Metric Embedding into the Hamming Space with the n-Simplex Projection

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2nd October 2019

Motivation & Preliminaries

- An efficient similarity search is nowadays necessary to process big volumes of complex data
- The similarity model: metric space (D, d)
- Transformations of the space (D, d) to Hamming space $(\{0,1\}^\lambda,\ h)$ are suitable to facilitate searching in big volumes of data
- Notation:
  - bit-strings are *sketches*
  - techniques transforming metric spaces to Hamming spaces are *sketching techniques*

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  - they require various costs of
    - transformation learning *(before the search)*
    - transformation of objects $o \in D$ to sketches
    - pre-processing of the searched dataset *(before the search)*
    - transformation of the query object $q \in D$ to the query sketch $sk(q)$ *(during the search)*

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Motivation to Propose Transformation Technique

- We propose a novel sketching technique, and we want to achieve a good trade-off between:

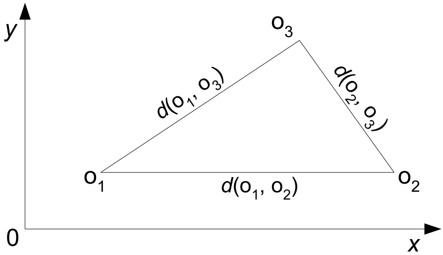
1. quality of the space approximation
2. applicability of technique
3. cost of the transformation learning
4. cost of the object transformation

Overview of the Proposed Sketching Technique

- Proposed *NSP_50* sketching technique:
  - exploits the *n-Simplex projection* to transform the given metric space to the Euclidean vector space
  - binarizes the Euclidean space to Hamming space

n-Simplex Property

- the n-Simplex projection is applicable to spaces with *n-point property*:
- the n-point property:
  - *„any n points $o_1, .., o_n \in D$ can be isometrically embedded into the $(n-1)$-dimensional Euclidean vector space"*
  - example: each metric space meets the 3-point property *(– due to the triangle inequality)*



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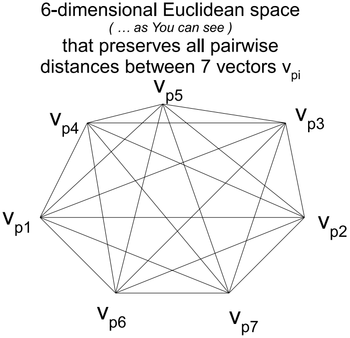
  - Example for $n = 7$: each p_i transformed to the 6-dimens. vector v_{pi}:



$$V_{p1} = (v_{11}; v_{12}; v_{13}; v_{14}; v_{15}; v_{16})$$
$$V_{p2} = (v_{21}; v_{22}; v_{23}; v_{24}; v_{25}; v_{26})$$
$$V_{p3} = (v_{31}; v_{32}; v_{33}; v_{34}; v_{35}; v_{36})$$
$$V_{p4} = (v_{41}; v_{42}; v_{43}; v_{44}; v_{45}; v_{46})$$
$$V_{p5} = (v_{51}; v_{52}; v_{53}; v_{54}; v_{55}; v_{56})$$
$$V_{p6} = (v_{61}; v_{62}; v_{63}; v_{64}; v_{65}; v_{66})$$
$$V_{p7} = (v_{71}; v_{72}; v_{73}; v_{74}; v_{75}; v_{76})$$

Figure: 7 pivots isometrically embedded into 6-dimensional Euclidean vector space

n-Simplex Projection

- (n+1)-point property guarantees it is possible to isometrically embed next object $o \in D$ while adding a dimension to the Euclidean space:
  - o is transformed to vector v_o
  - a new coordinate is added to all v_{pi} vectors
  - both is done in a way that all pairwise distances are still preserved

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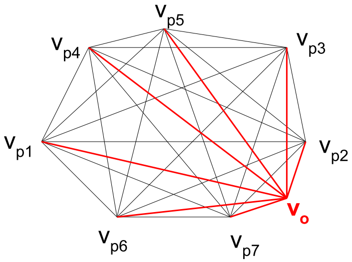

$V_{p1} = (v_{11}; v_{12}; v_{13}; v_{14}; v_{15}; v_{16}; 0)$
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$V_{p7} = (v_{71}; v_{72}; v_{73}; v_{74}; v_{75}; v_{76}; 0)$
$\mathbf{V_o = (v_1; v_2; v_3; v_4; v_5; v_6; v_7)}$

Please notice values added to vectors v_{pi} must be the same to preserve distances between these vectors

Contribution: *NSP_50* Sketching Technique

- We propose the *NSP_50* sketching technique that transforms metric spaces with the n-point property to Hamming space:
  - It selects n pivots
  - transforms all data-objects to n-dimensional Euclidean space by the n-Simplex projection
  - [1] evaluates the median value for each coordinate of vectors v_o, and binarize them:
    - sets 0 iff the value in the vector is smaller then the median
  - number of pivots n thus also defines the length of produced sketches

[1] before this step, we randomly rotate the Euclidean space to distribute the information over coordinates

Compared Sketching Techniques

- We compare the *NSP_50* technique experimentally and theoretically with other sketching techniques:
  - The *GHP_50* uses the *generalyzed hyperplane partitioning* (GHP) to split dataset into approx. halves. Each instance of the GHP determines the value of one bit in all sketches. The *GHP_50* produces sketches with low correlated bits.
  - The *BP_50* uses the *ball partitioning* (BP) to split data into halves to set values in a bit of all sketches. Also aims to produce sketches with low correlated bits.
  - The *PCA_50* use the principal component analysis to shorten vectors in the Euclidean space. Then it binarizes the vectors in a same way as *NSP_50*.

Properties of Sketching Techniques

- Proper analysis is in the paper
- The main features of sketching techniques:

*NSP*_50

- wide applicability
- good quality of space approximation
- cheap transformation learning
- λ distance computations and λ^2 flops to transform object

*GHP*_50

- very wide applicability
- still a good space approximation
- expensive transformation learning
- 2λ distance computations to transform object

*BP*_50

- very wide applicability
- very poor approximation quality when applied to complex spaces
- expectable transformation learning cost
- λ distance computations to transform object

*PCA*_50

- narrow applicability to Euclidean spaces *(could be partially extended)*
- very good space approximation
- cheap transformation learning
- $\lambda \cdot$ "*space dim*" flops to transform object

Experiments – Test Data

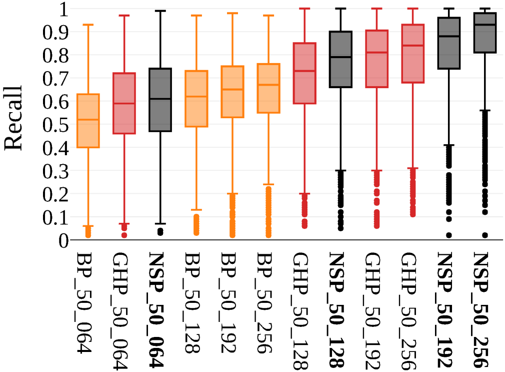
- We search for 100 nearest neighbours in 1 million datasets of image visual descriptors
  - *DeCAF* descriptors: Euclidean space of 4,096 dim. vectors extracted from the *Profiset image collection* using the Deep Convolutional Neural Network
  - SIFT descriptors from the *ANN dataset* that form the Euclidean space with 128 dimensions
  - *Adaptive-binning feature histograms* compared by the Signature Quadratic Form Distance (SQFD), extracted from the *Profiset image collection*. Each signature consists of, on average, 60 cluster centroids in a 7-dimensional space.

Experiments – Setup

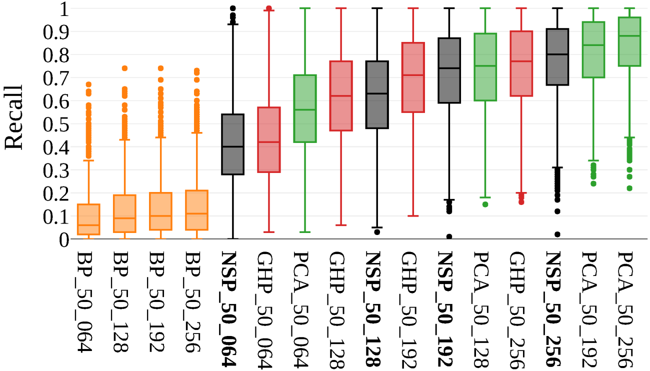
- We randomly select 1,000 query objects q for each dataset, and we compare the precise query answer with the approximate one:
- Approximate evaluation based on sketches:
  - For each q, we pre-select the *candidate set* $candSet(q)$ of 2,000 descriptors from the dataset (0.2 %) with the most similar sketches $sk(o)$ to the sketch of the query object $sk(q)$
  - We evaluate the distances $d(q, o), o \in candSet(q)$ to return 100 most similar objects from the candidate set
- Precise query answer:
  - 100 objects o from the dataset with minimum distances $d(q, o)$ to the query object q
- Comparison:
  - the recall expresses the relative size of the intersection of the approximate and the precise query answer

Experiments – Results SQFD dataset

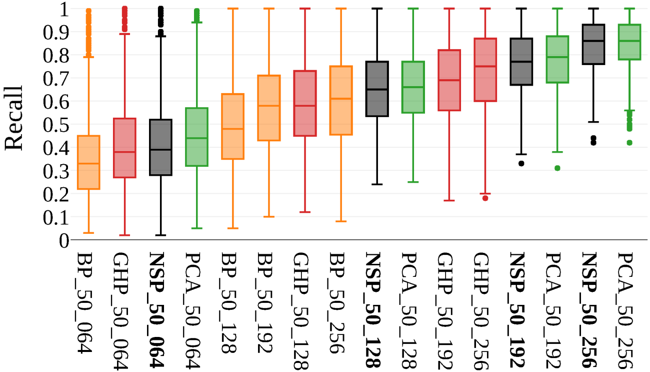
- Box-plots express the distribution of the recall values per query objects
- Suffix in the name of the sketching technique expresses the length of sketches (in bits)



Experiments – Results DeCAF dataset



Experiments – Results SIFT dataset



Conclusions

- We have proposed a novel sketching technique NSP_50
- based on the n-Simplex projection
- provides a good trade-off between applicability, quality and costs of transformation

Proper comparison

Comparison table:
http://www.nmis.isti.cnr.it/falchi/SISAP19SM.pdf