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Comparing the fit of New Keynesian DSGE models

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Abstract

The paper is focused on an analysis of model fit of Dynamic Stochastic General Equilibrium (DSGE) models following New Open Economy Macroeconomics (NOEM). Unlike most of the literature on the topic, this paper does not use Bayesian posterior odds ratio to analyze model fit to data; it uses alternative tools instead. In order to compare the results of the alternative tools to the standard posterior odds ratio, this paper uses the findings of Slanicay and Vašíček (2009), who compared model fit to data of several models with the tool Bayesian posterior odds ratio. The goal of the paper is to verify the results of Slanicay and Vašíček's (2009) model variants with different criteria than posterior odds and to compare the results with findings of their paper. The tools for the analysis are criteria based on root mean squared error (RMSE) and tools from the Global Sensitivity Analysis toolbox. Conclusions of this paper are the following: Habit persistence in consumption is found to be important and price indexation unimportant as in Slanicay and Vašíček (2009). Furthermore, model variants with foreign economy modeled as AR1 processes always perform better than the ones with structurally modeled foreign economy. This finding is in contradiction to the results of Slanicay and Vašíček (2009).

Keywords

Forecast quality, global Sensitivity Analysis, model fit, Bayesian posterior odds ratio, parameter importance.

JEL Classification: C32, C52, C53, E37

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1. Introduction

Contemporary literature extensively uses open economy Dynamic Stochastic General Equilibrium (DSGE) models to policy evaluation, forecasting, and other methodological issues. Large portion of these models belongs to the New Open Economy Macroeconomics (NOEM) class of models, which broaden classical approach by including nominal rigidities and market imperfections and derive (macroeconomic) models from microeconomic foundations.[1] Some authors call virtually the same extensions to models as following the New Keynesian (NK) paradigm.

Despite wide use of NK DSGE models, according to e. g. Justiniano and Preston (2004) and Slanicay and Vašíček (2009), there has been paid little attention to the fit of the model to actual data. Justiniano and Preston (2004) therefore apply Bayesian posterior odds ratio to several models in order to evaluate which model fits data best.[2] Similar approach is followed by Slanicay and Vašíček (2009) but the analysis is

[1] For a survey on New Open Economy Macroeconomics literature, see Lane (2001).

[2] Authors use data sets from Australia, Canada and New Zealand.

performed on Czech data (with EU12 as the foreign economy).

The relatively sparse literature devoted to study model fit mostly uses Bayesian posterior odds ratio as a standard analytic tool. Although the posterior odds ratio is a common criterion to compare models, its informativeness is limited – the result is just one number (the ratio). This paper aims at a broader approach to the topic.

The goal of the paper is to verify the results of Slanicay and Vašíček's (2009) model variants with different criteria than posterior odds and to compare the results with the findings of Slanicay and Vašíček (2009).[3]

The choice of alternative tools is motivated by their ability to explain the sources of the resulting fit. The researcher may be interested in questions like: Does the prior space of the model correspond to stable results? Which parameters cause the biggest conflicts in model relations that cause the deterioration of fit? Which parameters are the most important in different model specifications? Which model specification results in the lowest one-step-ahead prediction error? Which model requires the lowest possible variation in model innovations? These and other similar questions cannot be answered using posterior odds ratio as the only analytic tool. Luckily, all of these questions can be answered by using analytical tools used in this paper.

Even though the selected tools allow for deeper analysis, the ability to formulate conclusions about comparison of fit of Slanicay and Vašíček (2009) model variants is vital for the primary aim of the paper. The main contribution of the paper is a proposal of an innovative methodology to compare models according to their data fit without utilizing Bayesian posterior odds ratio. The usefulness of the innovative methodology can be found in two main areas. Firstly, the fact itself that the analysis does not use Bayesian posterior odds allows for comparison with the majority of papers that use Bayesian posterior odds. Secondly, the alternative analyses enable a deeper understanding of what lies behind the result that one model fits the data better that some other model.

The paper proceeds as follows. Section 2 introduces the model, data used and presents the linearized form of the model equations. Section 3 uses tools of Global Sensitivity Analysis toolbox to deeply analyze relations within model structures that affect the fit to data. Section 4 addresses directly the data fit by computing indices that measure data fit and quality of forecasting.[4] The final part of the paper summarizes the results.

2. Model

The macroeconomic models introduced in this section are derived from microeconomic foundations. The models presume seven types of representative agents. These are importers in the domestic economy[5] and domestic and foreign households, producers, and monetary authority.

Households maximize their utility function subject to their budget constraint. They derive positive utility from consumption and negative utility from labor. The budget constraint of the resources spent on consumption and money earned by working must be balanced in the long run.

Producers operate on monopolistically competitive markets and their production function has only labor input. They change price according to the Calvo-type price setting.

Importers import differentiated foreign goods and are also operating on monopolistically competitive market and they are also bound by Calvo-type price setting.

Monetary authorities behave according to a modified Taylor rule, i. e. they use their tool, nominal interest rate, to react to deviations of inflation and output from their required (target) levels.

None of the microfoundations and derivations or linearizations are introduced in this paper. This section just briefly introduces linearized model equations of all model variants that are used in the analysis. All model variants (with prior settings, data sets etc.) are taken over from Slanicay and Vašíček (2009), which can also be consulted for details on microfoundations, derivations and linearizations.

For original literature on very similar models, see Galí and Monacelli (2005) and Monacelli (2003). For full details of model linearization, see e. g. Justiniano and Preston (2004), Liu (2006), Musil and Vašíček (2006) or Remo and Vašíček (2008).

[3] Slanicay and Vašíček (2009a) came to similar results about importance of habit formation in consumption and price (and wage) indexation as Slanicay and Vašíček (2009), but on slightly modified model structures and estimated on US data set.

[4] Sections 4.1 and 4.2 do not use Global Sensitivity Analysis toolbox.

[5] In contrast to domestic importers, foreign importers are not modeled since the model is a small open economy (SOE) model, which means that none of the actions of the small domestic economy influences the behavior of the large foreign economy.

The section proceeds as follows. Subsection 2.1 explains some of the denotation used throughout the paper. Subsections 2.2 and 2.3 present linearized form of the model. Subsection 2.2 presents the domestic part and subsection 2.3 presents both *Monacelli* and *VAR* formulations of foreign sector, all linearized around the steady state. Subsection 2.4 introduces the eight used model variants and their notation and final subsection 2.5 mentions the data used for the analysis.

2.1 Denotation details

All variables are introduced as a logarithmic deviation from steady state, formally written $x_t = \log X_t - \log X$, where X is the value at steady state. Subscript t at a variable denotes relative time. Symbol E is a rational expectations operator. Symbol Δ denotes first difference so that e.g. $\Delta x_t = x_t - x_{t-1}$. AR1 shocks are denoted by ε s. Exogenous processes (innovations to equations) are denoted by ζ s. Greek letters without t subscripts denote model parameters.[6] Denotation of model's variables is explained in section 2.2.

Variables and parameters with a star superscript $\left(^*\right)$ denote foreign variables or corresponding parameters. Variables and parameters with a H subscript $\left(_H\right)$ relate to home-produced goods, whereas variables and parameters with a F subscript $\left(_F\right)$ relate to imported goods[7].

Notation in Figures lacks LaTeX/MathType characters, but the paraphrasing is mostly straightforward. For clarity, Figures have footnotes with hints on the paraphrasing (footnote 10, 15, and 16).

2.2 Domestic block

Goods market clearing condition is $(1-\alpha)c_t = y_t - \alpha\eta(2-\alpha)s_t - \alpha\eta\psi_{F,t} - y_t^*$, where law of one price gap is defined as $\psi_{F,t} = (e_t + p_t^*) - p_{F,t}$, c is consumption, y is output, s are terms of trade, y^* is foreign output, e is nominal exchange rate, p^* is foreign price index, $p_{F,t}$ is (domestic) price index of foreign goods.

Change in terms of trade equals $\Delta s_t = \pi_{F,t} - \pi_{H,t}$, where π is inflation (e.g. $\pi_{H,t}$ is inflation of home-produced goods).

Domestic firms' price setting equation is $\pi_{H,t} - \delta_H \pi_{H,t-1} = \beta E_t(\pi_{H,t+1} - \delta_H \pi_{H,t}) + \theta_H^{-1}(1-\theta_H) \cdot$ $\cdot(1-\beta\theta_H) mc_t$, where mc are firm's real marginal cost that follow equation $mc_t = \varphi y_t - (1+\varphi)\varepsilon_{a,t} + \alpha s_t +$ $+\sigma(1-h)^{-1}(c_t - hc_{t-1})$.

Real exchange rate definition is $q_t = e_t + p_t^* - p_t = \psi_{F,t} + (1-\alpha)s_t$. Importers' price setting equation is $\pi_{F,t} - \delta_F \pi_{F,t-1} = \beta E_t \cdot$ $\cdot(\pi_{F,t+1} - \delta_F \pi_{F,t}) + \theta_F^{-1}(1-\theta_F)(1-\beta\theta_F)\psi_{F,t}$.

Uncovered interest parity condition is $(i_t - E_t \pi_{t+1}) - (i_t^* - E_t \pi_{t+1}^*) = E_t \Delta q_{t+1} + \varepsilon_{s,t}$ (with using $\Delta e_t = \Delta q_t + \pi_t - \pi_t^*$).

Complete market assumption equation is $c_t - hc_{t-1} = y_t^* - hy_{t-1}^* + \sigma^{-1}(1-h)\left[\psi_{F,t} + (1-\alpha)s_t\right] + \varepsilon_{g,t}$. Identity for inflation definition is $\pi_t = \pi_{H,t} + \alpha\Delta s_t$.

Domestic block is closed with modified Taylor rule $i_t = \rho_i i_{t-1} + (1-\rho_i)[\psi_\pi \pi_t + \psi_y y_t] + \zeta_{M,t}$, where i is nominal interest rate, and three AR1 processes $\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \zeta_{g,t}$, $\varepsilon_{a,t} = \rho_a \varepsilon_{a,t-1} + \zeta_{a,t}$ and $\varepsilon_{s,t} = \rho_s \varepsilon_{s,t-1} + \zeta_{s,t}$.

2.3 Foreign block

There are eight variants of description of foreign sector. Four variants model foreign economy with structural equations (these are called *Monacelli*), another four variants describe foreign economy with AR1 processes (variants called *VAR*).

Monacelli

Structural relations representing basic behavioral characteristics of a foreign economy are natural counterparts of domestic-block equations:

$$y_t^* - hy_{t-1}^* = E_t(y_{t+1}^* - hy_t^*) - \frac{1-h}{\sigma}(i_t^* - E_t \pi_{t+1}^*) + \varepsilon_{g,t}^* - \varepsilon_{g,t+1}^*,$$

$$\pi_t^* - \delta_* \pi_{t-1}^* = \beta E_t(\pi_{t+1}^* - \delta_* \pi_t^*) + \theta_*^{-1}(1-\theta_*)(1-\beta\theta_*) mc_t^*,$$

$$mc_t^* = \varphi y_t^* - (1+\varphi)\varepsilon_{a,t}^* + \sigma(1-h)^{-1}(y_t^* - hy_{t-1}^*),$$

$$i_t^* = \rho_{i*} i_{t-1}^* + (1-\rho_{i*})[\psi_{\pi*} \pi_t^* + \psi_{y*} y_t^*] + \zeta_{M,t}^*,$$

$$\varepsilon_{a,t}^* = \rho_{a*} \varepsilon_{a,t-1}^* + \zeta_{a,t}^*, \qquad \varepsilon_{g,t}^* = \rho_{g*} \varepsilon_{g,t-1}^* + \zeta_{g,t}^*.$$

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[6] Exact meaning of all the parameters is not listed for two reasons. It is not vital for understanding paper's results and also to conserve space. Important parameters are discussed in section 2.4. Those interested in economic meaning of other (or all) parameters may consult Slanicay and Vašíček (2009).

[7] Described in a more elaborate way, F subscript denotes foreign-produced home-consumed goods (in another words, imported goods).

VAR

Foreign sector is described just by three AR1 processes:

$$y_t^* = \omega_y y_{t-1}^* + \zeta_{y,t}^*,$$
$$\pi_t^* = \omega_\pi \pi_{t-1}^* + \zeta_{\pi,t}^*,$$
$$i_t^* = \omega_i i_{t-1}^* + \zeta_{i,t}^*.$$

2.4 Model variants

As was mentioned above, the analysis uses eight model variants. They differ in the way the foreign economy is modeled and in the restrictions that are placed on certain parameters. Four model variants use structural *Monacelli* description (variants M1, M2, M3 and M4). Remaining four model variants describe foreign economy behavior with AR1 processes (variants called V1, V2, V3 and V4).

The original study Slanicay and Vašíček (2009) investigated the relevance of presence of habit persistence (parameter h) and price indexation (parameters δ) in a way of allowing the parameters to be non-zero or fixing them at zero value (and eliminating them effectively from the system). Slanicay and Vašíček (2009) then compared model fit with Bayesian posterior odds ratio.

Model restrictions for eight model variants are the same as in original paper Slanicay and Vašíček (2009) and are stated in Table1. For example, variant M2 is a model with structural foreign economy and allowed habit persistence. For another example, variant V4 is a model with foreign economy modeled as AR1 processes and with allowed habit persistence and price indexation.[8]

2.5 The data

Model consists of seven observable variables: y_t and y_t^* are modeled as (HP filter-) detrended log real GDP per worker for the Czech Republic (CR) and EU12, respectively. π_t and π_t^* are modeled as demeaned quarter-on-quarter inflation rate for the CR and EU12, respectively; i_t and i_t^* are modeled as demeaned nominal interest rate for the CR and EU12, respectively; q_t is modeled as (HP filter-) detrended log real exchange rate. All data are from Eurostat.

3. Global Sensitivity Analysis

This section presents results of Marco Ratto's Global Sensitivity Analysis (GSA) toolbox[9] applied on models of Slanicay and Vašíček (2009). Following subsections 3.1, 3.2, and 3.3 present results of separate GSA tools in a summarized manner. For cross-reference and exemplary purposes, all subsections present an example of actual output of GSA toolbox prior to summarization. Due to the length of the section, subsection 3.4 condenses main findings of subsections 3.1–3.3.

3.1 Stability analysis

Stability mapping helps to detect parameters X_i that are responsible for possible *bad behavior* of the model. Without burrowing into theoretic details (see Saltelli et al. (2008), Ratto (2008) or Čapek (2009)), the use is following: *Bad behavior* is either instability (model solution is unstable) or indeterminacy, both possibilities meaning that the solution of the model cannot be used for further needs. Stability mapping detects which parameters (and on which range) cause the solution of the model to be *bad*. Researcher can then suitably adjust prior space so that the instability/indeterminacy regions are eliminated.

Table 2 and Table 3 introduce results of stability mapping in columns two and three. Column 2 (stability region) separates the prior space into behavioral *good* part and non-behavioral (unstable and indeterminacy) *bad* parts. Models with structural description of foreign economy (M1–M4) exhibit that only some $3/4$ of the prior space is stable. Models with VAR-foreign economy are 10 percentage points better off with approximately 86 % of prior space stable. This

8 Martin Slanicay kindly provided me with Matlab codes to all model variants which ensures comparability of the results.

9 The toolbox is available online at http://eemc.jrc.ec.europa.eu/Software-DYNARE.htm, the tools are described in Saltelli et al. (2004 and 2008) and Ratto (2008 and 2009).

Table 1 Restrictions imposed on parameters in model variants

	M1	M2	M3	M4	V1	V2	V3	V4
restriction	$h=0$ $\delta_H=0$ $\delta_F=0$ $\delta_*=0$	$\delta_H=0$ $\delta_F=0$ $\delta_*=0$	$h=0$	–	$h=0$ $\delta_H=0$ $\delta_F=0$	$\delta_H=0$ $\delta_F=0$	$h=0$	–

can (in some aspects) lead to better model results and possibly to better model fit. One possible solution to the low portion of stable prior space (one that is offered by GSA) is demonstrated in the subsection *Expanding stability region*.

Column 3 of Table 2 and Table 3 describes, which parameters (and in which direction) influence the

Table 2 Global Sensitivity Analysis results, Monacelli model variants

Model	Stability region	Stability analysis	Mapping the fit	Parameter importance
M1	76 % S 2% U 22 % I	*unstable* θ_H, θ_F lower α slightly lower ρ_i slightly higher *indeterminacy* ψ_π, $\psi_{\pi*} < 1$	ρ_s higher: y, π, i, q ρ_{a*} higher: i^* ρ_{a*} lower: y^*, π^*	unimportant ρ_g, ρ_{g*} most important θ_H, φ
M2	76.3 % S 1.4 % U 22.3 % I	all the same as model M1	h higher: y ρ_{a*} higher: i^* ρ_{a*} lower: y^*	unimportant ρ_a, ρ_s most important θ_H, φ
M3	75.1 % S 0.9 % U 24 % I	*unstable* very small part – hardly recognizable *indeterminacy* ψ_π, $\psi_{\pi*} < 1$	δ_F higher i $\psi_{\pi*}$ lower all but π, i ψ_{y*} higher all but π, y ρ_s higher: π, y	unimportant ρ_g, ρ_{g*} most important θ_H, θ_F, θ_*
M4	75.8 % S 0.5 % U 23.7 % I	all the same as model M3	δ_F higher i ψ_{y*} lower q ρ_{a*} lower π^*	unimportant ρ_g most important θ_H, h

Table 3 Global Sensitivity Analysis results, VAR model variants

Model	Stability region	Stability analysis	Mapping the fit	Parameter importance
V1	85.7 % S 2.3 % U 12 % I	*unstable* θ_H, θ_F lower ρ_i slightly higher *indeterminacy* $\psi_\pi < 1$	ρ_s higher y ω_{y*} higher y^* $\omega_{\pi*}$ lower π^* ω_{i*} higher i^*	unimportant ρ_g, ρ_a, ω_{y*} most important θ_H, ψ_π
V2	86 % S 1.3 % U 12.7 % I	all the same as model V1	ρ_s higher y, π ω_{y*} higher y^* $\omega_{\pi*}$ lower π^* ω_{i*} higher i^*	most important θ_H, h
V3	86.5 % S 0.8 % U 12.7 % I	*unstable* very small part – hardly recognizable *indeterminacy* $\psi_\pi < 1$	δ_F higher i ρ_g higher π, i ρ_s higher y ω_{y*} higher y^* $\omega_{\pi*}$ lower π^* ω_{i*} higher i^*	unimportant ρ_g most important θ_H, ρ_i, θ_F
V4	86.4 % S 0.5 % U 13.1 % I	all the same as model V3	δ_F higher i ρ_s higher y ω_{y*} higher y^* $\omega_{\pi*}$ lower π^* ω_{i*} higher i^*	most important θ_H, ρ_i, h

solution of the model. Parameters mostly responsible for unstable model are θ_H and θ_F in their lower range. Parameters creating indeterminacy include reaction parameters in (domestic and foreign) Taylor rules for inflation (ψ_π and $\psi_{\pi*}$), if they are lower than 1.

Example of results from Ratto's GSA toolbox

An example of GSA toolbox results is in Figure 1, which is for model M1 and for unstable results.

In short, the underlying computation is following:[10] N Monte Carlo simulations are run over prior domain, which results in two subsets, $(X_i \mid B)$ of size n and $(X_i \mid \overline{B})$ of size $\overline{n}$, where $n + \overline{n} = N$. The two sub-samples may come from different probability density functions (PDFs) $f_n(X_i \mid B)$ and $f_{\overline{n}}(X_i \mid \overline{B})$. Corresponding cumulative distribution functions (CDFs) are $F_n(X_i \mid B)$ and $F_{\overline{n}}(X_i \mid \overline{B})$.

If $F_n(X_i \mid B)$ and $F_{\overline{n}}(X_i \mid \overline{B})$ differ for a given parameter X_i, the parameter may drive bad behavior of the model if its value falls within $\overline{B}$ subset. The shape of $F_{\overline{n}}(X_i \mid \overline{B})$ indicates, whether rather smaller or higher values of X_i drive the non-behavior. If the non-behavior CDF is to the left from behavior CDF, it indicates that rather smaller values of X_i are more likely to drive non-behavior. On the other hand, if the non-behavior CDF is to the right from the behavior CDF, it suggests that rather bigger values of X_i drive non-behavior.

Cumulative probability density functions shifted to the left off the dashed line in first two panels correspond to the observation in Table 2 that lower ranges of θ_H and θ_F are responsible for unstable results. Similar figures were drawn for both instability and indeterminacy and for all eight models, the results are summarized in Table 2.

Expanding stability region

Most of the models demonstrate prior space, of which as little as just $3/4$ is stable. Global Sensitivity Analysis can help with this problem. I'll show the procedure on model M1, which exhibits 76 % of prior space stable, 2% unstable and 22 % correspond to indeterminacy. Stability analysis suggests that ψ_π and/or $\psi_{\pi*} < 1$ cause indeterminacy. It also suggests that low ranges of θ_H and θ_F, slightly lower ranges of α and slightly higher ranges of ρ_i all contribute to unstable results.

The solution to the problem is to truncate prior densities at determinacy region.[11] Parameters ψ_π and $\psi_{\pi*}$ have both prior value 1.5 with standard deviation 0.15.[12] With these values, it is very unlikely that the estimation procedure could look for values below 1. We can cross-check the guess by looking at the real estimate, which is approximately 1.36 and 1.38, respectively. Shifting the lower bound of the truncation (from original 0.0001 to, say, 1) elegantly cuts off the part of prior space which corresponds to indeterminacy.

The procedure described for a case of indeterminacy is similar to the case of unstable results. As was mentioned above, low ranges of θ_H and θ_F tend to create unstable results. Both of these parameters have prior values 0.7 with standard deviation 0.1. Posterior estimates are higher than prior value (0.73 and 0.79, respectively), we can therefore shift the lower bound of truncation from original 0.0001 to 0.45. Again, the shape of prior density makes it almost impossible for the estimation algorithm to look at values as low as 0.45. Another parameter (partially) responsible for unstable results is parameter α. Prior value is 0.7 with even smaller standard deviation, 0.05.[13] Truncation of the prior density can therefore start at 0.55. Last parameter of interest is ρ_i, but there is little we can do about its prior density. Slightly higher values result in unstable results and, indeed, posterior estimates of ρ_i are very high (0.94 on (0.0001; 0.999) interval).

Carrying out just these five described truncations of redundant prior space results in very favorable shifts in the structure of the prior space. The final prior

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[10] Used notation is: X_i is i-th parameter, B is behavioral subset (part of domain that produces desirable results), $\overline{B}$ is non-behavioral subset (part of domain that produces undesirable results – instability or indeterminacy).

[11] Another possibility is to shift or to narrow prior distribution of the parameter, although such change might be hard to justify. The choice of suitable solution (if any) depends on the problem being solved, the software and optimization technique used and many other factors.

[12] All prior values are taken over from Slanicay and Vašíček (2009).

[13] Slanicay and Vašíček (2009) actually state that parameter α is calibrated at value 0.7. There has probably been a minor change in versions of the models. Either way, if α was really calibrated, it wouldn't add to the prior space at all and wouldn't be subject to stability analysis.

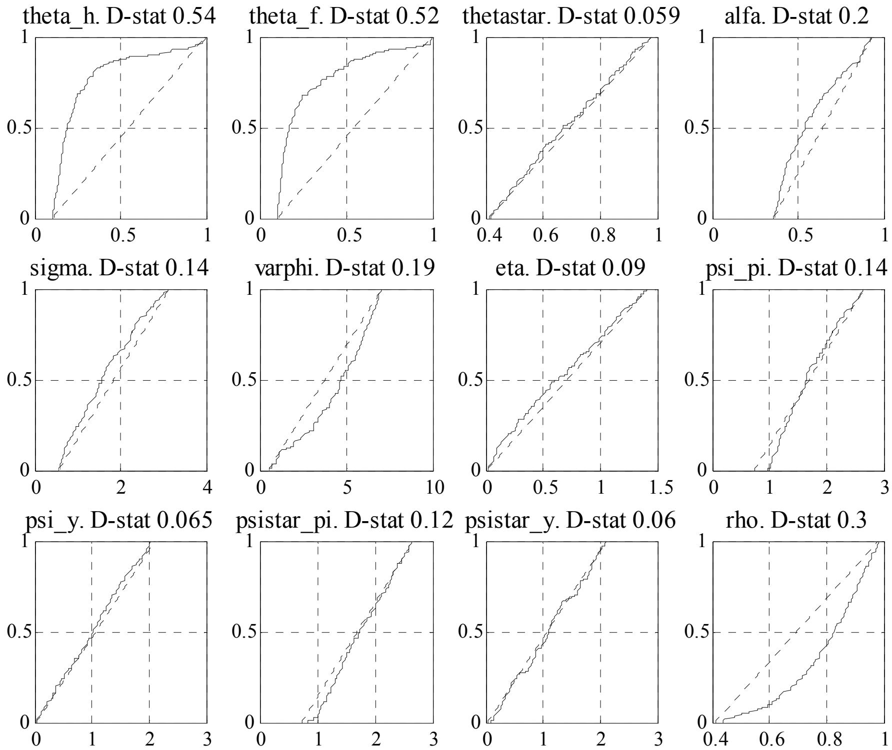

Figure 1 Example: Stability analysis results for model M1, unstable region
(Paraphrasing of the most important parameters is: theta_h is θ_H, theta_f is θ_F, rho is ρ_i, rho_s is ρ_s, rhostar_g is ρ_{g^*} and rhostar_a is ρ_{a^*}.)

space consists in 95.4 % of stable results, 0.3 % of unstable results and 4.3 % of indeterminacy. This means an improvement of 19.4 percentage points in stable results. Unstable results are reduced almost 7 times and indeterminacy region is now a fifth of what it was.

3.2 Mapping the fit

Since DSGE models consist of a number of observed variables which should fit the data as well as possible, mapping the fit may be a useful tool to learn about the linkages that drive the fit of trajectories of particular variables to data. Information provided by the results of mapping the fit can be used to unveil possible trade-offs and maybe also to amend model structure or to calibrate parameters properly in order to increase the fit of variables of interest.

Column 4 of Table 2 and Table 3 introduce results of mapping-the-fit analysis. Again, without technical details (those interested in details may consult Saltelli et al. (2008), Ratto (2008) or Čapek (2009)), the interpretation of the results is as follows: Let's use again model M1 for explanation. Corresponding cell (Table 2, column 4, row 2) lists 3 conflicts in data fit. "ρ_s higher: y,π,i,q" means that the four mentioned observable variables would prefer higher values of parameter ρ_s than its posterior distribution in order to fit data as well as possible. Because there are 7 observables, this result might seem odd, because *only* 3 observables shift posterior distribution towards lower-values whereas 4 observables would prefer higher values. Such situation nicely demonstrates one of the conflicts that exist in the particular estimate of the model. Remaining two entries in the corresponding table cell state "ρ_{a*} higher: i^*" and "ρ_{a*} lower: y^*,π^*". These entries demonstrate a conflict right away: i^* would prefer higher values of parameter ρ_a than its posterior values and observables y^*,π^* would prefer lower values of the same parameter.

Another group of conflicts that deserves mentioning is group of AR1 parameters in all models with VAR foreign economy (V1–V4). It is not unusual for such AR1 processes to demonstrate this type of behavior. The series that is described in an autoregres-

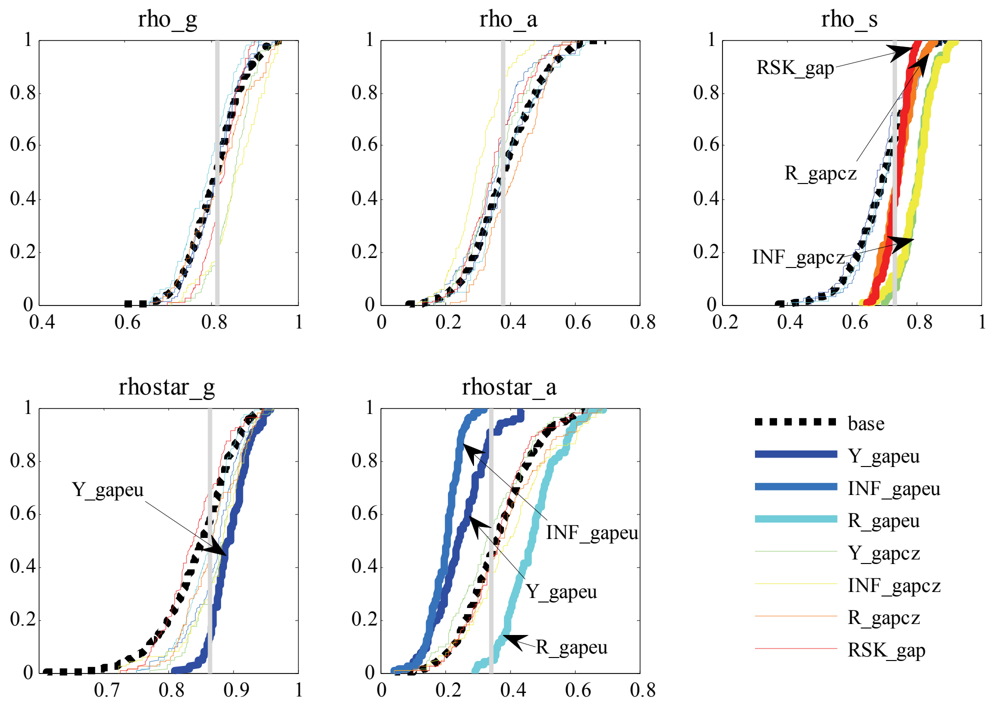

Figure 2 Example: Mapping-the-fit results for M1 model, selected 5 parameters
(The legend for this figure lists all seven observable variables. Y_gapcz is y, INF_gapcz is π and R_gapcz is i. Legend entries ending with *eu* are simply foreign counterparts. Last observable variable denoted RSK_gap is real exchange rate q.)

sive manner often prefers different value of the AR1 parameter then the rest of the model.

Generally, as Table 2 and Table 3 show, Monacelli-foreign models present greater variability in trade-offs, a lot of different parameters bear trade-offs for fit. Furthermore, if we assume away AR1 parameters in VAR-foreign economies, Monacelli-foreign models have much higher count of trade-offs.

As for the parameters of importance (habit persistence h and price indexation parameters δ_H and δ_F), h creates trade-offs in model M2, whereas δ_F creates trade-offs in models M3, M4, V3 and V4, that is, in all models where δ_F is allowed to be non-zero. The fact that price indexation creates trade-offs for fit wherever it is allowed to be non-zero seem rather to spoil model fit than improve it. Habit persistence is much better in this sense, since it only bears one trade-off in four models where it is allowed to be non-zero.

Example of results from Ratto's GSA toolbox

An example of mapping-the-fit results for M1 and five selected parameters is depicted in Figure 2.

The procedure of computation mapping-the-fit results is carried out as follows: (1) Structural parameters are sampled from posterior distribution, (2) RMSE (root mean squared error) of 1-step-ahead prediction is computed for each of observed series, (3) 10 % of lowest RMSE is defined as behavioral and B is defined as a subset of parameter values producing these behavioral results and (4) the calculations result in a number of distributions $f_j(X_i \mid B)$ that represent the contribution of parameter X_i to best possible fit of j-th observed series.

Plotting the distributions (or better the CDFs) is one step further to trace possible trade-offs. A trade-off is present when at least two distributions differ from posterior distribution (denoted in Figure 2 as base) and differ from each other.

Posterior mode (base) is depicted with black dotted line. Observables causing biggest trade-offs or conflicts are in bold. These conflicts can be found in Table 2 and are interpreted above. Similar figures were drawn for all parameters in all eight models, the results are summarized in Table 2.

3.3 Parameter importance

Results discussed in this section are outcomes of a part of GSA called Elementary Effects. For more detailed narrative on the topic, see Saltelli et al. (2008), Ratto

(2008) or Čapek (2009). Results for our eight models are in Table 2 and Table 3, column 5.

Elementary effects analysis can identify the most and the least important parameters in a model by investigating all possible relationships in the model and identifying, which parameter is un/important for that particular relationship. Parameters that are important play a significant role in many relationships among variables and – in some – they play a major role. Parameters that are unimportant may play major role for few relationships in the model and are virtually useless for explanation of most model relationships.

Not surprisingly, parameters that are unimportant are mostly AR1 parameters (ρ s and ω s), since they are usually only in one equation, which is not too interconnected with other equations of the system. Parameters that are most important in the models are θ_H, ρ_i, θ_F, h and φ. θs are the shares of non-optimizing agents, h is the habit persistence parameter, φ is the inverse elasticity of labor supply and ρ_i is the backward-looking parameter in the monetary rule. Price-setting of agents in domestic and foreign economy is therefore important part of the model. Habit persistence is important too (when allowed).

Example of results from Ratto's GSA toolbox

Example of GSA result for model M1 is in Figure 3. In this case, the theory underlying the results is called Elementary Effects. These effects are normalized measures of sensitivity of output to different input changes. In case the input change is fully recognized by the output, the normalized effect is 1. In case that the input change does not change output at all, the normalized effect is 0. The domain is searched for elementary effects by Morris sampling algorithm, which investigates many elementary effects for each parameter (input).

Elementary effects are summarized with boxplots with the following meaning: Lower bound of the box is lower quartile, upper bound of the box is upper quartile, central red line denotes median, dashed lines are whiskers which span to values not considered outliers and red dots are outliers.

In Figure 3, two parameters with boxplots placed closest to the top of the figure are θ_H and φ. These parameters therefore represent most important relationships in the model. Parameters with boxplots barely visible around zero are ρ_g and ρ_{g*}. Above these little boxplots there are a lot of red dots, parameters therefore represent a few important relationships but most relationships concerning these parameters are unimportant.

Similar figures were drawn for all eight models, the results are summarized in Table 2.

3.4 Section conclusion

Models with structural foreign economy seem to suffer from trade-offs for fit more than models with VAR-foreign economy. This result is intuitive, since models with structural foreign economy have more mutual relationships with other equations of the model. In layman's terms, more relationships are likely to bear more trade-offs. As for the parameters of importance, price indexation seems to conflict with model fit significantly. On the other hand, habit persistence interferes with model fit only slightly.

Lists of (un)important parameters do not differ much among the models. Habit persistence and price indexation are core research interests of Slanicay and Vašíček (2009). In this paper's calculations, the habit persistence parameter is one of the most important parameters in models M4, V2, and V4, but the parameters for price indexations are not among the most important parameters in any model.

4. Data fit and prediction quality

This section addresses the fit of the time series of the models without utilizing GSA toolbox. It conducts an analysis of fit of all observable time series and analysis of quality of prediction in these series.

4.1 Root Mean Squared Errors of one-step-ahead forecasts

Table 4 demonstrates values of RMSE (Root Mean Squared Error) of a one-step-ahead prediction, which can be considered a measure of quality of prediction and also a quality of data fit. Best results (lowest RMSE) among the eight models are indicated by a star (*), worst results are indicated by a dagger (†). Note that models V1–V4 demonstrated almost the same results for foreign economy, because foreign economy is described by simple AR1 processes (marked with gray shading) that are very loosely interconnected with the rest of the model.

In the sense of comparing RMSEs, the most successful models are V2 and V4 (both demonstrate two best predictions among the models – not counting grayed area). The least successful models are M1 and M4, both demonstrating two worst prediction results among the models. Models with foreign sector modeled as three AR1 processes therefore seem to predict generally better than models with structural foreign sector.

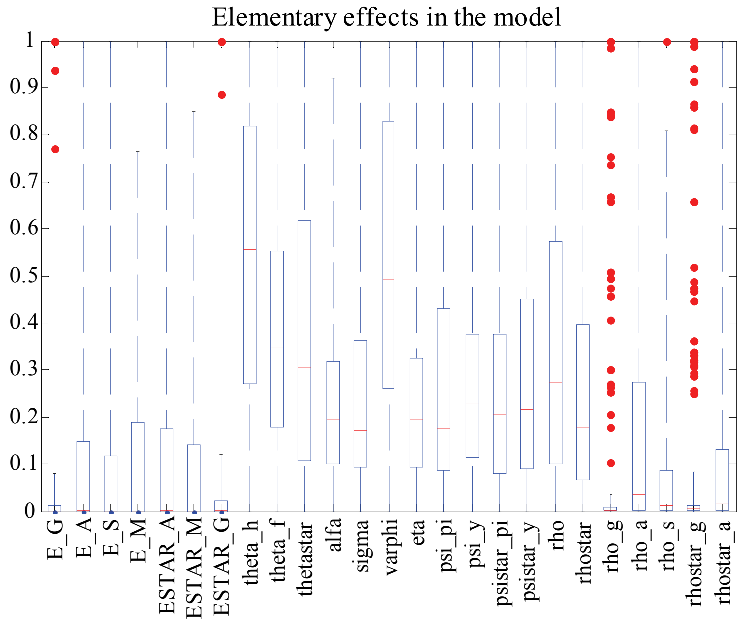

Figure 3 Example: Elementary effects in model M1
(First seven entries from the left denoted as E_G, E_A, E_S, E_M and ESTAR_A, ESTAR_M, and ESTAR_G are exogenous processes for domestic economy (E_. denote ζ s) and exogenous processes for foreign economy (ESTAR_. denote ζ^* s).)

4.2 Root Mean Squared Errors of smoothed shocks

Table 5 shows results of RMSEs calculated from smoothed shocks of the models. Comparability of smoothed shocks is limited if they enter the model in a different way. Foreign sector results in M models and V models are therefore not comparable (for V models, the cells are highlighted with gray shading). In this context, worst model is M1 with three worst results. Best model is hard to find because of limited comparability, but there are some candidates: M4 exhibits 3 best results (lowest RMSEs) and two worst results. V1 exhibits two best results and one worst (not counting data in grey area).

5. Conclusion

Since the goal of the paper was to verify the results of Slanicay and Vašíček (2009) model variants, the paper concludes with a list of Slanicay and Vašíček's main findings which are compared with author's own findings.

Slanicay and Vašíček (2009) came to the following conclusions:

1. Habit persistence in consumption (in utility function) considerably increases data fit.
2. Inclusion of price indexation in the models decreases their data fit.
3. Modeling foreign sector structurally or with AR1 processes produce ambiguous results: Some model specifications favor structural for

Table 4 Root Mean Squared Errors of one-step-ahead forecasts

variable	M1	M2	M3	M4	V1	V2	V3	V4
inflation	2.86†	2.59	2.50	2.30	2.77	2.48	2.40	2.14*
output	0.62†	0.35	0.62†	0.34	0.53	0.31*	0.54	0.31*
interest rate	0.27	0.30	0.28	0.32†	0.25*	0.25*	0.27	0.27
real exch. rate	1.46	1.58†	1.33*	1.52	1.35	1.50	1.33*	1.44
foreign int. rate	0.23	0.27	0.25	0.28†	0.11*	0.12	0.11*	0.12
foreign inflation	1.20	1.21	1.06*	1.07	1.39†	1.39†	1.39†	1.39†
foreign output	0.23	0.21	0.24†	0.20	0.10	0.09*	0.10	0.10

eign sector, some other favor AR1 foreign sector.

As for No. 1, this paper comes to similar results. Section 3.2 shows that the habit persistence interferes with model fit only slightly. Section 3.3 shows that habit persistence is among the most important parameters in models where it is allowed to be non-zero.

As for No. 2, again, this paper comes to similar results. Section 3.2 shows that the price indexation always conflicts with model fit and section 3.3 shows that price indexation parameters are not among most important parameters in any model.

Finally, as for No. 3, this paper comes to different results. Section 3.1 finds rather weak link to model fit and in that context, VAR-foreign economies perform better. Sections 4.1 and 4.2 both come to the result that in the sense of model fit, VAR-foreign models tend to err less. Contrary to the observation of Slanicay and Vašíček (2009), this paper does not find any model restriction, when Monacelli-foreign model performs better than its VAR-foreign counterpart.

To summarize the conclusions, by approaching to model comparison with different methodology, this paper comes to similar results as Slanicay and Vašíček (2009) with one notable exception concerning the foreign sector. Moreover, main conclusions of the paper are summarized from a number of individual analysis results, which enables deeper understanding of various factors that led to the results (unlike single number representing posterior odds ratio).

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Table 5 Root Mean Squared Errors of smoothed shocks

shock	M1	M2	M3	M4	V1	V2	V3	V4
ζ_a	12.34	14.00	13.69	16.60†	12.76*	13.37	13.61	14.02
ζ_g	2.41	1.38	2.41	1.36*	3.18†	1.91	3.12	1.85
ζ_M	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
ζ_s	0.84†	0.82	0.74	0.77	0.63*	0.67	0.63*	0.64
ζ_i^* / ζ_M^*	0.11†	0.10*	0.11†	0.10*	0.12	0.12	0.12	0.12
ζ_π^* / ζ_a^*	12.01	15.02	11.86*	15.77†	0.41	0.41	0.41	0.41
ζ_y^* / ζ_g^*	1.14†	0.50	1.10	0.49*	0.44	0.44	0.44	0.44

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