Changes in the behavior of economies in a DSGE model in the course of time

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Abstract. The contribution uses a New Keynesian Dynamic Stochastic General Equilibrium model as a tool for analysis of model behavior. The used macroeconomic model is a Small Open Economy model derived from microeconomic foundations and presumes four types of agents.

Aim of the contribution is to identify changes in behavior of modeled economies during dramatic economic events and explain observed changes in the mechanics of the model.

The method proceeds from Bayesian estimates of parameters and impulse response functions. In order to analyze changes in parameter estimates and impulse response functions in time, a number of estimation procedures are carried out. Each estimation involves the same baseline model with the same prior setting and varies only in the time-frame of the data set. Resulting changes in the estimates of parameters and different shapes of impulse response functions may point to changes in behavior of the system caused only by using different data span. Such model results may be explained in the context of changed behavior of economic agents.

Keywords: DSGE model, economic recession, change in model behavior, Bayesian estimates, impulse response function

JEL classification: C32, C52, E32, E43, E52, F41, F43
AMS classification: 91B51, 91B64

This paper presents some results of a continuous research on changes in behavior of economies represented by a DSGE model. Although some other time-frames may be also of interest, alotted space allows for single example – an actual one being contemporary recession.

1 Model – Introduction

This section introduces New Keynesian (NK) Dynamic Stochastic General Equilibrium (DSGE) model that is used for analysis in this paper. The model is derived from microeconomic behavior of particular economic agents. These include domestic and foreign households, domestic and foreign producers, domestic importers and domestic and foreign monetary authority.

The model is in small open economy (SOE) setting, so that it presumes two countries – small open economy that is influenced by a big closed economy. The small open economy is home economy, big large economy is the foreign economy.[1]

Most of the model assumptions are adopted from Lubik and Schorfheide [5]. Due to the lack of space, the model is not derived in this paper – similar models with details of derivation can be found in textbook Galí [2]. The model used in this paper differs from frequently cited basic models like in Liu [4], Justiniano and Preston [3] or already mentioned Galí's textbook [2] by households' utility function, incorporation of the world-wide technology shock and related transformations of marginal utility of income.[2]

When applicable, variables with H subscript (e.g. $X_{H,t}$) denote goods produced and activities associated with Home economy and variables with F subscript (e.g. $X_{F,t}$) denote goods produced and activities associated with

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[1] Strictly speaking, since the big foreign economy is modeled structurally, uncovered interest parity makes it possible for leaks from small economy to big economy. However, model results indicate that it does not occur.

[2] The difference mentioned last can be also found in Adolfson et al. [1, pp. 15–16].

Foreign economy. The location of economic activities is denoted with a star superscript (e.g. X_t^*) for the foreign economy and is left without extra notation for home economy.

2 Model – Log-linearized form

This section summarizes log-linearized model form; equations themselves are not written in full due to lack of space.

Starting with households, the system contains equations for evolution of marginal utility of income, the law of motion of habit stock, Euler equation, and the definition of inflation from domestic and imported inflation:

Behavior of producers yield the New-Keynesian Phillips Curve. Importers' optimization is analogous to that of producers and for the log-linearized system is utilized importers' Phillips curve.

Incorporation of open economy setting contributes to the system with a number of equations, namely definition of the depreciation rate of nominal exchange rate, differenced definition of terms of trade and combined definition of real exchange rate and LOP gap. The model also includes some equilibria equations that must hold. These include equation regarding international risk-sharing, uncovered interest parity (UIP) condition and log-linearized market clearing equation.

Foreign economy is modeled structurally so that there exist foreign households and producers that also show optimizing behavior. Optimizing behavior of foreign agents contribute to the log-linearized system with equations analogous to domestic case.

The model is closed by specifying monetary policy. Towards this end, standard Taylor-type rule is used. This formulation of monetary policy assumes that central banks respond to deviations of inflation from steady state, growth rate of output from steady state growth rate γ and possibly to deviations of nominal exchange rate depreciation from steady state.

The model is supplemented with AR(1) processes describing evolution of government expenditures (acting as a demand or market clearing shock) g_t, country-specific technology shock to production function (acting as a supply shock) and the evolution of z_t, which is growth rate of world-wide non-stationary technology shock.

3 Data

The data set used for estimation is depicted in Figure 1. There are four observed variables for domestic Czech economy in the first row and three observed variables for foreign EuroArea12 economy in the second row. Fourth panel is depreciation of nominal exchange rate – Czech Koruna.

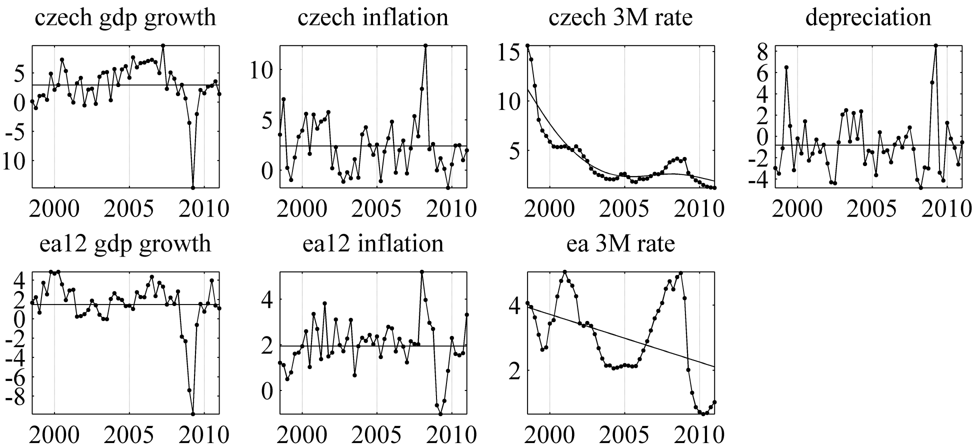

Figure 1: Data set

The data are quarterly and time is denoted at the end of period so that e.g. 2001.25 is first quarter of 2001 and 2008 is fourth quarter of 2007.

All variables are per cent, inflations and output growths are annualized, interest rates are per annum and depreciation rate is not annualized. Dotted line denotes original data and solid line denotes trend that is deducted prior to estimation. All variable except for interest rates use constant trend (mean) as a trend. Although this is standard procedure, it is not appropriate for domestic and foreign interest rates since these series show noticeable non-constant trends. Linear trend is sufficient for foreign interest rate but domestic interest rates exhibits even more curvature. After some experiments with polynomial trends and various filters, Hodrick-Prescott filter seems to be the best option.[3]

4 Parameter estimates with different time-frames

Since the research question at hand addresses changes in economy's behavior during recession, it is imperative to estimate the model on different time frames, namely on a time frame without recession and on a time frame that includes the recession.[4] Then, it is possible to compare the estimates and draw results as to the nature of the recession.

This intuitive idea of two model estimates and their comparison is clouded with two problems. One of the problems is that two estimates is not enough, because the choice of time frames would necessarily be arbitrary and moreover, the information acquired from comparison of just two results would be limited. Another problem, which is illustrated by Figure 2 stems from the choice of first observation, which may affect the estimate of the parameter to the extent that the choice of first observation may be far more important than the choice of the last observation. In another words, the choice of few oldest data could be in the estimate more important than inclusion or exclusion of crisis data.

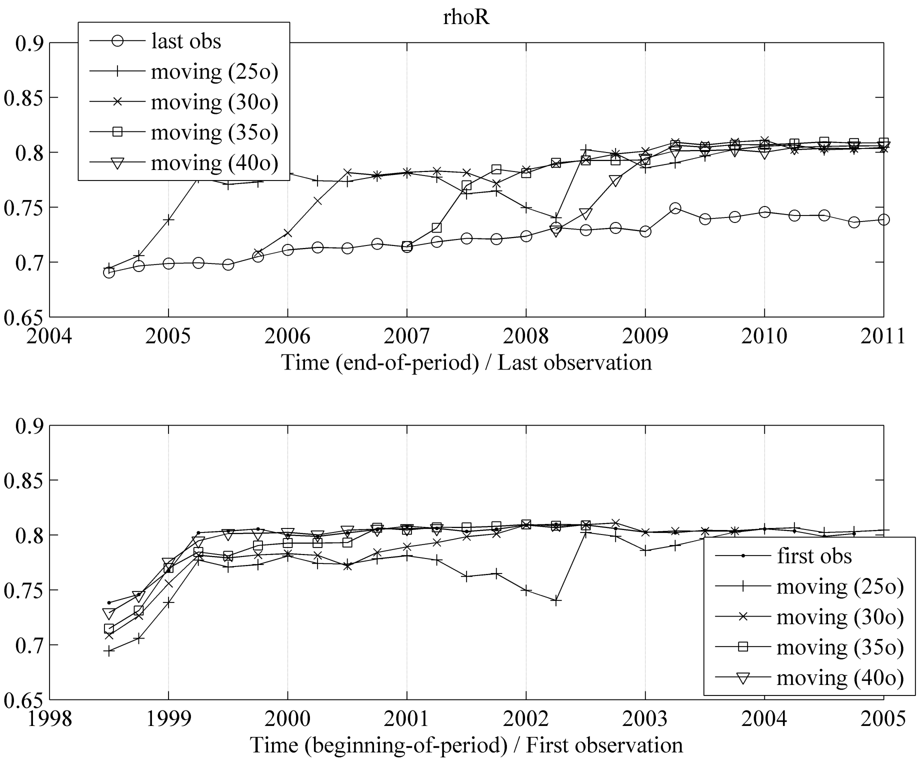

Figure 2: Estimate of backward-looking parameter in domestic Taylor rule ρ_R with different time-frames

These problems call for a more elaborate tool, which would minimize or eliminate the issues. In order to investigate sensitivity of the estimate to (a) the choice of the last observation, (b) the choice of the first observation and (c) the length of the time frame, a complex tool that addresses all of these variants is used.

[3]Sensitivity analysis of the determination of trends was conducted and model results does not change significantly when different method of detrending is used.

[4]Each time frame is defined by the choice of first observation and the last observation, the frequency always stays the same – all estimates are on quarterly data.

Due to lack of space, the methodology is not described in detail. On the other hand, the working of the procedure is explained on an example, which follows.

To illustrate the merits of the methodology, the example is focused on rather less intuitive case, which was very important for the remainder of the analyses in this paper. The chosen example is a case where the important information is in the beginning of the data series.

Figure 2 contains estimates of backward-looking parameter in domestic Taylor rule ρ_R and has the following structure – upper panel plots estimates where last observation varies ("last obs") and four different moving-window estimates with three different lengths of time frame. The length of the time frame is in brackets ("25o", "30o", "35o" and "40o".). The horizontal axis is time at the end of period. The last values of estimates are for 2011, which is the fourth quarter of 2010. The circle, plus, cross, square and triangle at time 2011 have the following meaning: "last obs" estimate at 2011 is an estimate of the parameter on time frame from the beginning (which is 1998.5) to the time denoted by the horizontal axis (2011). "moving (25o)" estimate uses 2011 as the last observation as well, but it uses just 25 observations, the beginning of the time frame is in this case 2005. "moving (30o)" begins at time 2003.75, "moving (35o)" begins at 2002.5 and "moving (40o)" begins at 2001.25 – all of the estimate have the end of the time frame at 2011.

The lower panel draws estimates with varying first observation (and fixed last observation) and again four different lengths of moving-window estimates. The difference also lies in the horizontal axis which is now the time at the beginning of the time frame. First value of estimates is for 1998.5, which is second quarter of 1998. Following explanation of the meaning of the dot, plus, cross, square and triangle address values at 1998.5. The "first obs" estimate naturally starts at 1998.5 and ends with the latest observation, which is 2011. "moving (25o)", "moving (30o)", "moving (35o)" and "moving (40o)" all start in 1998.5 and end in 2004.5, 2005.75, 2007, and 2008.25 respectively.

Now that it is clear what Figure 2 shows, this paragraph proceeds to interpretation of the contents of the Figure. Upper panel displays a gradual rise in "last obs" estimate but moving-window estimates all jump up at some point and end up in higher value of the parameter – "last obs" estimate ends at 2011 by value approximately 0.71 and all moving-window estimates end up with parameter value 0.8. This result indicates, that there might be some important information somewhere in the beginning of the time series. The lower panel confirms the suspicion because all the displayed estimates exhibit a rise of the parameter estimate by 0.1 in the first 4 observations. It means that no matter the length of the time frame, the exclusion of observations from 1998.5 to 1999 lead unambiguously to a rise in the point estimate of the parameter. In this context, the gradual rise in "last obs" estimate is illusory. New added information in the "last obs" estimate just dilute the strong information in the first three observations.

Similar situation as in the case of parameter ρ_R occurred with some other parameters (namely Calvo parameters $\theta_{\cdot}$), which indicates, that data in time frame from 1998.5 to 1999 belong to different historic era, which was not under investigation and the data in question were therefore discarded from the set for further analyses.

Further analyses are therefore focused on a truncated data set that should represent homogeneous structure.

5 Changes in parameter estimates during recession

After discarding of the data that represented different economic era, the remaining interval from first quarter of 1999 to the last quarter of 2010 was used for the analyses. Since there is an assumption of homogeneous structure (supported by previous analysis), there is no need to carry out "first obs" and moving-window estimates. Figure 3 therefore depicts "last obs" estimates.

The two depicted parameter estimates show foreign central bank's reaction to output growth ψ_2^* (`psi2star`) in the left panel and AR1 persistence parameter in the world-wide technology shock ρ_Z (`rhoZ`) in the right panel. The Figure also depicts two widths of confidence bands. The wider confidence band is 90% band and the narrower band varies and is stated in the title of the panel in question. The width of the narrower confidence band is chosen so that such confidence bands do not overlap for at least two different periods. This can be a measure of how much the point estimate changes realtive to the amount of uncertainty associated with the estimate.

Left panel shows that the reaction of foreign central bank to output growth diminishes from fourth quarter of 2007 and stabilizes after "crisis quarter" 2009.25. This can be explained by looking at the data – foreign output drops, but the interest rates start to go down not before fourth quarter of 2008 and then it drops in first quarter of 2009. This can be interpreted as central bank's negligence towards output growth. However, this conclusion is not

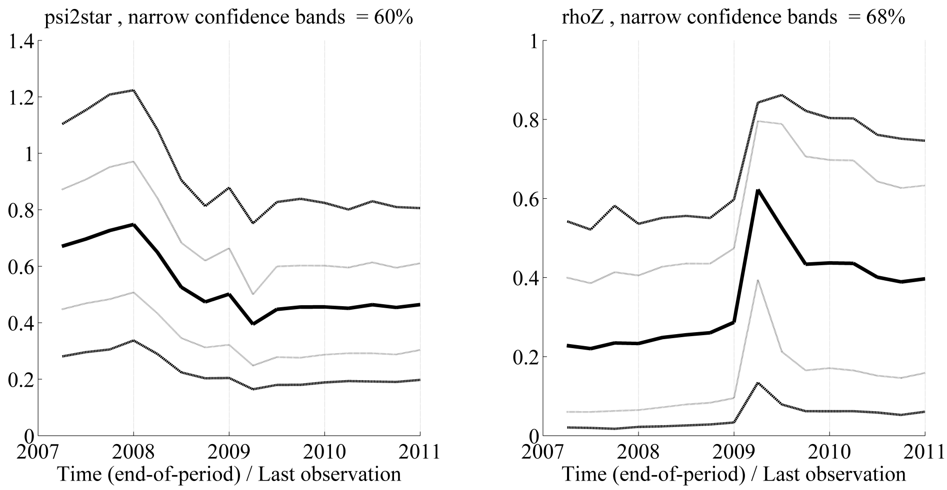

Figure 3: Parameter estimates with different time-frames. Left panel: foreign central bank's reaction to output growth ψ_2^*, right panel: AR1 persistence parameter in the world-wide technology shock ρ_Z, wide confidence bands = 90%

surprising considering the evolution of inflation in first quarters of 2008 – the inflation was quite high and inflation is primary objective of foreign central bank. It therefore had to disregard from output decline and focus on the danger of inflation.

The width of the narrower confidence bands can also be interpreted in a way that 60% confidence bands mean that the confidence bands do not overlap with significance level 0.4. With this significance level, it is possible to say that the reaction of the central bank's changed as a result of recession.

Another example is in the right panel of Figure 3 depicting the evolution of the persistence parameter in the world-wide technology shock. The point estimate of the parameter jumped from pre-crisis value 0.2 to 0.6 in "crisis quarter" 2009.25. The persistence of whatever hits both economies from outside is therefore much more long-lasting. In the mechanics of the model, this increase was used to describe the economic crisis, which was understood by the model workings as negative world-wide technology shock. As the crisis subsided over time, it was not as deep as the model expected it to be and the persistence parameter went back down to 0.4.

The left panel therefore shows a case of changing preferences of one of the economic agents in the model and the right panel shows an important part of model's explaining current economic crisis.

6 Impulse response functions with different time-frames

Figure 4 displays impulse response functions as a result of an innovation with value -0.6 to the system. In this particular case, the unitary innovation is put to an AR1 process z_t describing evolution of growth rate of world-wide technology shock. The value 0.6 was estimated as the standard deviation of the shock and negative value was used to better mimic negative shock causing economic crisis.

Bottom-right axis in panels in Figure 4 is labeled "lag" and it is the lag of the impulse response function. Time 0 corresponds to the time of the innovation, time 1 is one period later and the lag goes up to 15. Each black line in the figure is an ordinary impulse response function that can be found in numerous scientific literature.

Bottom-left axis "time (end-of-period)" addresses different time frames on which the model was estimated. First observation for the estimation was always first quarter of 1999. Last observation varies from first quarter of 2007 to fourth quarter of 2010 and bottom-left axis denotes the time of last observation. More concretely, impulse response functions in the left-hand part of the figure are responses calculated on time frame from 1999.25 to 2007.25. It might be considered as the "oldest" impulse response function. Each other impulse response function more to the right corresponds to impulse response function calculated on time frame longer by one quarter. The second impulse response function from the right therefore corresponds to time frame from 1999.25 to 2007.5. The impulse response function most to the right uses most up-to-date data and is calculated on time frame from 1999.25 to 2011.

The z-axis denoted on the right of the figure is the value of the impulse response functions, which is common

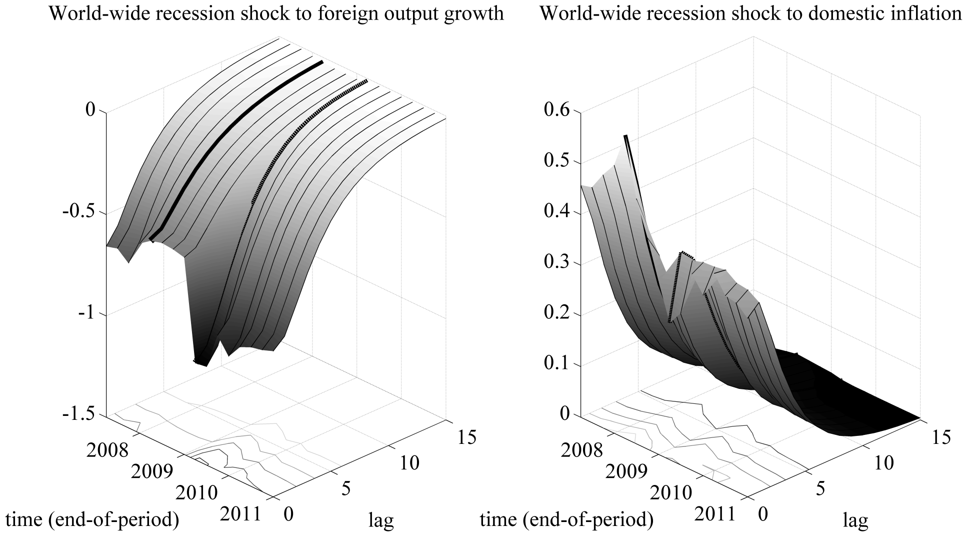

Figure 4: Impulse response functions with different time-frames

in depicting of impulse response functions.

Impulse responses function highlighted in bold are of special interest, since they exhibit biggest difference between each other. The bold impulse response function more to the left (the "older" one) corresponds to pre-crisis time-frame from 1999.25 to 2008.25. The second bold impulse response function is calculated on time frame ending in first quarter of 2009, that is, the period where is the oncoming recession most apparent in the data.

Changing structure of the economy described by model behavior exhibits approximately twice as low drop in foreign output in crisis than it was before crisis as a result of a negative world-wide technology shock. These two highlighted impulse response functions are also shown in left panel of Figure 5, together with 90% confidence bands. Dashed impulse response function corresponds to data set containing crisis period, the solid line depicts pre-crisis impulse response function.

Confidence bands are depicted by shading and dotted borders. Darker shading in the background corresponds to "crisis" dashed impulse response function, whereas lighter shading in the front belong to "pre-crisis" impulse response function. This depiction shows that point estimate of the IRF in crisis is outside pre-crisis confidence bands. On the other hand, upper part of the confidence bands is almost the same and major parts of the confidence intervals therefore overlap.

The right panels of Figures 4 and 5 show the same type of results that were discussed in previous paragraphs. The investigated case was again a world-wide technology shock, but this time the reaction under scrutiny was domestic inflation.

Right panel of Figure 4 displays that the reaction of inflation is much lower in crisis and moreover, the shape of the point estimate of the impulse response function is different. Again, in order to investigate the case in greater detail, right panel of Figure 5 can be used. It is now visible, that the reaction of inflation in crisis is almost nonexistent at the time when the shock occurs. In 3–4 quarters, the inflation rises by 0.3 percentage points. The reaction of inflation before crisis is quite different – inflation jumps by 0.6 percentage points and then it gradually diminishes. However, the confidence bands show a great deal of uncertainty about the point estimates of the impulse response functions. However, the confidence bands show that positive reaction of inflation is much more probable in period before crisis than in economic crisis, when the lower band of confidence interval drops in negative values.

7 Conclusions

The paper presents innovative approaches towards investigation of changes that may occur in the behavior of modeled economies during economic recession and also offers some economic examples.

Section 4 briefly introduced a methodology that could help unveiling different economic eras. The working

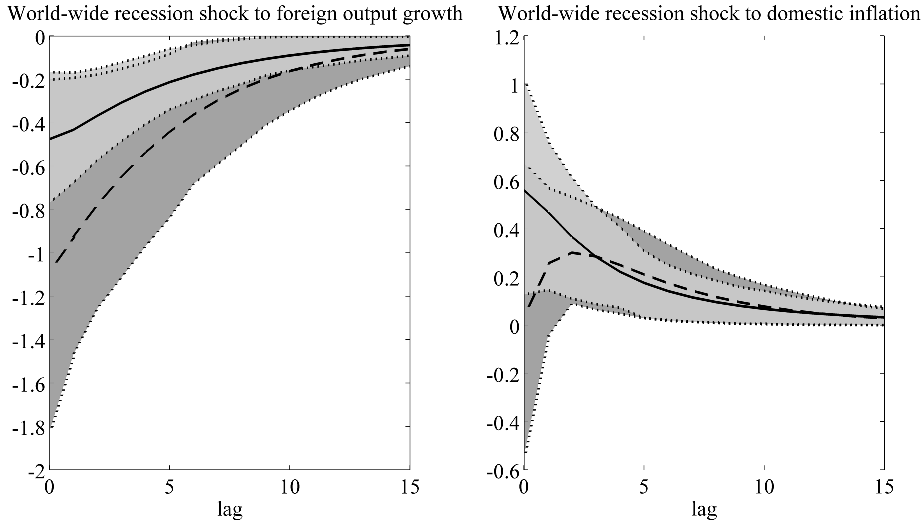

Figure 5: Impulse response functions with different time-frames with 90% confidence bands. Dashed line with background darker shaded confidence bands correspond to "crisis" time frame from 1999.25 to 2009.25. Solid line with lighter shaded confidence bands correspond to "pre-crisis" time frame from 1999.25 to 2008.25.

of the procedure is presented on a case with a resulting change to the data set, which was truncated by first three observations. Section 5 presents two exemplary results of changes in estimated parameter values together with the uncertainty of the point estimates. It turns out that e.g. foreign central bank lowers its preference about output growth while it engages in monetary policy. Section 6 uses parameter sets estimated on different time-frames a computes impulse response functions with different time-frames. 3-D graph of the impulse response functions unveil changes in the reactions of variables in question. Attention is also paid to uncertainty associated with the impulse response functions. The results show a significantly different reaction of foreign output growth to a world-wide technology shock. Another case shows a different shape of impulse response function of domestic inflation. Both of these cases indicate changes in the structure of the economies represented by the model.

Acknowledgements

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