

DSGE Model Sensitivity to Current Recession

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Abstract. The goal of the paper is to investigate whether the behavior of a DSGE model changes as crisis data are incorporated into the information set.

The paper comes to two major findings: a drop in the estimate of habit persistence in consumption h and a jump in the persistence of the world-wide technology shock ρ_Z when crisis data are incorporated. The main cause for the harsh changes in estimates of these parameters in the first quarter of crisis year 2009 is probably a two-percentage-points drop in Euroarea 3-months rates, which is a biggest quarter-to-quarter change in the time series ever.

Moreover, the changes are fueled by movements in foreign inflation and output growth, which (together with the interest rate) contradict the workings of the Taylor rule. In the eyes of the model, the European central bank holds its interest rates unreasonably low.

Keywords: Global Sensitivity Analysis, economic crisis, crisis data, recursive estimate, shock decomposition, RMSE analysis

JEL classification: C32, C52, E32, E43, E52, F41, F43

AMS classification: 91B51, 91B64

1 Motivation

The goal of the paper is to investigate sensitivity of a DSGE model to current recession. More concretely, the motivation of the work is, whether the behavior of the model changes as crisis data are incorporated into the information set.

The economic hypotheses questioning whether the model behavior should or should not change can be formulated in both directions. On one hand, if crisis data demonstrate the same economic linkages (albeit with more variation) as pre-crisis data, the model output should not change as crisis data are added into the information set.¹ On the other hand, if crisis data demonstrate behavior that was not apparent in pre-crisis data, the model output will likely differ.

The goal of the paper is therefore to find out which of these hypotheses suit this model and this crisis better and, if there are any changes in behavior, the aim is also to try to explain them.

Following section 2 briefly introduces the workhorse for following analyses. Section 3 continues by introducing the model with explanation of shocks in the model and the data used. Sections 4, 5 and 6 use various analytic tools to unveil the connections and changes in the model and section 7 puts these findings together to form a big picture of what happened and how it can be interpreted. Section 8 concludes and 9 opens possible discussion topics towards the linkages of the realizations of the used model and the real behavior of the European Central Bank.

All the computations were made in MATLAB with use of Dynare and GSA packages.

2 Model

The model is a small-scale Dynamic Stochastic General Equilibrium (DSGE) model following New Keynesian paradigm. Large part of the model is adopted. Original model is in Lubik and Schorfheide [4]. As authors themselves state, closest predecessor is a model of Monacelli [5].

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¹This hypothesis abstracts from trivial rise in standard deviations of shocks in question.

Model structure in Lubik and Schorfheide [4] is adjusted in this article from the original two large open economies setting to a small open economy (SOE) setting. Such setting corresponds better to the studied economies, which is the large economy of the Euro area and the small open economy of the Czech Republic.

Derivation of the model from microeconomic foundation is not introduced here because of lack of space and because most of model assumptions, derivations and linearizations are similar to existing literature.² However, since one of the assumptions (and related derivations) is not among the most usual in the existing literature, I'll try to draw attention to this topic in the next subsection.

2.1 Households

Households maximize intertemporal utility function subject to a sequence of budget constraints according to a Lagrangian

$$L = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - h\gamma C_{t-1})^{1-\tau}}{A_{W,t}} - N_t \right) \right] \\ + \Lambda_t (W_t N_t + D_t - T_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t} - E_t[Q_{t,t+1} D_{t+1}]) + \Lambda_{t+1}(\cdot) + \dots$$

where Lagrange multiplier Λ is the marginal utility of income. $C_t - h\gamma C_{t-1}$ is effective consumption under habit formation. Parameter γ is the steady state growth rate of non-stationary world-wide technology shock $A_{W,t}$. Parameter β is a discount factor, τ is the relative risk aversion, h is the habit persistence parameter. N_t is labor input, W_t is the nominal wage, $Q_{t,t+1}$ is the stochastic discount factor and D_t represents payments from a portfolio of assets so that $E_t[Q_{t,t+1} D_{t+1}]$ corresponds to the price of portfolio purchases at time t . T_t are taxes paid by the household, $P_{H,t}$ is the domestic price index, $P_{F,t}$ is price index of imported consumption goods and $C_{F,t}$ is domestic consumption of imported foreign goods.

The main differences to standard New Keynesian DSGE models like in Galí [3] is that households derive utility from effective consumption and not the consumption itself. The world-wide shock $A_{W,t}$ is also introduced in a non-standard way and the last main difference is the usage of Lagrange multiplier Λ in subsequent calculations.

2.2 Rest of the model

The model also consists of other economic agents such as producers or importers. These behave in a standard manner, i.e. they operate on monopolistically competitive markets and set their prices according to Calvo-type price setting. Producers face production function with only labor input. Another agent in the economy is the central bank that sets nominal interest rate to avoid deviations of inflation, economic growth and nominal exchange rate changes from their long run trends. Other model equations just define remaining economic entities and their relations such as the law of one price, the uncovered interest parity and other.

2.3 Linearization

This subsection presents the linearized form of the model, which is taken to data in the next sections of the paper. All variables are in log-deviations from steady state, e.g. $\tilde{x}_t = \log x_t - \log \bar{x}$. Tildes above variables are not used for higher clarity of layout.

Due to lack of space, only equations that were changed by the author are stated in this subsection. Remaining equations can be found in the original model Lubik and Schorfheide [4]. Changes made by the author to mimic SOE setting are just omissions of some terms of equation crossed by a slash (/ / /) and are mentioned in a commentary to respective equations.

Real exchange rate definition is $s_t = \psi_{F,t} - (1 - \alpha)q_t - \alpha q_t^*$ where the term $-\alpha q_t^*$ is left out in SOE setting.

²See e.g. mentioned articles Lubik and Schorfheide [4], Monacelli [5], or textbook Galí [3].

Market clearing condition is $y_{H,t} = c_t + g_t - \frac{\alpha}{\tau} s_t - \alpha(1 - \alpha)\eta(q_t / q_t^*)$ where term $-q_t^*$ is left out in SOE setting.

2.4 Linearization: Foreign economy

This subsection presents linearized equations of the foreign economy, which is a large closed economy. The number of equations is considerably lower, because large economy is by no means influenced by small economy. Main channels e.g. bilateral nominal exchange rate, terms of trade and exports from the small economy to the large economy are therefore nonexistent. Since some of the equations are not used at all, this subsection lists all relevant linearized expressions for the large foreign economy.

Forward-looking Phillips curve is $\pi^* = \beta E_t \pi_{t+1}^* + \kappa_F^* m c_t^*$, where $\kappa_F^* = \frac{1 - \theta_F^*}{\theta_F^*} (1 - \theta_F^* \beta)$ and marginal cost $m c_t^* = -\lambda_t^* + \phi q_t^* - A_t^*$. Parameter θ_F^* is Calvo parameter for foreign producers and the term $-\alpha q_t^*$ is left out in SOE model setting.

Marginal utility of income is natural counterpart of domestic economy equation $-\lambda_t^* = \frac{\tau}{1 - h\beta} c c_t^* - \frac{h\beta}{1 - h\beta} E_t [\tau c_{t+1}^* + z_{t+1}]$, with effective consumption $(1 - h) c c_t^* = c_t^* - h c_{t-1}^* + h z_t$. Market clearing is much simpler due to SOE setting: $y_{F,t}^* = c_t^* + g_t^* + \frac{\phi}{\tau} q_t^* / \alpha(1 - \alpha)\eta(q_t / q_t^*)$

Monetary policy rule for the foreign economy is without link to exchange rate: $r_t^* = \rho_R^* r_{t-1}^* + (1 - \rho_R^*) [\psi_1^* \pi_t^* + \psi_2^* (\Delta y_t^* + z_t) + \psi_3^* \Delta e_t]$.

3 Shocks & data

The model contains 8 shocks, 5 of them are introduced as AR1 processes $(z_t, A_t, A_t^*, g_t, g_t^*)$ and three are direct innovations (to Taylor rules and nominal depreciation equation)

The model also contains 7 observable variables. These are output growth, inflation and nominal interest rate for domestic and foreign economies and depreciation rate for domestic economy. All data are in quarterly frequency. Interest rates are per annum, output growth and inflation are annualized and nominal exchange rate depreciation is per quartal. The domestic economy is represented by the Czech economy, foreign economy is Euroarea with 12 countries (this aggregation was chosen mainly due to the completeness of historical time series in the Eurostat database).

Figure 1 depicts used time series together with a Hodrick-Prescott (HP) trend, which is used to detrend the time series so that the model is in gap form. Time notation is such that e.g. the first quarter of 2009 (2009q1) corresponds to the value 2009.25 on the x-axis and the fourth quarter of 2009 (2009q4) corresponds to the value 2010.00. Full data sample spans from 1996q2 to 2009q4. First used observation

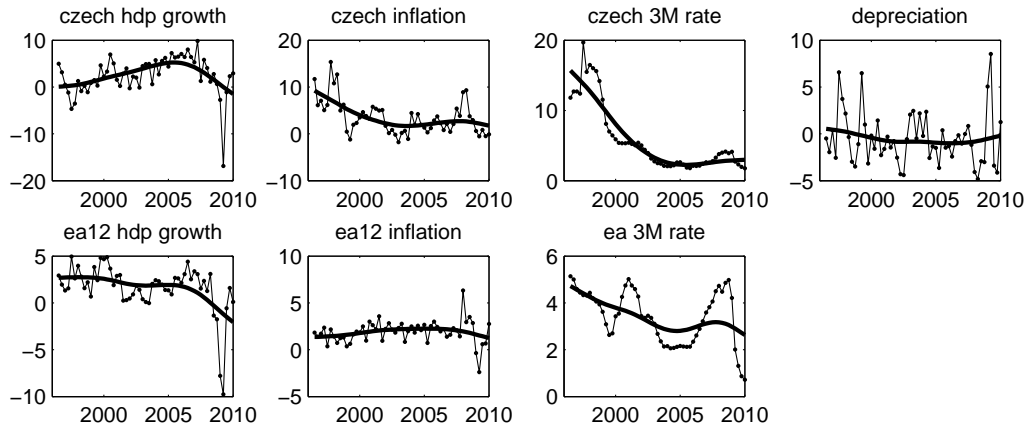


Figure 1: Data time series (observable variables). Black solid line is HP trend.

is 1998q2 (partly due to presample analysis used to construct priors for Monte Carlo estimation, partly due to incorporation of inflation targeting in 1998, partly due to time series variation).

4 Recursive estimates

This subsection introduces the most important results of the recursive estimate analysis, which are depicted in Figure 2. Black central line in Figure 2 represents the value of parameter posterior mean. More concretely, each point in time t is an estimate of the parameter on data from 1998q2 to t . Dash-dotted lines are 95% confidence bands of parameter estimates. Greyed area emphasizes period of crisis (since 2008q1).

Number of observations changes from 19 to 47, which means 29 "recursive" estimates of the whole model. Ends of sample vary from 2002q4 (19 observations) to 2009q4 (47 observations). Four selected parameters are the ones with the largest changes in the posterior mean during time.

4.1 Summary for $\sigma_\epsilon^{\Delta e}$ and θ_F^*

First panel in Figure 2 depicts the evolution of the estimate of parameter $\sigma_\epsilon^{\Delta e}$ which is a standard deviation of the innovation to the nominal exchange rate depreciation equation. Upward movement of the estimate of this parameter is explained by higher variation in nominal exchange rate during crisis. As it can be seen in Figure 1 panel 4, Czech economy underwent serious depreciation shock. The link between variation in nominal exchange rate and this parameter is almost direct, since nominal exchange rate movements are almost unexplained by the model and most of the variation comes from innovation to the equation. This fact can be observed in the left panel of Figure 4 and is also mentioned in Lubik and Schorfheide [4] in the sense that this generation of small-scale New Keynesian models do not explain variation in nominal exchange rates.

Second panel of Figure 2 depicts movements in the estimate of Calvo parameter of foreign producers θ_F^* . The estimate of the parameter drops from approx. 0.55 prior to crisis to approx. 0.45 in the beginning of the crisis due to great variation in inflation (see panel 6 in Figure 1). The link is intuitive since major changes in inflation push firms to change their prices more often which is exactly what Calvo parameter shows.

The estimate of the parameter moves back to 0.6 in 2009q1 which is the date when the biggest drop in economic activity occurred. This movement is hard to explain per se, because many other changes took place in 2009q1.

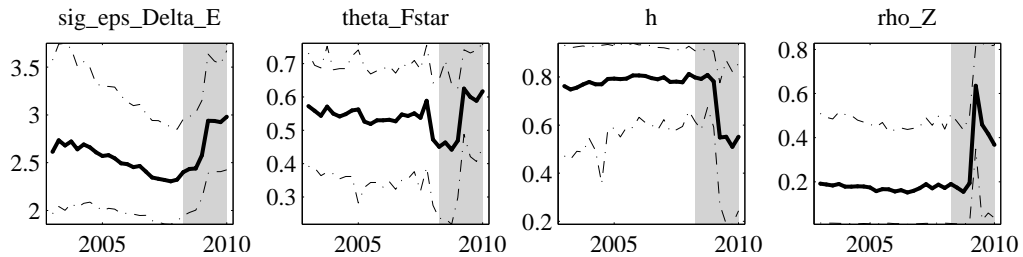


Figure 2: Recursive parameter estimates. Solid line = posterior mean. Dash-dotted line = 95% conf. bands.

4.2 Summary for h and ρ_Z

Third and fourth panels in Figure 2 are interconnected in joint explanation. Panel 3 shows a steep drop of habit persistence parameter from 0.8 to below 0.6 – this means that crisis data presume lower persistence in consumption. Panel 4 shows even more radical change in parameter estimate. The persistence of the world-wide technology shock jumped up from 0.2 to over 0.6, which would mean that crisis data prefer higher persistence in this kind of shock.

In order to see the whole picture of what happened in the crisis and why it happened, we need to use also other tools. One of them is Global Sensitivity Analysis (GSA) [7], another one is the shock decomposition.

5 Global Sensitivity Analysis

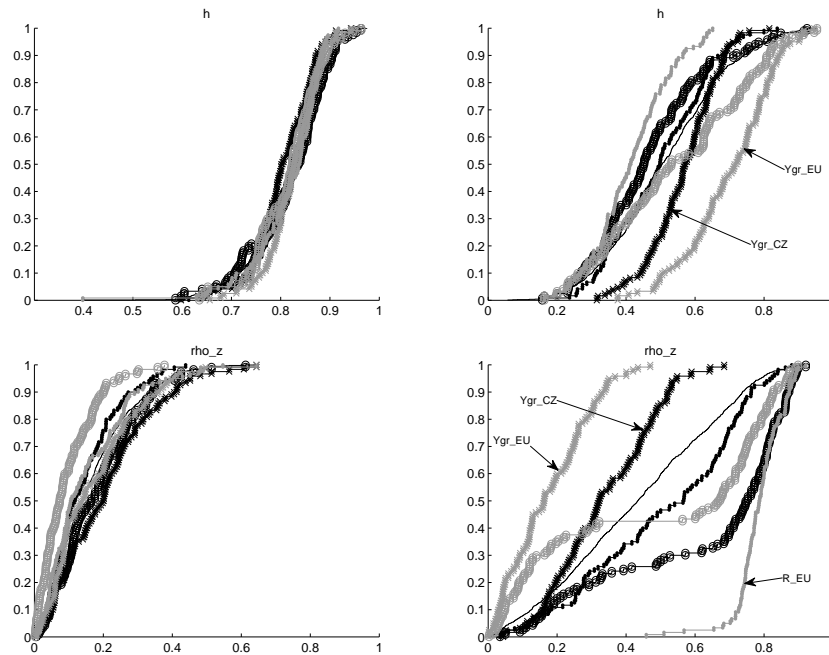


Figure 3: Root Mean Squared Error analysis (trade-offs for fit), the lines are cumulative distribution functions (CDFs) of the filtered samples corresponding to the best fit for each singular observed series. Thin solid line is posterior CDF. Other black lines are for the Czech economy, all gray lines are for the foreign economy. Economy growth rates are marked with a cross (\times), inflation rates are marked with a circle (\circ) and interest rates are marked with a bullet (\bullet). Left panels: data set without crisis data. Right panels: data set with crisis data.

This section utilizes GSA toolbox [7], concretely the Root Mean Squared Error (RMSE) analysis. This analysis is described (with examples of use) in Saltelli [8] and [9], with more economic background in Ratto [6] and with greater detail in Čapek [1]. In short, the lines in Figure 3 denote cumulative distribution functions (CDFs) that represent the values of the parameter(s) that ensure the best possible fit for variable of concern.

Left panels in Figure 3 show CDFs for data set truncated before current economic crisis, these panels thus show the behavior of the estimated model prior to crisis. Apparently, CDFs overlap perfectly (in case of h , NW panel) or tightly (in case of ρ_Z , SW panel). In other words, in order to obtain the best possible fit of the six observed variables, the estimates of parameters h and ρ_Z should simply stay where they are (at their posterior means). The model estimated on the truncated data set presents therefore no trade-offs for fit.

On the other hand, right panels show almost no overlap of CDFs, which correspond to a model with full data set that include crisis data. The biggest trade-offs are marked with a text arrow. The northeastern panel presents trade-offs for parameter h : There are two curves that are visibly shifted away from the others. These are marked with gray and black with \times s, which – according to the figure description – means that these are CDFs for domestic and foreign output growth. In other words, these observable variables would fit data better if the posterior mean of habit persistence parameter h was higher. This situation also presents a trade-off: most observable variables prefer rather lower values of parameter h (at its posterior mean), whereas domestic and foreign output growth prefer higher value of h .

The southeastern panel displays trade-offs for persistence in the world-wide technology shock ρ_Z . The trade-offs are even higher than in the case of habit parameter h , because only the domestic interest rate prefers the value of parameter ρ_Z that is actually estimated. All other parameters have different preference, either upward (mainly foreign interest rate) or downward (mainly domestic and foreign output growth).

Putting these findings together with subsection 4.2, we see that the decrease in the posterior mean of h creates some trade-offs. More concretely, most observables are in line with the lower value of h but mainly the domestic and foreign output growths do not prefer the lower value of h . In case of parameter ρ_Z , the situation is somewhat similar. The jump of parameter ρ_Z creates huge trade-offs and again, the domestic and foreign output growths do not prefer the changed (in this case increased) value of the parameter. The main driving force of the parameter is the foreign interest rate. In this sense, we can see that the domestic and foreign output growths are rather consistent with prevailing values of parameters, no matter if the crisis hit or not. On the other hand, the variable most significantly reacting to crisis data is the foreign interest rate. In order to untangle the reasons for this behavior, we'll need yet another tool and that is the shock decomposition.

6 Shock decomposition

Shock decomposition is a tool that shows which innovations and in what strength contribute to the behavior of a variable of interest. Figure 4 depicts shock decompositions for the nominal exchange rate depreciation Δe and the foreign interest rate R^* . We shall concentrate on the latter now.

Last 4 observations (2009q1–2009q4) play a key role since large negative innovation to the world-wide technology shock occurred at 2009q1. The persistence of this innovation to the shock is of major importance. Last 4 observations show that innovation to the world-wide technology shock (EPS_Z) mostly contribute to negative values of the foreign interest rate. As the shock vanishes, the innovation to the foreign monetary rule (EPS_Rstar) has to take over and contribute to existing low (and negative) values of the (detrended!) foreign nominal interest rate. To mimic the argumentation of section 5, if the world-wide shock was more persistent (ρ_Z was higher), the innovation to foreign monetary rule wouldn't have to take over the role of the world-wide technology shock. In this sense, the foreign interest rate prefers more persistent shock to explain its very low values.

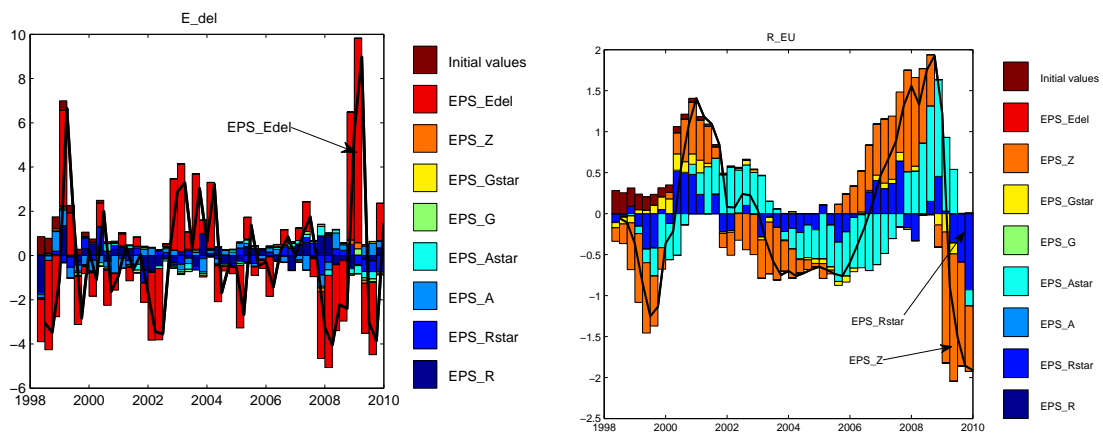


Figure 4: Shock decomposition of the rate of change of nominal exchange rate Δe (left panel) and of foreign nominal interest rate (right panel). Innovations that contribute to these variables are in the legend on the right of the chart: **EPS_Edel** is the innovation to the rate of change of the nominal exchange rate, **EPS_Z** is the innovation to the world-wide technology AR1 shock, **EPS_G** (and **EPS_Gstar**) is the innovation to the AR1 shock to domestic (and foreign) market clearing condition, **EPS_A** (and **EPS_Astar**) is the innovation to the country-specific technology AR1 shock and **EPS_R** (and **EPS_Rstar**) is a direct innovation to the domestic (and foreign) monetary rule.

Knowing what happened and why, one welcomes an economic hypothesis that could explain such behavior. One is offered in the next subsection.

7 The big picture

Why the trade-offs emerged during the crisis and what is the connection to the changes in the parameter estimates we observe? This section offers an explanation.

Starting with a look at the data (Figure 1), we observe (panels 1–3) that the crisis hit at 2009q1, is followed by rather quick recovery (in terms of economic growth) but without inflationary pressures. Such development in reality (and in the model as well) justifies low domestic interest rates.

On the other hand, foreign development is rather different (panels 5–7). The development of economic growth is similar to domestic economy, but (unlike domestic inflation) foreign inflation rises and in the last quarter (2009q4) is even above its long-run trend. Such situation in the model’s logic calls for a rise in interest rate to confront the rising inflation. However, such intervention did not happen. In the last 4 quarters, the foreign interest rate steadily falls. Such wedge in the logic of the model must be explained somehow. The first call the model did was to dramatically increase the persistence of the world-wide technology shock and lowered the habit persistence in consumption. Both of these changes work in the model as means of prolonging the crisis itself, which would explain the low (and falling) interest rates. The fact that the crisis is not as long as would monetary policy expect is then offset in other equations by other shocks and innovations.

On the whole, the reason for the radical changes in parameters h and ρ_Z are caused by the model disagreement in the equation of foreign Taylor rule. Or, in another words, in the logic of the model, the foreign monetary authority is far too benevolent on the interest rates given the evolution of economic activity and the evolution of inflation.

8 Conclusion

By inspecting the differences in the model variants with and without crisis data we observe several changes in behavior. Two ”minor” changes concern parameters $\sigma_\epsilon^{\Delta e}$ and θ_F^* . Posterior mean of the estimate of the parameter $\sigma_\epsilon^{\Delta e}$ went up during crisis in order to capture higher variation in the series of nominal exchange rate depreciation. As for parameter the Calvo parameter for foreign economy θ_F^* , the estimate is lower during crisis because of unprecedented changes in foreign inflation.

Two major changes in the behavior of the model relate to a drop in the estimate of habit persistence h and a jump in the persistence of the world-wide technology shock ρ_Z . The main cause for the harsh changes in the estimates of these parameters in the first quarter of crisis year 2009 is probably a two-percentage-points drop in Euroarea 3-months rates, which is a biggest quarter-to-quarter change in the time series ever.

Moreover, the changes are fueled by movements in the foreign inflation and the output growth, which (together with the interest rate) contradict the workings of the Taylor rule. In the eyes of the model, the European central bank holds its interest rates unreasonably low.

9 Discussion

Foreign quarter-to-quarter annualized inflation is in 2009q4 approx. 2.8%, which is above 2% ECB target. Since according to [2] ECB sees the inflation fairly below its target with rather weak inflationary pressures, there are some differences in the measures of inflation. This model uses EA12 aggregation, whereas ECB is interested in the inflation in the whole Euroarea (currently EA16). Since the 4 countries are small,³ the change in inflation between EA12 and EA16 is not large. Greater problem is in comparison of quarter-to-quarter (q-o-q) inflation (which is naturally in the quarterly model) versus year-to-year (y-o-y) inflation that is followed by the ECB. Since y-o-y infation is far more persistent, current rise of q-o-q inflation will eventually raise y-o-y inflation after few quarters. This is a probable explanation towards the difference in the model scenario and reality.

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³13th admitted country was Slovenia in 2006, Cyprus and Malta were admitted in 2008 and Slovakia joined in 2009.

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