Errata for PhD Dissertation

Malak Khouchen. *Classical Description of Fundamental Strings and D-Branes and their Relation to the AdS/CFT Correspondence.* Masaryk University, Brno, Czech Republic.

The following list of errors was prepared based on the reviews received on my dissertation work. After submitting my dissertation I prepared this list of corrections on the basis of reviewers' suggestions. I chose to list them ordered by the page number where they were found.

Page 21, par. 5, line 3: The sentence "generates an infinite set of charges that are conserved thus proving the classical integrability of the model." should be replaced by "generates an infinite set of charges that are conserved. If these conserved quantities are in involution, this proves the classical integrability of the model."

Page 22, par. 1, directly after equation (2.45): The following sentence should be added "where the subscripts on the R-matrix show in which space the given matrix acts non-trivially (while in the other space they act as the identity operator)."

Page 24, par. 1, line 1: Added the reference [147]: "gives the Bethe ansatz equations [147] for J magnons along chain of length L".

Page 25, par. 2, line 6: Reference numbers [56,57] should be replaced by reference number [148]: "The string sigma model in $AdS_3 \times S^3$ supported by mixed three-form fluxes has recently been proved to be integrable [148]".

Page 26, par. 5, line 2: The sentence "$\gamma^{\alpha\beta}$ is the worldsheet metric with $\det \gamma = -1$" should be replaced by "$\gamma^{\alpha\beta} = h^{\alpha\beta}\sqrt{-h}$ is the Weyl-invariant combination of the worldsheet metric $h_{\alpha\beta}$ with $\det \gamma = -1$".

Page 27, equation (2.65) should be replaced by:

$$\begin{aligned}
&\partial_\alpha(\gamma^{\alpha\beta} A^{(2)}_\beta) - \gamma^{\alpha\beta}[A^{(0)}_\alpha, A^{(2)}_\beta] + \frac{1}{2}\kappa\epsilon^{\alpha\beta}\big([A^{(1)}_\alpha, A^{(1)}_\beta] - [A^{(3)}_\alpha, A^{(3)}_\beta]\big) = 0 \; , \\
&P^{\alpha\beta}_-[A^{(2)}_\alpha, A^{(3)}_\beta] = 0 \; , \\
&P^{\alpha\beta}_+[A^{(2)}_\alpha, A^{(1)}_\beta] = 0 \; , \qquad (2.65)
\end{aligned}$$

Page 31, par. 6, line 3: The sentence "where $\eta_{\alpha\beta}$ is defined by $ds^2 = -d\tau^2 + ds^2$." should be replaced by "where $\eta_{\alpha\beta}$ is defined by $ds^2 = -d\tau^2 + d\sigma^2$. "

Page 40, par. 3, line 3: The sentence " The relevant action consists of two parts" should be replace by "The relevant effective action consists of two parts".

Page 52, par. 3, line 1: The sentence “where $\gamma^{\alpha\beta}$ is the worldsheet metric with $\det\gamma = -1$” should be replaced by “where $\gamma^{\alpha\beta} = h^{\alpha\beta}\sqrt{-h}$ is the Weyl-invariant combination of the worldsheet metric $h_{\alpha\beta}$ with $\det\gamma = -1$”.

Page 54: At the end of this page, I added the following subsection to chapter 3 named "**3.4.2 Recent Developments of the η-Deformed Model**":
“In the last few years, several studies have been published to give a better understanding of the η-deformed model and a considerable progress has been made. An important question for example turns out to be whether or not the corresponding model is type IIB string sigma model. In general, there is no a priori reason why the deformed η-model should define a scale invariant 2d theory and, moreover, why it should preserve the conformal (Weyl) invariance and hence still correspond to a consistent superstring theory as the undeformed $AdS_5 \times S^5$ model does.
In [149,150], it was shown that the deformed background (3.73) fails to satisfy the standard IIB supergravity equations which indicates that the corresponding sigma model is not Weyl invariant, i.e. does not define a critical string theory in the usual sense. An approach followed to find out can be to work out the quadratic fermionic action starting from the η-deformed action and some conveniently chosen representative of the coset $PSU(2,2|4)/SO(1,4) \times SO(5)$. Then we need to find a field redefinition which brings this action into the Green-Schwarz canonical form. This would allow us to identify the background fields and further check if they satisfy the equations of motion of type IIB supergravity and, in particular, to find a solution for the dilaton.
The model in [149] is based on a solution of the modified classical Yang-Baxter equation R-matrix corresponding to the distinguished Dynkin diagram of the $psu(2,2|4)$ superalgebra.
In the corresponding model case, there is no independent information about the dilaton, and there exists no dilaton field that completes background fields to a type IIB solution [149]. The RR couplings were shown not to satisfy the equations of motion of type IIB supergravity.
In fact, the targetspace of the η-model derived in [124] does not solve the type IIB supergravity equations, but rather a generalisation of them [150]. The background fields do not solve the type II supergravity equations, they instead satisfy a set of generalised type II supergravity equations that depend on a background Killing vector. These generalised equations ensure scale invariance for the sigma model, but are not enough to have the full Weyl invariance, which is present only when the target space satisfies the standard equations of supergravity.
Later, in [151], it was shown that a further unimodularity condition is needed to ensure the Yang Baxter deformation has a valid string theory (supergravity) description. For the η-model, the requirement that the target space is a supergravity solution translates into a simple condition on the R-matrix which enters the definition of the deformation. The η-model has an interpretation as a string sigma model precisely for the so-called unimodular R-matrices. The unimodularity condition takes the form:

$$r_{ij}[b_i, b_j] = 0 \tag{1}$$

This is trivially satisfied by abelian r-matrices ($[b_i, b_j] = 0$), but for non-abelian r-matrices this is non-trivial. Thus in the construction considered in [149], the R-matrix is not unimodular in agreement with the fact that the background fields do not solve the type II supergravity equations.

The unimodularity condition or the Weyl invariance condition of the η-deformation was derived in [151] from the requirement that the spinor derivatives of the dilaton to coincide with the dilatinos. The condition has the following form:

$$\hat{\mathcal{K}}^{MN}\text{STr}[[T_M, R(T_N)\}Z] = 0 , \quad \forall Z \in \mathfrak{f} \tag{2}$$

where T_M are basis of $\mathfrak{f}$ and $\hat{\mathcal{K}}^{MN}$ is the inverse of the Lie algebra metric defined by the supertrace:

$$\mathcal{K}_{MN} = \text{STr}[T_M T_N] \tag{3}$$

The approach followed was comparing the general η-model form to the standard type II Green-Schwarz superstring and working out the superspace torsion to extract the background superfields (the supervielbein, NS-NS three-from, R-R bispinor, dilatino and gravitino field strength). It was further proved that the η-model has a standard type II supergravity solution as target space if the Killing vector superfield K_a appearing in the generalized supergravity equations is zero [151]. In fact, it must be that it vanishes order by order in the deformation parameter.

In [152], η-deformation of $AdS_2 \times S^2 \times T^6$ and $AdS_5 \times S^5$ superstring sigma models were studied using a certain class of solutions to the non-split modified classical Yang-Baxter equation (mCYBE) known as Drinfel'd-Jimbo R-matrices which are constructed from a Dynkin diagram and the associated Cartan-Weyl basis. The Drinfel'd-Jimbo R-matrix annihilates the Cartan generators and multiplies the positive and negative roots of the superisometry algebra by $-i$ and $+i$ respectively. It was shown that for the two considered models, the unimodularity condition (no Weyl anomaly) is satisfied if and only if all the simple roots of the corresponding Dynkin diagram are fermionic. Therefore, supergravity backgrounds corresponding to R-matrix associated tothe Dynkin diagram with all fermionic simple roots were constructed for $AdS_2 \times S^2 \times T^6$ and $AdS_5 \times S^5$ and it was confirmed that they satisfy the type IIB supergravity equations.

In a recent paper as well [153], a two-parameter integrable deformation of the $AdS_3 \times S^3 \times T^4$ superstring was constructed. The superisometry algebra of this model has a group-product structure $\hat{G} \times \hat{G}$ and thus allows for a two-parameter integrable deformation. The shape of the deformation is again governed by the R-matrix solving the non-split mCYBE on the superisometry algebra of the undeformed background. Such R-matrices include those of Drinfel'd-Jimbo type. There are now two real deformation parameters η_L and η_R controlling the strength of the deformation in the left and right $\hat{G}$ copy. Two supergravity backgrounds corresponding to two different unimodular Drinfel'd Jimbo R-matrices associated to the fully fermionic Dynkin diagram were derived. The unimodularity condition for this type of Lie superalgebras is equivalent to that the supertrace of the structure constants built out of the R-bracket should vanish.

The complexified algebra $(p)sl(2|2)$ admits three inequivalent Dynkin diagrams associated to different R-matrices that lead to different backgrounds. R-matrices as-

sociated to the fully fermionic Dynkin diagram give rise to two distinct full supergravity solutions. The two inequivalent unimodular R-matrices are of the type $\mathcal{R} = \text{diag}(R_L, R_R)$ where R_L and R_R are Drinfel'd Jimbo R-matrices satisfying the non-split modified classical Yang-Baxter equation on $\mathfrak{psu}(1,1 \mid 2)_L$ and $\mathfrak{psu}(1,1 \mid 2)_R$ respectively.
The bosonic background is common to the two choices of R-matrix, with the metric and closed B-field. However, the dilaton and R-R sector depend on the choice of R-matrix.
This is worth comparing to the solutions found in [117] where two solutions to the supergravity equations of motion with bosonic background and supported by a R-R three form flux have been constructed. In [117], it was shown that the metric of the deformation of $AdS_3 \times S^3 \times T^4$ space can be extended to a 10d type IIB supergravity solution. There is a one-parameter family of solutions with the free parameter $a = a(\kappa)$ of the three scalar equations of the 6d theory. Nevertheless, the full solution supported by the dilaton, RR scalar and RR 3-form strength exists only for two members of this family, $a = 0$ and $a = 1$. This is the deformed background that we use in chapters 4 and 5. The consistent 6d truncated on 4-torus Lagrangian of the full 10d type IIB supergravity Lagrangian is:

$$\mathcal{L}_6 = e^{-2\Phi}[R + 4(\nabla\Phi)^2] - \frac{1}{12}F_{mnp}F^{mnp} - \frac{1}{2}(\partial C)^2 \tag{4}$$

The equations of motion of the scalars can be written as:

$$\nabla^2 C = 0, \quad R + 2\nabla^2\Phi + \frac{1}{2}e^{2\Phi}\partial_m C \partial^m C = 0, \quad \nabla^2(\frac{1}{2}C^2 + e^{-2\Phi}) = 0. \tag{5}$$

Expanding in powers of $\kappa\rho$, a unique solution was found which depends on one parameter a. To find a 3-form flux F_{mnk} supporting (together with the scalar fields) the given metric through the Einstein equation, one has to specify the form of its stress tensor T_{mn}. Given the corresponding six-dimensional background (metric, dilaton,etc.) and computing the effective stress tensor in Einstein equation that should be representing the contribution of the 3-form field, this T_{mn} should satisfy the following conditions in order for F_{mnk} to exist:

$$\text{tr}T = 0\,, \quad \text{tr}T^3 = 0\,, \quad \text{tr}T^5 = 0 \tag{6}$$

It was shown that the expected stress tensor for the 3-form RR field $F_3 = dC_2$ satisfies these relations only for the special values $a = 0, 1$ of the parameter $a(\kappa)$. The small-κ limit solutions can then be extended to solutions with general κ to give the resulting exact solutions:

$$
\begin{aligned}
&\mathbf{a{=}0}: \quad e^{-2\Phi} = \frac{(1-\kappa^2\frac{\rho^2}{L^2})(1+\kappa^2\frac{r^2}{L^2})}{[1-(\frac{\kappa\rho r}{L^2})^2]^2}, \quad C=0\,, \\
&C^{(2)} = \frac{1}{L}\frac{1}{1-(\kappa\frac{\rho r}{L^2})^2}[\rho^2(dt+\kappa L d\varphi)\wedge(d\chi+\kappa\frac{r^2}{L^2}d\psi) - \\
&-r^2(Ld\varphi-\kappa dt)\wedge(d\psi+\kappa\frac{\rho^2}{L^2}d\chi)]\,, \\
&\mathbf{a{=}1}: \quad e^{-2\Phi} = \frac{(1-\kappa^2\frac{\rho^2}{L^2})(1+\kappa\frac{r^2}{L^2})}{[1+\frac{\kappa^2}{L^2}(r^2-\rho^2+r^2\rho^2)]^2}, \quad C=0\,, \\
&C^{(2)} = \frac{\sqrt{1+\kappa^2}}{L(1+\frac{\kappa^2}{L^2}(r^2-\rho^2+\rho^2 r^2))}[\rho^2 dt\wedge d\chi + \\
&+\kappa[r^2-\rho^2+\frac{1}{L^2}(\rho r)^2]dt\wedge d\varphi + \frac{\kappa}{L}(\rho r)^2 d\chi\wedge d\psi - r^2 L d\varphi\wedge d\psi]\,,
\end{aligned}
\tag{7}
$$

So, the solutions of the supergravity equations of motion corresponding to the two R-matrices in [153] are the analogues of these $a=0$ and $a=1$ solutions. The dilatons are exactly the same, but the R-R fluxes are different. This is due to the fact that in [117] solutions of the supergravity equations of motion are only supported by a three-form flux while in [153] the deformed backgrounds possess both a three-form and a five-form flux."

Page 115: Added the omitted citations:

[147] F. Levkovich-Maslyuk, The Bethe ansatz, J. Phys. A49(2016),[arXiv:1606.02950].

[148] A. Cagnazzo and K. Zarembo, B-field in AdS_3/CFT_2 Correspondence and Integrability ,JHEP 1211, 133 (2012),arxiv:1209.4049.

[149] G. Arutyunov, R. Borsato, and S. Frolov. Puzzles of η-deformed $AdS_5 \times S^5$. JHEP, 12:049, 2015.

[150] G. Arutyunov, S. Frolov, B. Hoare, R. Roiban, and A.A. Tseytlin. Scale invariance of the η-deformed $AdS_5 \times S^5$ superstring, T-duality and modified type II equations. Nucl. Phys. B, 903:262–303, 2016.

[151] R. Borsato and L. Wulff. Target space supergeometry of η and λ-deformed strings. JHEP, 10:045, 2016.

[152] B. Hoare and F. K. Seibold. Supergravity backgrounds of the η- deformed $AdS_2 \times S^2 \times T^6$ and $AdS_5 \times S^5$ superstrings. JHEP, 01:125, 2019.

[153] F. K. Seibold. Two-parameter integrable deformations of the $AdS_3 \times S^3 \times T^4$ superstring. JHEP, 10:049, 2019.