ACCELERATING METRIC FILTERING BY IMPROVING BOUNDS ON ESTIMATED DISTANCES

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METRIC FILTERING

Topic

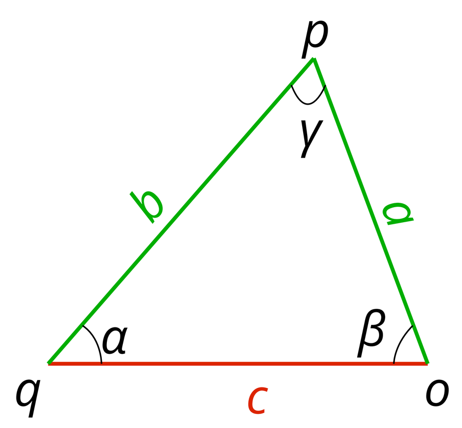
- Let q, o, p are objects in metric space (D, d)

  The upper-bound:
  $d(q,o) \leq d(q,p) + d(o,p)$

  The lower-bound:
  $d(q,o) \geq |d(q,p) - d(o,p)|$

Isometric Embedding to Euclidean Space

- The isometric embedding of q, o, p into 2D Euclidean space is always possible, → let's compute angles in the triangle
- Notation:



Tight Bounds on c?

- Two angles out of α, β, γ must be zero, and one 180° to have any of the bounds on c tight
- This practically never happens in contemporary data
- α, β, γ are always (nearly precisely) limited:

$$\Omega_{\min} \leq \alpha, \beta, \gamma \leq \Omega_{\max}$$

We enhance triangle inequalities with information about range of angles $\Omega_{\min}$, $\Omega_{\max}$ in triangles to improve the tightness of the bounds on unknown distances.

If the angles limitation $\Omega_{\min}, \Omega_{\max}$ holds, then newly derived bounds on c are correct.

NEWLY DERIVED BOUNDS ON C

New Bounds on c – Examples

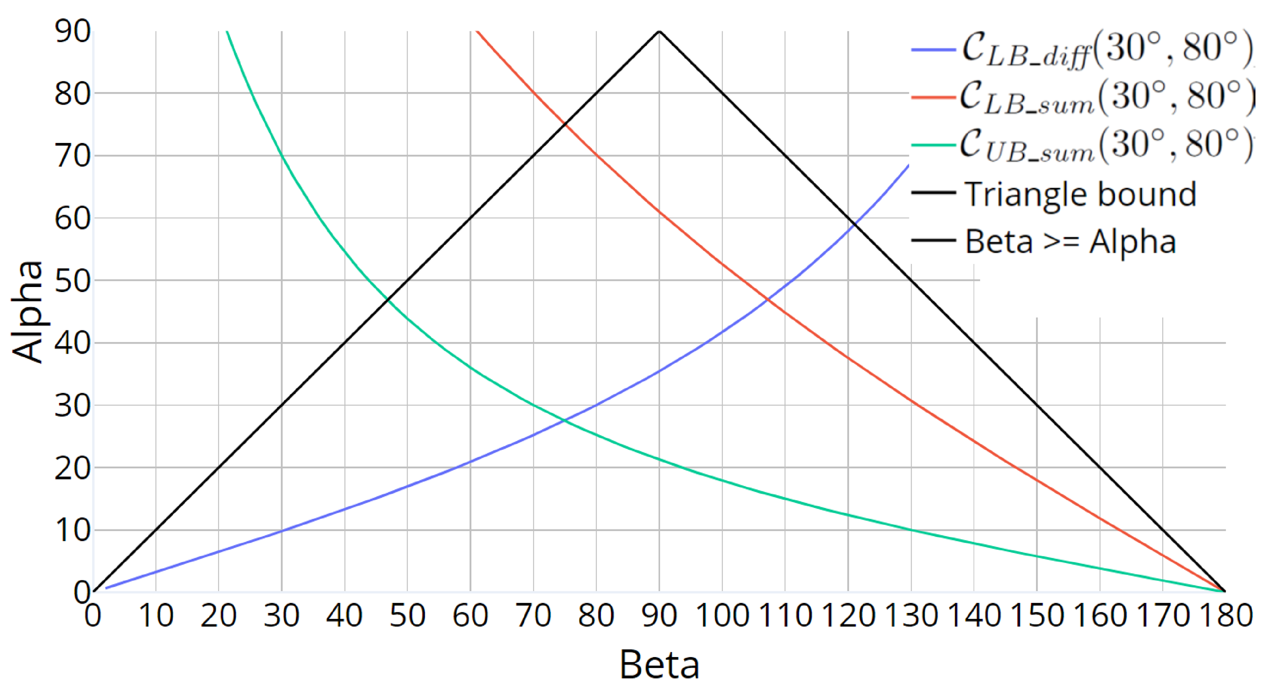
$[\Omega_{\min}, \Omega_{\max}]$	Upper-bound	Lower-bound	(Very new) lower-bound
$[0°, 180°]$	$c \leq (a+b) \cdot 1$	$c \geq \lvert a-b \rvert \cdot 1$	$c \geq (a+b) \cdot 0$
$[60°, 60°]$	$c \leq (a+b) \cdot 0.5$	*indef. exp.*	$c \geq (a+b) \cdot 0.5$
$[20°, 100°]$	$c \leq (a+b) \cdot 0.815$	$c \geq \lvert a-b \rvert \cdot 1.347$	$c \geq (a+b) \cdot 0.174$
$[20°, 80°]$	$c \leq (a+b) \cdot 0.742$	$c \geq \lvert a-b \rvert \cdot 1.532$	$c \geq (a+b) \cdot 0.174$
$[25°, 120°]$	$c \leq (a+b) \cdot 0.869$	$c \geq \lvert a-b \rvert \cdot 1.294$	$c \geq (a+b) \cdot 0.216$
$[25°, 90°]$	$c \leq (a+b) \cdot 0.752$	$c \geq \lvert a-b \rvert \cdot 1.570$	$c \geq (a+b) \cdot 0.216$
$[30°, 100°]$	$c \leq (a+b) \cdot 0.778$	$c \geq \lvert a-b \rvert \cdot 1.580$	$c \geq (a+b) \cdot 0.259$
$[30°, 80°]$	$c \leq (a+b) \cdot 0.684$	$c \geq \lvert a-b \rvert \cdot 1.938$	$c \geq (a+b) \cdot 0.259$

Consequences of Imprecise Angles Limitation $\Omega_{\min}, \Omega_{\max}$

- In practice, metric spaces guarantee just a trivial limitation $[\Omega_{\min}, \Omega_{\max}] = [0°, 180°]$
  - We still can set $[\Omega_{\min}, \Omega_{\max}]$ experimentally
  - Then, we lose the certainty that the bounds on c are correct

- We define the decomposition of all triangles into classes that correspond to the correctness of new bounds on distance c
- Each bound on c is either *'tight'*, *'correct but not tight'*, or *'wrong'*
- 15 classes of triangles exist (Some combinations do not exist)
- This allows to set $[\Omega_{\min}, \Omega_{\max}]$ to provide extremely precise bounds on c (more precisely: such that they almost do not lead to false negatives and false positives)

- Visualisation:



- x-axis expresses the size of angle β
- y-axis expresses the size of angle α
- We assume $\beta \geq \alpha$, and $\alpha + \beta \leq 180°$ without a loss of generality (see the video). Thus focus on a space below black lines
- Coloured curves assume $[\Omega_{\min}, \Omega_{\max}] = [30°, 80°]$
  - Decomposition of triangles according to the correctness of new bounds on c
  - All bounds on c are correct iff the point describing a triangle is between colour curves
- All triangles $\triangle a, b, c$ can be depicted as single points:

RESULTS

Visualisation of Real-life Data

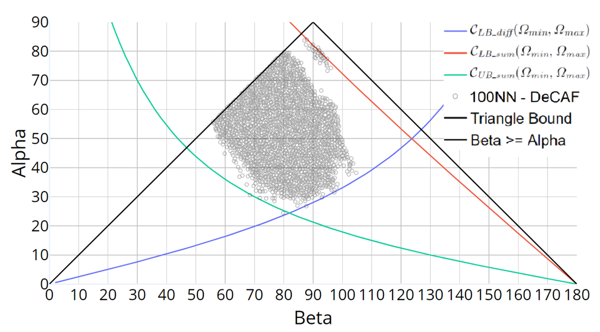


Fig.: DeCAF descriptors

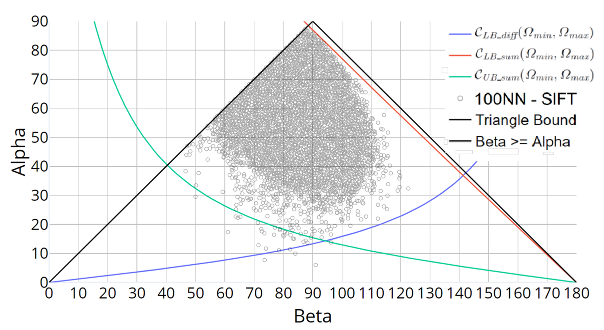


Fig.: SIFT descriptors

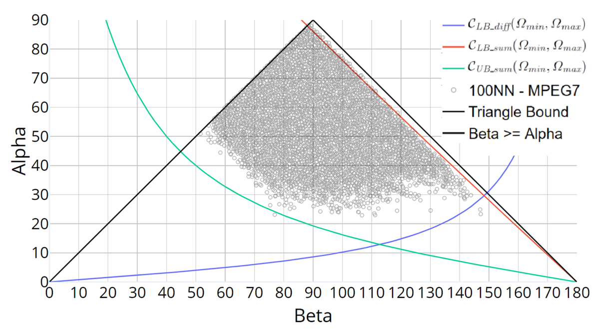


Fig.: MPEG7 descriptors

Black points: triangles sampled from the real life data, with focus on nearest neighbours.

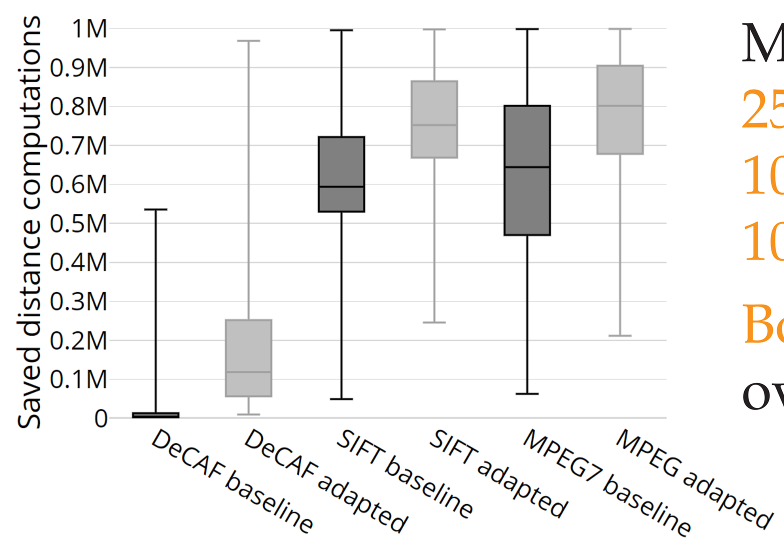
- Coloured curves visualise angles limitation $[\Omega_{\min}, \Omega_{\max}]$ used in experiments

Derived Bounds & Experiments

Selected angles limitation $[\Omega_{\min}, \Omega_{\max}]$, and new bounds on c

$[\Omega_{\min}, \Omega_{\max}]$	$[\Omega_{\min}, \Omega_{\max}]$	$[\Omega_{\min}, \Omega_{\max}]$
$[28°, 90°]$	$[8°, 86°]$	$[30°, 80°]$
$c \geq 1.66 \cdot \lvert b-a \rvert$	$c \geq 0.07 \cdot (b+a)$	$c \leq 0.68 \cdot (b+a)$
$[12°, 84°]$	$[3°, 88.5°]$	$[20°, 85°]$
$c \geq 1.26 \cdot \lvert b-a \rvert$	$c \geq 0.03 \cdot (b+a)$	$c \leq 0.76 \cdot (b+a)$
$[8°, 86°]$	$[4°, 88°]$	$[40°, 90°]$
$c \geq 1.16 \cdot \lvert b-a \rvert$	$c \geq 0.04 \cdot (b+a)$	$c \leq 0.71 \cdot (b+a)$

Increase of saved distance computations:



Metric filtering with 256 pivots, 1M data, 1000 query objects, 10NN queries.

Box-plots: values over particular query objects

Dark: pure triangle inequalities, light: new bounds

- The evaluation with new bounds is approximate. Nevertheless:
  - 999 out of 1000 answers to 10NN query contain ALL 10 NN in case of all datasets
  - The median query answer size is 10 in case of all datasets

https://disa.fi.muni.cz/~xmic/2020_09_Video_SISAP_Angles