Estimating Output Gap in the Czech Republic: DSGE Approach

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Abstract. In our contribution, we estimate the output gap that is consistent with a fully specified DSGE model. The output gap is defined in this framework as a deviation of actual output from its flexible-price equilibrium level. The flexible-price equilibrium corresponds to the state of the economy with more efficient allocation. These estimates are thus useful indicators for monetary policy. Our output gap illustrates Czech business cycles which are rather different to other estimates (e.g. HP filter). This result may be typical for economies in transition. Moreover, our results for the Czech economy show that the turning points in the gaps are accompanied by government changes.

1 Introduction

There are various methods and models to estimate the potential output and output gap. One way is to use pure statistical estimates (e.g. HP filter). Such an approach lacks of economic content. Production function method estimates an aggregate production function with equilibrium inputs. "More economic" approaches estimate potential output within the framework of the Phillips curve. These models exploit reduced-form equations and the resulting output gaps are non-accelerating inflation measure. In our contribution, we estimate the output gap (using Bayesian techniques) that is consistent with a fully specified DSGE model. The output gap is defined in this framework as a deviation of actual output from its flexible-price equilibrium level. The flexible-price equilibrium corresponds to the state of the economy with more efficient allocation. These estimates are thus useful indicators for monetary policy.

Our contribution is structured as follows. The next section provides a brief description of the basic New Keynesian DSGE model which is used for our analysis. Section 3 explains used data and estimation techniques. In Section 4, our baseline estimation results are presented and robustness of our analysis is checked. Section 5 concludes this contribution.

2 The model

The model is a simple variant of the standard New Keynesian (NK) DSGE model used by Hirose and Naganuma [5]. The model consists of optimizing households and monopolistic firms facing price stickiness. Monetary policy follows an interest-rate feedback rule. All real variables are detrended by a non-stationary trend component of the productivity process $\bar{A}_t$ with the constant growth rate γ^*. This assumption guarantees stationarity of the model. We will present only the log-linearized version of the model in this contribution. Basic equations, derivation of first-order conditions and details to the log-linearization of corresponding equations are provided by Herber [3] and Herber and Němec [4]. For further references and extensions of basic NK DSGE model see Galí [2].

Our system consists from following log-linearized equations:

$$u^*_{C,t} - y_t = E_t u^*_{C,t+1} - E_t y_{t+1} + r_t - E_t \pi_{t+1} \tag{1}$$

$$w_t - p_t = d_t + \eta n_t - u^*_{C,t} + y_t \tag{2}$$

$$u^*_{C,t} = \frac{1}{1-\beta h}\left[(1-\tau)\Big((1+\beta h^2)y_t - h y_{t-1} - \beta h E_t y_{t+1}\Big) + d_t - \beta h E_t d_{t+1}\right] \tag{3}$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\omega)(1-\omega)}{\omega}\varphi_t + \frac{1-\omega}{\omega}(z_t - \beta\omega E_t z_{t+1}) \tag{4}$$

$$y^f_t = a_t + \frac{1}{1+\eta} u^{*f}_{c,t} - \frac{1}{1+\eta} d_t \tag{5}$$

$$u^{*f}_{C,t} = \frac{1}{1-\beta h}\left[(1-\tau)\Big((1+\beta h^2)y^f_t - h y^f_{t-1} - \beta h E_t y^f_{t+1}\Big) + d_t - \beta h E_t d_{t+1}\right] \tag{6}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\Big[\psi_\pi \pi_t + \psi_y (y_t - y^f_t)\Big] + \epsilon_{r,t} \tag{7}$$

$$d_t = \rho_d d_{t-1} + \epsilon_{d,t} \tag{8}$$

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \tag{9}$$

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \tag{10}$$

Lower case letters are logarithms of the corresponding upper cases. The notation is very intuitive but all the details may be found in Herber [3] or Herber and Němec [4]. We have seven equations for endogenous variables and three equations which describe exogenous processes. The system includes four shocks – $\epsilon_{r,t}, \epsilon_{d,t}, \epsilon_{z,t}$ a $\epsilon_{a,t}$. Previous equations may be read as follows: Equation (1) is a New Keynesian IS curve (Euler equation for the consumption). Equation (2) represents labor supply curve of the households. Equation (3) express marginal utility of consumption. Equation (4) is a New Keynesian Phillips curve. Equation (5) defines potential output. Equation (6) reflects marginal utility of consumption in the case of flexible-price framework. Equation (7) is a modified Taylor rule. Equation (8) is the $AR(1)$ process of demand shock. Equation (9) is the $AR(1)$ process of cost shock. Equation (10) is the $AR(1)$ process of productivity shock. We will use the log-linearized versions of both production function and real marginal costs:

$$y_t = a_t + n_t, \tag{11}$$

$$\varphi_t = w_t - p_t - a_t. \tag{12}$$

We estimate 15 parameters. These parameters are described in the Table 1.

Table 1. Parameters

Parameter	Description	Range	Density
τ^{-1}	Intertemporal elasticity in consumption	$\mathbb{R}^+$	Gamma
β	Discount factor	$(0,1)$	Beta
h	Habit persistence	$(0,1)$	Beta
η^{-1}	Elasticity of substitution between labor and consumption	$\mathbb{R}^+$	Gamma
ω	The share of non-optimizing firms	$[0,1]$	Beta
ψ_π	Elasticity of the interest rate with respect to inflation	$\mathbb{R}^+$	Gamma
ψ_y	Elasticity of the interest rate with respect to output gap	$\mathbb{R}^+$	Gamma
ρ_r	Interest rate smoothness	$[0,1)$	Beta
ρ_d	Persistence in the demand shock	$[0,1)$	Beta
ρ_z	Persistence in the supply shock	$[0,1)$	Beta
ρ_a	Persistence in the productivity shock	$[0,1)$	Beta
σ_r	Standard deviation of the monetary shock	$\mathbb{R}^+$	Inv. Gam.
σ_d	Standard deviation of the demand shock	$\mathbb{R}^+$	Inv. Gam.
σ_z	Standard deviation of the supply shock	$\mathbb{R}^+$	Inv. Gam.
σ_a	Standard deviation of the productivity shock	$\mathbb{R}^+$	Inv. Gam.

3 Estimation

We use Bayesian techniques (Random-Walk metropolis Hasting algorithm) and Kalman filter algorithm in order to estimate both the parameters and unobserved potential output. The model is thus identified by using the Dynare toolbox for Matlab [6]. We use quarterly seasonally adjusted macroeconomic data of the Czech Republic from the second quarter 1996 to the fourth quarter 2008. Observable variables are quarter-on-quarter productivity growth, inflation and nominal interest rate. Productivity is obtained from the output growth of real GDP divided by the labor force. The inflation rate is based on CPI and interest rate is the 3M PRIBOR. Inflation and nominal interest rate are annualized. One of the alternative models uses data of the inflation target. The variables in the model are expressed as deviation from their steady-states. We have added measurement equations that relate the model variables to the data (instead of prefiltering the observed data):

$$Y_t^{obs} = \gamma^{ss} + y_t - y_{t-1}, \tag{13}$$

$$\Pi_t^{obs} = \pi^{ss} + 4\pi_t, \tag{14}$$

$$R_t^{obs} = rr^{ss} + \pi^{ss} + 4r_t, \tag{15}$$

where γ^{ss} is equilibrium growth rate of a non stationary technology shock, π^{ss} is annualized inflation at the steady-state and rr^{ss} is real interest rate at the steady-state. Priors are presented in the Table 2. They are mostly in line with

Table 2. Estimates - parameters and shocks

Parameter	Prior mean	Prior s.d.	Post. mean	90% HPDI	
τ	1.10	0.50	2.3128	1.5128	3.0692
h	0.80	0.10	0.9661	0.9317	0.9982
η	1.50	0.30	1.5442	1.0440	2.0247
ω	0.60	0.20	0.7944	0.7163	0.8774
ψ_π	1.50	0.30	1.3157	0.9985	1.5823
ψ_y	0.30	0.15	0.2706	0.0627	0.4794
ρ_r	0.65	0.25	0.7290	0.6603	0.7990
ρ_d	0.50	0.25	0.8308	0.7253	0.9414
ρ_z	0.50	0.25	0.5724	0.2029	0.9779
ρ_a	0.70	0.25	0.9336	0.8790	0.9953
γ^{ss}	0.50	0.30	0.6725	0.5067	0.8467
π^{ss}	3.50	0.50	3.6368	2.8588	4.3909
rr^{ss}	1.80	0.30	1.7559	1.2991	2.1924
σ_r	0.30	∞	0.3355	0.2771	0.3946
σ_d	0.50	∞	1.0901	0.4321	1.7915
σ_z	0.50	∞	0.4230	0.1050	0.9410
σ_a	0.80	∞	1.4366	0.8301	2.0816

the estimates of similar estimates for the Czech Republic (see Musil and Vašíček [7] or Remo and Vašíček [8]). Prior hyperparameters of π^{ss} and rr^{ss} are obtained from the historical averages of inflation and real interest rate.

4 Results

Table 2 reports posterior distributions of the structural parameters. The posterior mean of the parameter τ is 2.31, intertemporal elasticity of substitution τ^{-1} is thus 0.43. Households prefer actual consumption to future consumption. The parameter h is 0.97, i.e.past consumption plays crucial role in current decisions. The parameter η is 1.54. Elasticity for labor supply is therefore 0.65. This low elasticity is typical for the Czech labor market because of low flexibility of the labor force. Price rigidity is related to the parameter ω. The mean value 0.8 means that only 20% of firms change their price every quarter. The mean contract duration is thus five quarters.

The reaction function of the Czech National Bank is based on the parameters ρ_r, ψ_π a ψ_y. Interest rate rule may be written as

$$r_t = 0.73r_{t-1} + 0.36\pi_t + 0.07gap_t + \epsilon_{r,t}. \tag{16}$$

The smoothing parameter is relative high and plays the most important role in monetary decisions. Central bank places five times less weight on the output gap relative to inflation variations. The autoregressive coefficients for the shocks represent persistency of the shocks.

Figure 1 plots the posterior mean of smoothed estimate of the output gap and its 90% confidence intervals. This trajectory may be compared with the gap obtained using Hodrick-Prescott filter. Using U.S. data we were able to replicate the original results presented by Hirose and Naganuma [5] which are similar compared with other commonly used estimates. Our output gap, on the other hand, illustrates Czech business cycles which are rather different to the estimate using HP filter.

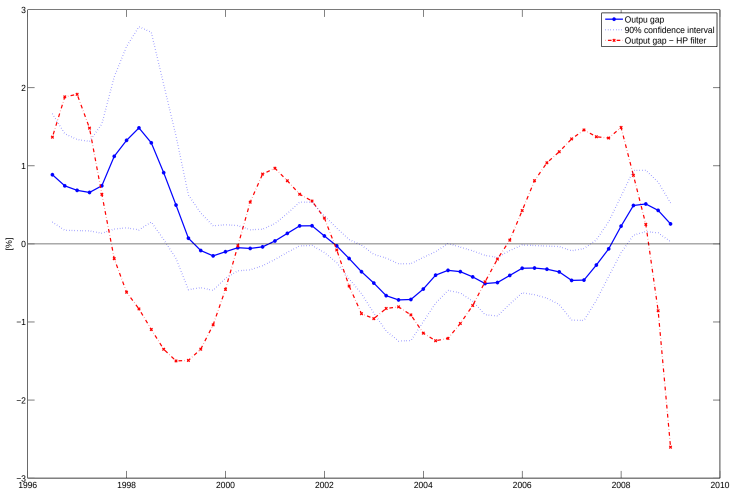

Fig. 1. Output gap and confidence intervals

Our approach shows the potential of the economy with flexible-prices that would prevail in the absence of cost shocks. We may present alternative estimates which measures output gap as the deviation of the actual output from its equilibrium (trend) level. In this case, the dynamics of the gap should be in accordance with HP filter estimates. This may be shown on the Figure 2. Resulting trajectories differ in levels because our data are detrended by steady-state growth rate of a technology shock (parameter γ_{ss}).

As argued by Woodford [9], an optimal monetary policy replicates flexible-price equilibrium, which can be the best outcome of the economy where government offsets monopolistics distortions by appropriate transfers. The output gap that is defined here should be a useful measure for monetary policy makers (from a welfare perspective). Monetary policy is able to stabilize inflation and welfare of the households (approximated by the output gap). These goals are conformable with each other.

New Keynesian approach to the output gap offers relevant economic background compared to the "traditional" estimates. Output gap is linked to real marginal costs which influence inflation within the Phillips curve framework. There is a strong relationship between output gap and inflation which may be used by monetary authority.

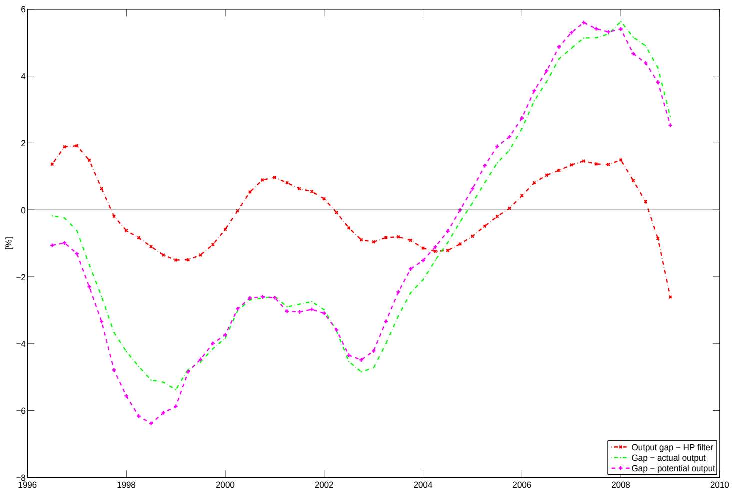

Fig. 2. Actual output gap and potential output gap

What says our output gap in the Figure 1? Its trajectory indicates the existence of rigid prices in the Czech economy. On the one hand, these rigidities have damped impact of recession in the period since 1997 and 2001 (positive output gap). On the other hand they restrain economic growth in the periods of the booms (negative output gap since 2005). Our results for the Czech economy show that the turning points in the gaps are accompanied by government changes (1998, 2002 and 2006).

4.1 Prior sensitivity and model alternatives

Our results may be influenced by the choice of prior hyperparameters. We have specified two alternative prior sets based on tight and loose prior. Our choice concerns following parameters: τ, h, η, ω, ψ_π, ψ_y, γ^{ss}, π^{ss} and rr^{ss}. In the case of "tight prior" the prior standard deviation equals a half of the base one. In the case of "loose prior" the standard deviations has been doubled. As expected, for most of the parameters, deviations of the posterior estimates from the prior means were remarkable under the loose prior. The tight prior leads the posterior estimates close to the prior. The output gap is similar to the gap under the loose prior at the beginning of the period and to the gap under loose prior in the second half of the period. The output gap under the tight prior exhibits smaller volatility due to the smaller values of h and ω. The dynamic of all trajectories is almost the same within the whole period. Our baseline estimate may be taken as a relevant "compromise" estimate.

We have made three alternative specifications of the model which may be more appropriate to describe the Czech economy. The first model (Model 1) assumes time-varying steady-state growth rate of technology shocks γ_t^{ss}. The second model (Model 2) assumes time-varying steady-state inflation π_t^{ss}. The last model (Model 3) takes into account inflation targeting. In our case, inflation gap will be defined as the difference between actual inflation and its target level.

The results are not sensitive to the changes in measurement equations. Obtained output gaps have similar dynamics in comparison with the basic model. They differ only in levels of the gaps and in the confidence intervals which are larger. Interesting results are offered by Model 3 which takes into account inflation targeting. Central bank was not able to achieve its inflation target at the beginning of the period of inflation targeting. This fact leads to higher volatility of the inflation gap in our model. The Phillips curve and the dynamics of the real marginal costs are the mechanisms through which the volatility of the output gap increases. For more detailed discussion about our alternative models and sensitivity analysis (including the trajectories of time-varying steady-state values) see Herber [3] or Herber and Němec [4].

5 Conclusion

In our contribution, we estimated the output gap which is consistent with the fully specified DSGE model. This gap is a useful measure for welfare of the economy. The output gap is a plausible indicator for monetary policy. Structural parameters may provide an economic interpretation for movements of the estimated output gap. Using U.S. data we were able to replicate the original results presented by Hirose and Naganuma [5] which are similar compared with other commonly used estimates. Our output gap, on the other hand, illustrates Czech business cycles which are rather different to other estimates (e.g. HP filter). This result may be typical for economies in transition. Rigidities in the Czech economy have damped the fall of economy in the periods of recession but they have hindered the potential growth in the periods of booms. Moreover, our results for the Czech economy show that the turning points in the gaps are accompanied by government changes.

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