the potential output as the permanent trend component of the real GDP. The main drawback is that the resulting output gap has no

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structural interpretation due to the atheoretical nature of this approach. Alternatively, one can use a partial equilibrium perspective of a production function approach and define the potential as the output consistent with a current level of technologies and normal utilisation of capital and labour. This approach allows at least for analysis of contributions of each factor to total production, see Hájková and Hurník (2007) for application on the Czech data. Even better economic interpretation can be obtained by combining the output gap with a Phillips curve relation and a multivariate filter, as is for the Czech data done by Beneš and N'Diaye (2004).

This paper aims to quantify a model-based measure of the output gap for data of the Czech economy. In contrast to the previously mentioned methods, we use a more recent approach to the output gap estimation, which is based on a fully specified New Keynesian dynamic stochastic general equilibrium (DSGE) model. At least since the seminal paper of Smets and Wouters (2003), DSGE models have exhibited comparable ability to fit the data and at to forecast as reduced-form vector autoregression models, at least for the medium-term horizon.[2] But the main strength of DSGE models stems from their ability to tell theoretically coherent stories about data development, as is stressed by Edge et al. (2008). DSGE models are derived from optimisation problems of economic agents, and they are thus more immune to the Lucas (1979) critique of reduced-form models for the lack of microfoundations. Dynamics of variables in a DSGE model, including the output gap, is driven by a set of shocks which have a clear structural interpretation.

We quantify three different measures of the output gap, which can be analysed within a New Keynesian DSGE model: the natural level gap, the efficient level gap, and the trend level gap. We argue that from these three measures the trend level output gap, defined as a deviation from a flexible price level of output which is generated only by permanent technology growth shocks, provides the most plausible description of the economy.

Our analysis is done by using a modified version of a small open economy DSGE model with a real wage rigidity, which was originally proposed by Sheen and Wang (2016). Our model and the output gaps are estimated by using information from a set of sixteen observable variables. A similar analysis on the output gap quantification within a larger-scale DSGE model was done for example by Smets and Wouters (2003) for the Euro area, by Edge et al. (2008) for the United States, and by Fueki et al. (2016) for Japan.

Vetlov et al. (2011) then present an analysis of output gaps based on DSGE models used in central banks of Hungary, Euro Area, and of the Czech Republic. A crucial difference between Vetlov et al. (2011) and our work is that the trend level flexible price output in our model is driven by permanent growth shocks identified from the data. Vetlov et al. (2011) specify the trend level flexible price output only as a simple linear trend of the

[2] On the other hand, as fittingly pointed out an anonymous referee, vector autoregression models or dynamic factor models usually outperform DSGE models for nowcasts or short-term forecasts.

real GDP. Our specification thus allows for the more flexible development of the potential, with a linear trend as a limit case.

The second published paper on the DSGE output gap estimation for the Czech economy comes from Herber and Němec (2009). These authors used a closed economy model originally proposed by Hirose and Naganuma (2010), which contains only four structural shocks and three observable variables (output, inflation, and interest rate), and the authors focus on the analysis of the natural level output gap. Our model is more complex and contains 18 structural shocks, which allow for more detailed structural interpretation of the output gap development.[3] For example, we show that the open economy shocks which are missing in the model of Herber and Němec (2009) played an important role during the 2008-2009 recession. We estimate not only the natural output gap (which essentially measures the relevance of nominal rigidities), but also the efficient level output gap (measuring the relevance of nominal rigidities and imperfect competition), and the trend level output gap (measuring the business cycles fluctuations around the growing steady state).

The remainder of this paper is organised as follows. Section 2 briefly discusses various notions of potential output and the output gap, which can be analysed within a DSGE model. Section 3 sketches a basic structure of our model. In section 4, we present estimates of the parameters and of the output gaps, investigate their cross-correlations with key macroeconomic variables and historical shock decomposition, and analyse responses of the model economy to UIP risk premium and labour market shocks. Section 5 concludes.[4]

2. Output gap measures in DSGE models

The output gap is defined as the percentage deviation of actual output from some measure of potential output. Vetlov et al. (2011, p. 9-10) and Fueki et al. (2016, p. 2-3) distinguish among three different notions of potential output, which can be analysed within a New Keynesian DSGE model:

i) The *natural* level of output that would prevail under flexible prices and wages generated by the imperfectly competitive economy, which is affected by all structural shocks from a rigid economy. The related output gap thus measures the relevance of nominal rigidities (i.e., sticky prices and wages).

ii) The *efficient* level of output that would prevail under flexible prices and wages generated by the perfectly competitive economy. The steady-state markups and markup shocks are therefore zero in this economy, but remaining shocks from rigid economy

[3] There is a general tendency to have a rich set of shocks in current DSGE models. On the other hand, empirical evidence shows that only several factors are needed to explain the business cycle frequency fluctuations in advanced economies (Andrle et al., 2017). As pointed out an anonymous referee, one thus should be very careful when interpreting data via estimated structural shocks.

[4] The technical appendix to this paper then contains a detailed description of the model's log-linearized equations, and complete lists of calibrated and estimated parameters.

affect the flexible economy. The related output gap measures the relevance of nominal rigidities and imperfect competition.

iii) The *trend* level of output (or long run natural output) that would prevail under imperfect competition with flexible prices and wages, but where the economy is affected only by the permanent technology shocks that determine its stochastic balanced growth path. The related output gap measures the business cycles component of output (i.e., all temporary fluctuations around the growing steady state) and corresponds more closely to traditional measures of the output gap, which are obtained from the univariate time series filters or production function approach.

Fueki et al. (2016, p. 2) argue that concepts of the natural and efficient output gaps are dominated by the trend level gap, at least from the practical point of view. Firstly, the trend level gap more closely corresponds to the policymakers' view that the changes in potential are driven mostly by the permanent technology progress. Secondly, this concept of output gap may, in comparison with the remaining concepts, be more informative about inflationary pressures in the economy. Finally, this concept is less sensitive to structural specification and interpretation of the model.[5] This last feature plays an important practical role since as modern New Keynesian DSGE models are becoming more complex and include more exogenous shocks, it is not straightforward to determine which shocks should affect the potential output.[6] The answer is simple in case of the trend level gap – potential output is affected only by the permanent technology growth shocks.

Another distinction in DSGE output gap literature is due to the alternative treatment of the history of state variables of the model economy. The *unconditional* potential output is computed, assuming that the prices in the flexible economy had always been flexible in the past, and the state variables are computed, taking into account this history. The *conditional* potential output is computed by assuming that prices were sticky in the past, but became flexible in the present period and are expected to remain flexible in the future; for a detailed discussion see Vetlov et al. (2011, p. 12-13). Neiss and Nelson (2003) then argue that the concept of the conditional potential output may in some situations imply some counterintuitive monetary policy recommendations, while the unconditional potential output does not. For this reason, we use the concept of the unconditional output gap in this paper.[7]

[5] For example Hirose and Naganuma (2010) in their seminal paper report that the natural level output gap is not robust to specification of the monetary policy rule.

[6] Typical problematic shocks are shocks to wage equations in Calvo sticky wage models, which can be structurally interpreted both as inefficient wage markup shocks, but also as efficient labour supply preference shocks. This problem is mentioned even in a seminal paper of Smets and Wouters (2007, p. 591).

[7] Another advantage of the unconditional output gap stems from the fact that it is in practice much easier to compute – a model builder just specifies a parallel block of equation for the flexible-price economy, and solves jointly the whole system consisting of the economy with rigidities and of the flexible-price economy.

3. Structure of the model economy

Overview

We use a modified version of the model originally proposed by Sheen and Wang (2016). It has a standard New Keynesian structure with a rich set of nominal and real rigidities such as sticky prices, rigid wages, variable capital utilisation, investment adjustment costs, and habit persistence in consumption. In particular, it combines a New Keynesian closed economy setting of Christiano et al. (2005) with small open economy features of Adolfson et al. (2007). Sheen and Wang (2016) then add the more elaborate structure of the labour market block, with explicitly modelled unemployment.

To depict some important features of the Czech economy, we introduced several modifications to the original model of Sheen and Wang (2016), which was designed for the commodity-rich Australian economy. Firstly, because of the relatively high import intensity of the Czech export sector, we modified the model structure and incorporated the import content to the production of exports. Secondly, we follow Pedersen and Ravn (2013) and use more structural specification of the foreign economy block, instead of the original VAR(1) reduced form used by Sheen and Wang (2016). This specification, together with the import intensity of exports, allows for more precise identification of open economy structural shocks, which are essential to explain the behaviour of the economy, especially before and during the 2008-2009 recession. Thirdly, we introduce the job vacancies as an additional observed variable by using a variant of the matching function, which facilitates the estimation of parameters of the hiring costs function. Fourthly, we introduce time-varying inflation target into the model, to depict changes in inflation target of the Czech National Bank. And finally, we changed the specification of the monetary policy rule to be more consistent with the inflation targeting of the Czech National Bank.

The remaining part of this section presents a sketch of our model. Its detailed description including a precise formulation of the agents' optimisation problems can be found in Sheen and Wang (2016), and a complete set of the log-linearised equilibrium conditions is provided in Appendix C to this paper.

Model

There are five types of firms operating in our model – domestic goods producers, consumption importers, investment importers, export importers, and exporters. Domestic goods firms produce using capital and labour services and sell their product to a retailer, who transforms the intermediate product into a homogeneous final good, that is sold to the households. The production function for intermediate producer i is given by

$$Y_{i,t} = z_t^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} N_{i,t}^{1-\alpha} - z_t \varphi$$

where $Y_{i,t}$ is the product of the intermediate firm, z_t is a unit-root permanent technology, ϵ_t is a stationary technology process, $K_{i,t}$ is capital service used in production, and $N_{i,t}$ is labour service used in production. Parameter α denotes capital share in production, and φ is a fixed cost scaled by the permanent technology.

The final domestic good Y_t is composed of a continuum of i differentiated intermediate goods through a constant elasticity of substitution (CES) technology

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{1}{\lambda_t^d}}\right]^{\lambda_t^d}, 1 \leq \lambda_t^d < \infty$$

where λ_t^d is the time-varying markup for the domestic goods market, which in log-linear form follows AR(1) process.

Each intermediate firm is subject to price stickiness. Following the Calvo (1983) approach, each firm faces a probability $(1 - \xi_d)$ that it can re-optimise its price. Otherwise, the firm indexes its price to the domestic inflation rate π_{t-1}^d (with weight κ_d) and to the central bank inflation target π_t^T (with weight $1 - \kappa_d$). Log-linearisation of the optimality condition for maximisation of the intermediate domestic firms' profit yields the New Keynesian Phillips curve

$$\hat{\pi}_t^d - \hat{\pi}_t^T = \frac{\beta}{1 - \beta\kappa_d} E_t[\hat{\pi}_{t+1}^d - \rho_\pi \hat{\pi}_t^T] + \frac{\kappa_d}{1 - \beta\kappa_d}[\hat{\pi}_{t-1}^d - \hat{\pi}_t^T] + \frac{\kappa_d \beta(1 - \rho_\pi)}{1 - \beta\kappa_d}\hat{\pi}_t^T + \frac{(1 - \xi_d)(1 - \beta\xi_d)}{\xi_d(1 + \beta\kappa_d)}[\hat{\lambda}_t^d - \widehat{mc}_t^d]$$

where a hat denotes log-deviation from steady state, β is households' discount factor, κ_d is indexation parameter for domestic goods and ρ_π measures persistence of inflation target. The stationarized marginal cost of the intermediate producers mc_t^d depends on the rental rate of capital r_t^k, gross nominal interest rate R_t, wage rate w_t, the labour hiring cost g_t, and on the (exogenous) growth rate of the permanent technology progress μ_t^z

$$mc_t^d = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{(r_t^k)^\alpha}{\epsilon_t} E_t(R_{t-1}w_t + g_t - \beta(1-\delta)\mu_{t+1}^z \pi_{t+1}^d g_{t+1})^{1-\alpha}$$

We now briefly describe the import and export sectors. A continuum of consumption, investment and export goods importing firms buy a homogenous foreign good at given world price and differentiate it through a CES brand naming technology. These firms are also subject to the Calvo-type of price stickiness. Following, for example, Smets and Wouters (2002) or Adolfson et al. (2007), we use the local currency pricing. Let index $a \in \{mc, mi, mx\}$ denote consumption, investment, and export-good importers. Their New Keynesian Phillips curves are given by

$$\hat{\pi}_t^a - \hat{\pi}_t^T = \frac{\beta}{1 - \beta\kappa_a} E_t[\hat{\pi}_{t+1}^a - \rho_\pi \hat{\pi}_t^T] + \frac{\kappa_a}{1 - \beta\kappa_a}[\hat{\pi}_{t-1}^a - \hat{\pi}_t^T] + \frac{\kappa_a \beta(1 - \rho_\pi)}{1 - \beta\kappa_a}\hat{\pi}_t^T + \frac{(1 - \xi_a)(1 - \beta\xi_a)}{\xi_a(1 + \beta\kappa_a)}[\hat{\lambda}_t^a - \widehat{mc}_t^a]$$

Instead of the domestically produced exports in the original Sheen and Wang (2016) model, we introduce an aggregate export good given by the CES aggregate of domestically produced export goods X_t^d and imported export goods X_t^m

$$X_t = \left[(1-\omega_x)^{1/\eta_x}(X_t^d)^{(1-\eta_x)/\eta_x} + \omega_x{}^{1/\eta_x}(X_t^m)^{(1-\eta_x)/\eta_x}\right]^{\eta_x/(1-\eta_x)}$$

where ω_x is a share of imports in exports, and η_x is the elasticity of substitution across exported goods.

The exporting firms buy the aggregate export good X_t, differentiate it through the CES brand naming technology, also subject to the Calvo price stickiness. The New Keynesian Phillips curve for exporting firm then has the same structure as for importers, only with the index $a \in \{x\}$.

There are thus five New Keynesian Phillips curves in the model, determining inflation in the domestic, imported consumption, imported investment, imported exports, and export sectors. Parameters ξ_d, ξ_{mc}, ξ_{mi}, ξ_{mx} and ξ_x then determine the degree of price stickiness in each sector. The flexible price setting environment corresponds to a situation when all these parameters are equal to zero.

Preferences of a representative household j in our model economy can be described by means of the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln\left(C_{j,t} - bC_{j,t-1}\right) - \zeta_t^N A_L \frac{N_{j,t}^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{j,t}}{z_t P_t^d}\right)^{1-\sigma_q}}{1-\sigma_q} \right]$$

where $C_{j,t}$, $N_{j,t}$, and $Q_{j,t}/P_t^d$ denote levels of aggregate consumption, labour supply and real cash holding of the jth household. Consumption is subject to habit formation measured by parameter b, constants A_L and A_q affect steady-state values of employment and real money holdings, and σ_L and σ_q are elasticity parameters. ζ_t^c and ζ_t^N are then consumption and labour supply preference shocks.

Aggregate consumption of the households is given by the CES aggregate of domestically produced goods C_t^d and imported consumption goods C_t^m

$$C_t = \left[(1-\omega_c)^{1/\eta_c} (C_t^d)^{(1-\eta_c)/\eta_c} + {\omega_c}^{1/\eta_c} (C_t^m)^{(1-\eta_c)/\eta_c}\right]^{\eta_c/(1-\eta_c)}$$

where ω_c is a share of imports in consumption, and η_c is the elasticity of substitution across consumption goods. Also total investment is given by the CES aggregate of domestic and imported investment goods (I_t^d and I_t^m), with a share of imports in investment ω_i and the elasticity of substitution η_i

$$I_t = \left[(1-\omega_i)^{1/\eta_i} (I_t^d)^{(1-\eta_i)/\eta_i} + {\omega_i}^{1/\eta_i} (I_t^m)^{(1-\eta_i)/\eta_i}\right]^{\eta_i/(1-\eta_i)}$$

The accumulation of physical capital stock of households $\widetilde{K}_t$ is described by the law of motion

$$\widetilde{K}_{t+1} = (1-\delta_k)\widetilde{K}_t + \Gamma_t \left[1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + \Delta_t$$

where $[1 - \tilde{S}(\cdot)]$ describes the capital installation technology, Γ_t is stationary investment-specific technology shock, and Δ_t permits trade of installed capital among households. The households can also change the utilisation rate of their physical capital stock $u_t = K_t/\widetilde{K}_t$ by paying the capital utilisation cost.

The optimal investment decision of households on domestic and foreign bonds yields a risk-adjusted UIP condition, whose log-linear form is given by

$$\frac{1}{\bar{R}_t}\left(\hat{R}_t - \hat{R}_t^*\right) = E_t \Delta \hat{S}_{t+1} - \tilde{\phi}_a \hat{a}_t + \tilde{\phi}_t$$

where $\hat{R}_t^*$ is the foreign interest rate, $\hat{a}_t$ denotes the net foreign asset position, $E_t \Delta \hat{S}_{t+1}$ is the expected nominal exchange rate depreciation, $\tilde{\phi}_t$ is exogenous risk premium shock, and $\tilde{\phi}_a$is the parameter.

The wage setting in the model is described by a combination of the Nash bargaining, and of the real wage rigidity. The Nash Bargaining between the households and the intermediate firms yields the Nash bargaining wage

$$w_t^* = g_t + \frac{\zeta_t^N A_L N_t^{\sigma_L}}{\psi_t^z} - \beta(1-\delta)E_t\left[\frac{\psi_{t+1}^z}{\psi_t^z \mu_{t+1}^z \pi_{t+1}^d}(1 - x_{t+1})g_{t+1}\right]$$

where ψ_t^z denotes the stationarized Lagrange multiplier from the utility maximisation problem of households, x_t is labour market tightness variable, and δ is parameter denoting the average job separation rate. The real wage rigidity is then constructed as the weighted average of the Nash bargaining wage and of the lagged real wage

$$w_t = f w_{t-1} + (1-f)w_t^*$$

where parameter f measures the degree of the rigidity.

We introduce the job vacancies into the original Sheen and Wang (2016) model through a variant of the matching function, as is suggested by Blanchard and Galí (2010)

$$\frac{V_t}{H_t} = \epsilon_t B x_t^{\vartheta} \zeta_t^x \equiv g_t$$

where V_t and H_t denote the growth rates of vacancies and hirings of the intermediate firm, ϑ is elasticity of hiring costs with respect to labour market tightness, ζ_t^x is the hiring cost shock, and parameter B determines the steady state hiring costs.

The monetary policy in the original Sheen and Wang (2016) model responded to the lagged inflation, output gap, and real exchange rate. To be more consistent with the inflation targeting of the Czech National Bank, we follow Andrle et al. (2009) and implement a regime of inflation forecast targeting into our model

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)[\hat{\pi}_t^T + \phi_\pi(E_t \hat{\pi}_{t+1}^c - \hat{\pi}_t^T)] + \varepsilon_t^R$$

where parameter ρ_R measures persistence in monetary policy, ϕ_π measures response to deviations of expected CPI inflation from the inflation target, and ε_t^R represents i.i.d. monetary policy shock.[8]

[8] Andrle et al. (2009) in the g3 model use targeting of deviations of CPI inflation from its target four periods ahead. We use only one period ahead expected inflation, because our model quite often did not satisfy the Blanchard-Kahn conditions once we used more than one perod ahead expected inflation in the monetary policy rule.

We use a semi-structural specification of the foreign economy block, instead of the reduced form VAR(1) model used in the original model of Sheen and Wang (2016). Equations of the foreign economy block are reduced-form versions of the domestic consumption Euler equation and the New Keynesian Phillips curve. The third equation is the Taylor rule. This approach allows for more precise identification of the foreign economy shocks, which are then better comparable with the domestic structural shocks. There are four foreign economy block shocks – monetary policy shock, IS curve demand shock, asymmetric permanent technology shock, and Phillips curve supply shock.

Our model contains in total eighteen structural shocks: labour supply preference shock, investment-specific technology, five markups (domestic, imported consumption, imported investment, imported exports, and exports), consumption preference, permanent technology growth, temporary technology, risk premium, hiring cost, monetary policy, inflation target, and four foreign economy block shocks.

4. Estimation

Estimation of parameters

For further analysis, we firstly stationarize all real variables in our model by the unit-root technology process z_t, log-linearise all model equations, solve the whole system numerically using Dynare and estimate its parameters by using the Bayesian approach. We use quarterly data for the Czech Republic from 2001Q1 to 2018Q4. All data were taken from the Czech National Bank ARAD database, except job vacancies and estimates of the NAIRU which were taken from the OECD database. We use the following set of sixteen observable variables: the real GDP, consumption, investment, imports and exports, the real wages, CPI inflation rate, GDP deflator, nominal interest rate (PRIBOR), the CNB's inflation target, CZK/EUR nominal exchange rate, unemployment rate, unfilled job vacancies, the euro area real GDP, CPI inflation, and nominal interest rate (EURIBOR). We map the stationarized growth rates of observable variables with variables in the theoretical model according to the measurement equations described in Appendix C. The growth rates and interest rates are stationarized by demeaning, and a stationarized measure of unemployment is constructed as the deviation of observable unemployment from the NAIRU estimate.[9]

We add the measurement errors to each observable variable. This allows us to account for possible model misspecification and also simplifies several numerical issues connected with maximisation of the model's posterior mode. We use the Kalman filter to evaluate the model likelihood, and the Metropolis-Hastings Markov Chain Monte Carlo method to perform the posterior simulations. We take two chains, each with two million draws, of which 50% draws are discarded to eliminate the impact of initial values. The variance of the candidate distribution from which we simulate the draws is set to achieve an acceptance rate around 30%. The convergence of the chains is checked using the Brooks and Gelman (1998) diagnostics.

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[9] This approach was previously used for example by Elbourne et al. (2015).

We calibrate many parameters before the estimation; their values are presented in Appendix A. These parameters are usually related to the steady-state values of observable variables and are therefore set to match the sample means. The remaining parameters of our model are estimated, including persistence and volatility of the shocks, and the measurement errors. Appendix B presents the prior distribution for these parameters, which to a large extent corresponds to values used by Ryšánek et al. (2012) and Malovaná (2015) for the Czech economy. For parameters bounded between 0 and 1, we use the beta distribution, for strictly positive parameters, we use the inverse gamma distribution, and we use the normal distribution for unbounded parameters. Complete results of the estimation are for the sake of space also presented in Appendix B.

Table 1 Estimation results

Parameter	Description	Prior mean	Posterior mean	90% HPDI	
f	real wage rigidity	0.5	0.23	0.08	0.38
ϑ	hiring cost elasticity	1	1.04	0.48	1.58
B	steady state hiring cost	1	0.53	0.21	0.89
b	habit formation	0.8	0.78	0.69	0.89
$\tilde{\phi}_a$	risk premium	0.01	0.006	0.005	0.007
	Calvo lotteries				
ξ_d	domestic	0.66	0.51	0.41	0.60
ξ_{mc}	imported consumption	0.66	0.91	0.87	0.95
ξ_{mi}	imported investment	0.66	0.51	0.40	0.62
ξ_{mx}	imported exports	0.66	0.67	0.55	0.79
ξ_x	export	0.66	0.79	0.72	0.86
	Backward indexation				
κ_d	domestic	0.33	0.12	0.02	0.21
κ_{mc}	imported consumption	0.33	0.10	0.02	0.19
κ_{mi}	imported investment	0.33	0.36	0.12	0.60
κ_{mx}	imported exports	0.33	0.40	0.17	0.62
κ_x	export	0.33	0.20	0.05	0.35
	Monetary policy				
ρ_R	interest rate smoothing	0.85	0.93	0.91	0.95
ϕ_π	inflation response	2	2.47	1.87	3.02

Source: Own computations.

We will briefly discuss some selected results from the estimation of the parameters, which are shown in Table 1.[10] The real wage persistence parameter f is estimated at 0.23, indicating quite a low degree of real wage rigidity in the Czech economy. The hiring costs elasticity ϑ is estimated close to 1, which is calibration used by Sheen and Wang (2016). Parameter B implies the point estimate of the hiring costs to GDP ratio evaluated at the steady state 0.1 per cent, which is quite small value in comparison with the estimate of 0.97 per cent obtained by Sheen and Wang (2016) for Australia.

The estimates of the sticky price Calvo lotteries ξ imply a high degree of nominal price rigidity in imported consumption and export sectors, with the average price duration of 11, respectively 5 quarters. Price stickiness in domestic, imported investment and imported exports sector is substantially lower, with the average duration between 2-3 quarters. Except for the imported consumption goods, these results are approximately consistent with the micro evidence of Murárik (2011), who found durations between 5-11 months for the whole consumption basket in the Czech economy. Interestingly, a high degree of price rigidity in imported consumption and export sectors was also found by Sheen and Wang (2016), with ξ_{mc} at 0.91 and ξ_x at 0.89.

Values of indexation parameters κ between 0.1-0.4 then imply a slightly higher relative role of indexation to the inflation target over the lagged inflation in the Phillips curves. Similar results were obtained by Sheen and Wang (2016) for Australia or by Ryšánek et al. (2012) for the Czech economy. The estimates of the monetary policy rule reveal a substantial degree of interest rate smoothing, with parameter ρ_R equal to 0.93, and that the Czech National Bank adjusts the interest rate more than proportionally in response to deviations of the expected inflation from the target.

Output gap estimates

Figure 1 plots posterior mean of the two-sided smoothed estimates of the various DSGE output gaps. For comparison, we also present the deviation of the real GDP from its HP-filtered trend (dotted line).[11] The light grey areas represent periods of a negative real GDP year-on-year growth. We present output gaps defined as the deviation of output from the permanent technology trend level of output (line with circles; the dark grey area represents its 90% highest posterior density interval), from the natural level of output (line with asterisks), and from the efficient level of output (dash-dot line).[12]

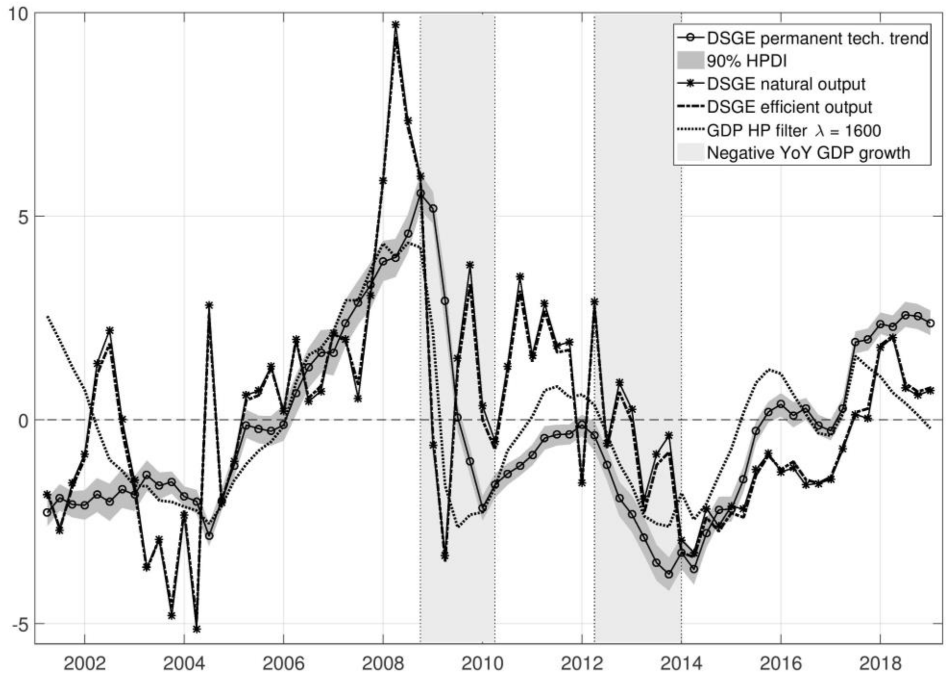
[10] The estimates presented in Table 1 and in Appendix B correspond to version of the model with the permanent trend technology output gap. Estimates of the parameters for the model with the alternative output gap measures were practically the same, and are not shown here for the sake of space.

[11] We set the smoothing parameter λ in HP filter equal to 1600.

[12] In order to practically calculate the model consistent output gaps, we expand the model equations (which were briefly presented in section 3) with their flexible price and wage versions. The flexible price setting equations are obtained by calibrating the Calvo lotteries parameters ξ and the real wage rigidity parameter f equal to 0. For estimation of the efficient level of output we also eliminate effects of imperfect competition by setting the steady state markups to one.

Visually, DSGE permanent trend output gap, natural output gap, and efficient output gap estimates follow relatively closely each other, but the permanent trend output gap appears to be much less volatile than the remaining two measures.[13] Differences between natural and efficient output gaps are rather negligible due to our quite low calibration of the steady-state markups. All three measures of the output gap indicate that the Czech economy was strongly overheating at least during the 2006-2008 period. The trend output gap peaks at 5.5% above the potential during 2008Q3. The natural output gap and the efficient output gap imply that the economy was even 9.7% above the potential, which seems unintuitively high.

Figure 1 DSGE output gaps for the Czech economy



Source: ARAD database and own computations.

All three output gap measures then interpret the beginning of the 2008-2009 crisis as the return of the overheated economy to its potential. The trend output gap got to the potential at 2009Q2, reached -2.16% two quarters later, and almost got back to the po-

[13] This (according to our opinion desirable) property is result of the fact that the trend potential output responds only to the permanent technology growth shocks, while the remaining two measures of potential react to all structural shocks.

tential at 2011Q4. The natural and efficient output gaps estimates fluctuated around zero during the whole 2009-2011 period.

The 2012-2013 recession is by all three measures identified as the period of a negative output gap. The trend output gap was decreasing since 2011Q4, reached -3.8% at 2013Q3, and returned to the potential at 2015Q2. The Czech economy is according to our estimates overheating at least since 2017Q2. Our most recent estimate of the trend output gap is 2.4% for the last quarter of 2018.

Note also that the HP-filtered output gap relatively closely follows the trend level output gap, with several notable differences. The HP filter, in comparison with the DSGE estimates, implies a much faster return to the potential after the 2012-2013 recession. Secondly, the HP-filtered output gap is positive at the beginning of our sample, while our DSGE estimates indicate that the economy was below its potential. Finally, the HP filter unintuitively implies that the Czech economy was not overheating in 2018. The last two points can be attributed to the well-known beginning and end of the sample bias of time series filters, for details see Baxter and King (1999).

Historical shock decomposition

The historical shock decomposition presented in Figure 2 allows us to discuss the main driving forces of the trend output gap.[14] The historical shock decompositions for the natural and efficient output gaps provided qualitatively very similar story,[15] and are for the sake of space not presented here. According to our results for the trend output gap, 2006-2008 overheating can be to a large extent attributed to the cost shocks (with the dominant role of import and export markups). Technology and demand shocks also contributed to the rise of output of 5.5% above the potential.

From the quantitative point of view, our model very likely overestimates the role of permanent technology shocks and thus underestimates the impact of foreign shocks before and during the 2008-2009 recession, which was imported from abroad to the Czech economy. The permanent technology in our model depicts a common growth trend of all real variables, including the foreign output, and thus captured a common slump of those variables during the Great Recession. Due to this property of our model, the estimates point to a very fast return of the economy to its potential during the first and second quarter of 2009, explained by adverse permanent technology shocks. The

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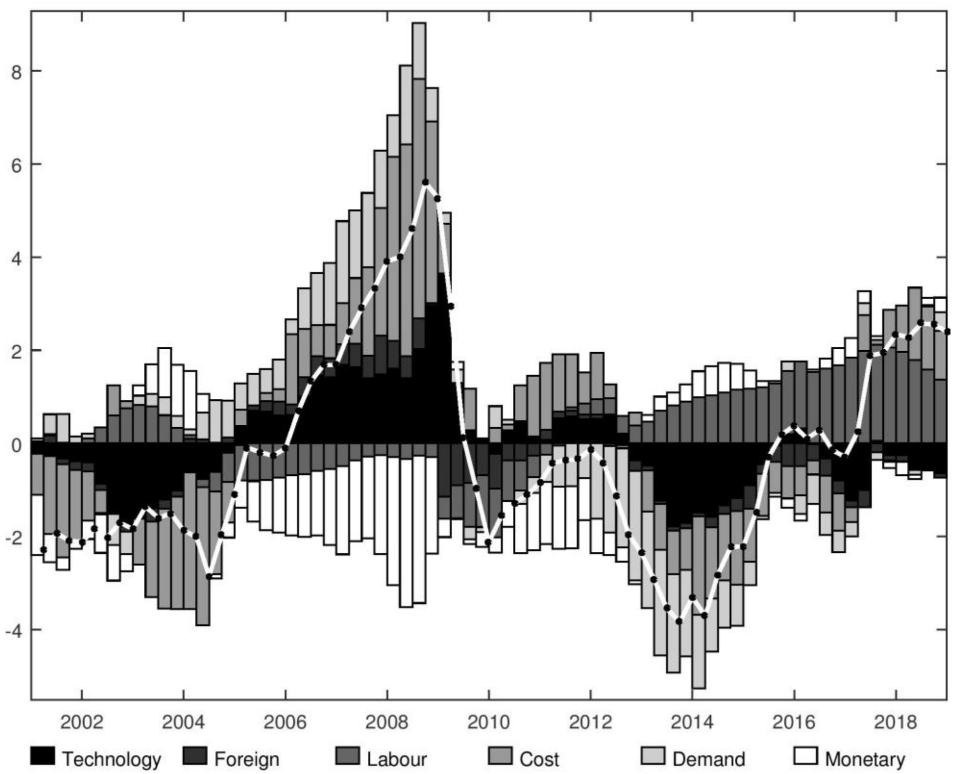
[14] White line with black dots in the figure represents our output gap estimate, and the grey rectangles represent impact of structural shocks, which are aggregated into the following six groups: Technology shock (consisting of the unit root permanent technology shock and temporary technology shock), Foreign shock (asymmetric foreign technology, foreign demand, supply, and monetary policy), Labour market shock (labour supply, hiring cost), Cost shock (domestic markup shock, imported consumption markup, imported investment markup, imported exports markup, export markup), Demand shock (consumption preference, investment shock), Monetary shock (monetary policy, UIP risk premium, inflation target and impact of initial conditions).

[15] However, there were some quantitative differences regarding a relative importance of various shocks due to a higher volatility of the natural and efficient output gap estimates. The natural and efficient level output gaps tend to hihlight a role of various markup shocks.

subsequent decline of output 2% below potential is then attributed to a sequence of the foreign economy and labour market shocks. More intuitive would be a stronger role of foreign shocks relative to technology during the 2008-2009 period, which would give us deeper cyclical development of the output gap during the 2008-2009 period.

The Czech economy got almost back to the potential at the end of 2011, but the adverse domestic demand and cost shocks (whose impact was positive until this period) bring the second recession. The recession can be attributed primarily to a restrictive fiscal policy of the former Czech government, as captured by the adverse demand (consumption and investment) shocks. Return of the economy to its potential during the year 2015 can be attributed to the labour market shocks. Since the end of 2017, also cost shocks (mainly pro-inflationary domestic markup) contributed to the rise of the Czech output 2.4% above the potential.

Figure 2 Historical shock decomposition of the trend output gap

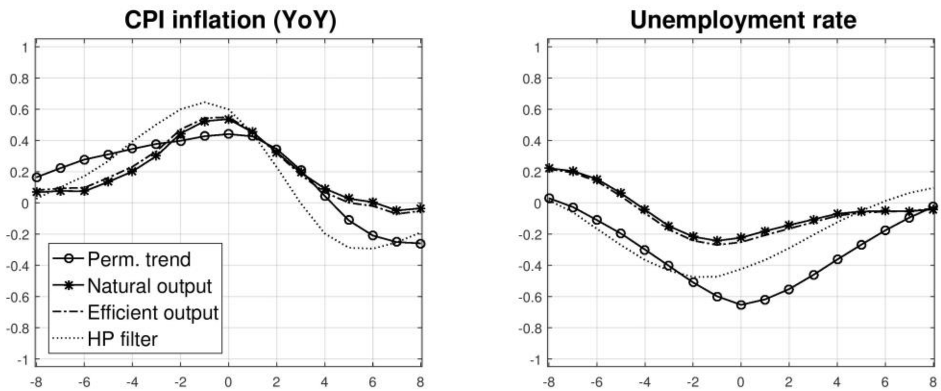


Source: ARAD database and own computations.

Cross-correlations of output gaps and data

Figure 3 presents the unconditional correlations between our estimates of the output gap and year-on-year CPI inflation and unemployment rate. Purpose of this analysis is to investigate the possible relationship of the output gap measures with other macroeconomic indicators. The results show that the output gap measures lead CPI inflation by one quarter; a similar result for the Czech economy reports Vetlov et al. (2011, p. 28). This positive correlation of inflation and output gap is also consistent with a standard New Keynesian Phillips Curve relation. Secondly, the permanent trend output gap exhibits a strong contemporaneous correlation with the unemployment rate. This highlights the importance of information from the labour market data for estimates of this measure. The HP filter gap leads unemployment by one quarter, similarly as for the natural and efficient output gaps.

Figure 3 Cross-correlations between data and output gaps[16]



Source: ARAD database and own computations.

Behaviour of the economy – impulse response functions

Finally, this section discusses the impulse responses to an orthogonal one-standard-deviation UIP risk premium (Figure 4) and labour supply preference (Figure 5) shocks of three variants of our model. In overall, the behaviour of the model is quite intuitive, especially for the trend level output gap variant.

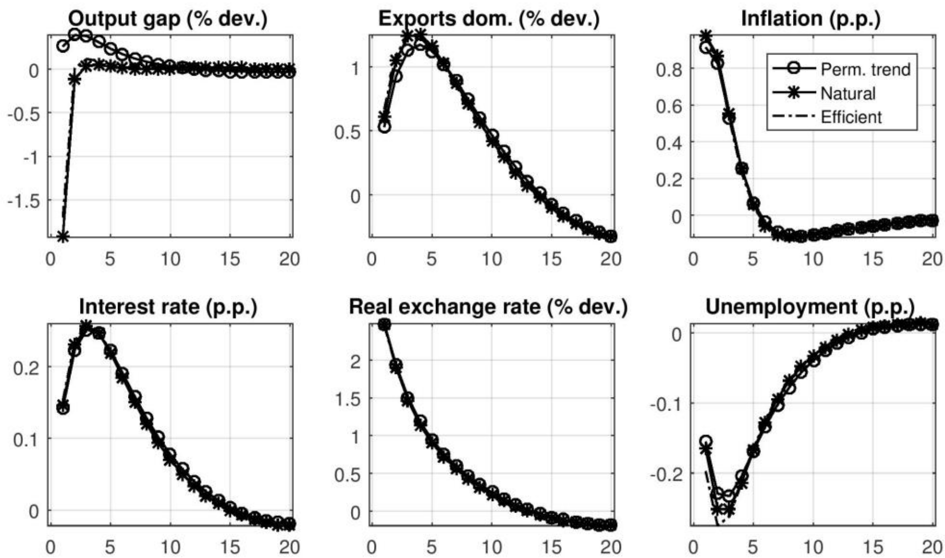
Investigation of the UIP risk premium shock is motivated by a recent period of the exchange rate commitment adopted by the Czech National Bank from November 2013 till April 2017. Its moderate positive impact on the output gap is visible in Figure 2.[17] Following the UIP risk premium shock of one standard deviation, the EUR/CZK nominal exchange rate depreciates by 2.7 percentage points (p.p.), the CPI inflation rate increases by 0.9 p.p. (quarter on quarter units, annualised), and the real exchange rate

[16] The figure shows correlations between variables in period *t*, and output gap measures in periods from *t*-8 to *t*+8.

[17] UIP risk premium shock was aggregated together with the monetary policy, inflation target and initial conditions into the Monetary shock.

depreciates almost 2.5% above its steady state. The weaker currency has a strong impact especially on domestically produced exports, which with some delay increase by 1.17%. Higher exports are at the expense of domestic consumption and especially the investment, so the trend level output gap increases only by 0.4%. The central bank should react to higher inflation by an increase in the interest rate of 0.25 p.p. Note that the natural and efficient output gaps unintuitively decrease in response to the UIP shock. It is caused by a strong contemporaneous reaction of output in the theoretical economy without rigidities (2.2%), relative to the moderate and gradual reaction of output in the economy with rigidities (0.5%).[18]

Figure 4 Impulse responses to the one-standard-deviation UIP shock



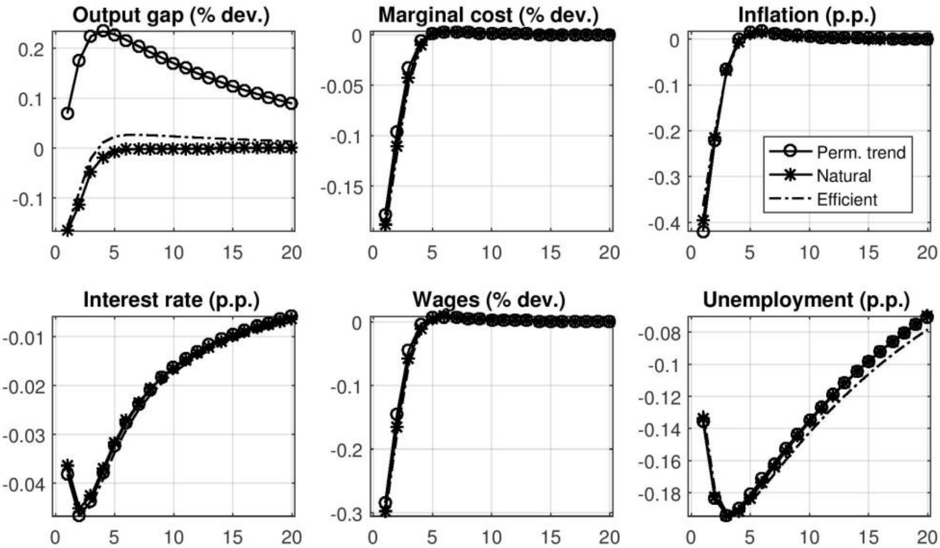
Source: Own computations.

Following a positive labour supply preference shock in Figure 5, the unemployment rate declines by 0.19 p.p. and the real wages drop by 0.3%. The wages play an important role in the domestic marginal costs, which drop by 0.19%. As a result, the inflation decreases by 0.4 p.p. (quarter on quarter, annualised). But since this change in inflation is relatively short-lived and the central bank reacts with some delay only on deviations of expected inflation from its target, the response of the interest rate is only negligible. Increased labour supply has a positive impact on consumption, investment, and exports, and gradually brings the trend level output gap 0.23% above its potential. The natural

[18] The efficient and natural output gaps are defined as difference of the output in the model economy with and without rigidities.

and efficient output gaps again unintuitively decrease in response to the shock due to a strong contemporaneous reaction of output in the economy without rigidities, relative to the output in an economy with rigidities.

Figure 5 Impulse responses to the one-standard-deviation labour supply shock



Source: Own computations.

5. Conclusion

In this paper, we quantified the output gap of the Czech economy by using a small open economy DSGE model with labour market frictions. We analysed three different concepts of the output gap, which can be constructed within a standard New Keynesian model: the natural level output gap, the efficient level output gap, and the permanent trend level output gap. We found that all three concepts of output gap provide a similar picture of the development of the Czech economy, and also exhibit a similar pattern of the dynamic cross-correlations with the inflation and unemployment rate.

Nevertheless, we think that the permanent trend output gap is the most favourable measure due to the following findings. It exhibits the lowest volatility from all three measures. From the conceptual point of view, it captures all temporary fluctuations around the steady state that growths only due to permanent technology progress. The trend output gap also provided the most intuitive description of the Czech business cycle and reacted more intuitively on the UIP and labour supply shocks. Following the UIP risk premium shock, the real exchange rate depreciates almost by 2.5%, the CPI inflation rate increases by 0.9 percentage points, and the trend level output gap increases by 0.4%. The labour supply shock decreases the unemployment rate by 0.19 percentage points and gradually brings the economy 0.23% above its potential.

We provided the structural interpretation of the development of the Czech output gap using the historical shock decomposition. Our model shows that the 2006-2008 overheating (with a peak of 5.5% above the potential) can be to a large extent attributed to the cost shocks (with the dominant role of import and export markups). Our model interprets the 2008-2009 recession primarily as the return of the overheated economy to its potential, driven by adverse technology and foreign economy shocks. By the end of the year 2009, the Czech economy was 2% below its potential according to our estimates. The 2012-2013 recession brought the economy 3.8% below potential and was caused mainly by a combination of adverse domestic demand and cost shocks, in contrary to the 2008-2009 recession. The subsequent recovery, associated with the lowest unemployment rates in Czech history, has been then driven by a positive labour market shocks. The labour market data allow for proper identification of the labour market shocks, which played a crucial role during the recovery and thus brought important information about the state of the Czech economy. This motivates development and application of DSGE models with labour market rigidities and explicitly observable unemployment data, such as ours.

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