AN INVESTIGATION OF STUDENT UNDERSTANDING OF
BASIC CONCEPTS IN SPECIAL RELATIVITY

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Abstract

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This dissertation reports on a systematic investigation of student understanding of the concepts of time and space in special relativity. During the investigation we have identified persistent difficulties with the definitions of the position and time of an event and with the concept of a reference frame. Many students do not think of a reference frame as a system of observers that determine the same position and time for any event. Instead, they interpret statements of the frame-dependence of the time of an event to mean that observers at different locations receive signals from events at different times. When asked to describe measurement procedures for spatial quantities, students do not spontaneously apply the formalism of a reference frame, but instead tend to associate events with moving objects in a manner consistent with indiscriminate application of length contraction. Traditional instruction in relativity appears to have little effect on these ideas, which are present among students from the introductory to the graduate level in physics. We have applied the results from this research to guide the design of instructional materials to address some of the specific difficulties that we identified.

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My graduate work has been some of the most enjoyable and rewarding of my life so far, and I am deeply grateful to those who helped make it such an experience.

My two closest supervisors have taught me that to do this work well is to serve truth, beauty, and human potential. Stamatis Vokos has been the mentor that most people only dream of; with him I find my best self and put her to work. Peter Shaffer has been my compass, and although the North Star will move off axis in the distant future, his heading will stay true. With these two I have learned to cherish mistakes; to combine courage with humor; to strengthen my soul. May I someday give such gifts to a student.

Lillian C. McDermott and Paula R.L. Heron have worked tirelessly on my behalf, and I look forward to continuing collaborations with them from near and far. Other members of the Physics Education Group, past and present, have made substantial contributions to the work described in this dissertation. Andrew Boudreaux was my daily collaborator at the beginning of this project and continues to help shape the work to this day; I thank him not only for his intellectual effort, but also for his friendship. I am grateful to Brad Ambrose and Tara O'Brien-Pride for their insightful preliminary work on student understanding of special relativity. Other colleagues not only subjected early and late versions of the research tasks to intense and invaluable scrutiny, but also made the lab an inviting and stimulating place to work. I am especially indebted to Chris Border, Lezlie Salvatore DeWater, Elain Fu, Chris Kautz, Michael Loverude, Donna Messina, Luanna Ortiz, Mel Sabella, John Thompson, and Karen Wosilait. Steve Kanim, in addition, kept me from rearranging the furniture once too often. Edwin Taylor has given liberally of his insight, experience, humor, and quotations, including those found at the beginning of each chapter of this dissertation. Thanks are due for the ongoing cooperation of the instructors in whose physics classes the research tasks and curriculum have been administered and refined, especially James Bardeen and E. Norval Fortson.

I thank my parents, Jean and Steve Scherr, for their support and confidence in me and their genuine interest in my work. Finally, I am infinitely grateful to my husband, Dale, for his tremendous patience, and for changing with me along the way.

To Dale

"Bless what forces us to invent goodness every morning"

– Marge Piercy, *Amidah*

CHAPTER ONE:

INTRODUCTION TO THE DISSERTATION

> *I do not define time, place, space, and motion, [because they are] well known to all.*
> – I. Newton[1]

A. MOTIVATION FOR THE STUDY

There is growing national interest in increasing the exposure of students in introductory courses to modern physics topics, such as relativity. Proponents of enlarging the scope of the curriculum argue that in the beginning of the 21st century the content of introductory classes should reflect some of the major intellectual breakthroughs of the 20th century. Others hold that the list of topics that must be covered is already too daunting. Physics education research can play a pivotal role in this debate. Whether students first encounter modern physics concepts at the introductory level or in advanced courses, it is important to identify what students can and cannot do after instruction and what steps can be taken to help deepen their understanding of the material. In addition, analyzing the ways in which students undergo the transition between understanding phenomena to which they have immediate access and understanding phenomena that lie outside their everyday experience can help us identify reasoning skills that are needed for the study of advanced topics.

This dissertation describes a systematic investigation of student understanding of basic topics in special relativity. This work is part of an ongoing study by the Physics Education Group at the University of Washington. Over the last five years, the Group has been investigating student understanding of key ideas in Galilean, special, and general relativistic kinematics. Extensive research had already been conducted on student understanding of non-relativistic kinematics in the laboratory frame.[2] One of the major goals of this project was to expand this

research base to relativity in order to provide a guide for the development of instructional materials by ourselves and others.

This investigation began as an effort to identify student difficulties with standard problems[3] in special relativity and evolved into an investigation of student difficulties with the ideas of time, position, events, and reference frames. This evolution was the result of the repeated realization that many of the difficulties that students have with ideas from special relativity, such as length contraction and the relativity of simultaneity, are due in part to an underlying failure to apply the formalism of a reference frame in determining the position and time of an event. The structure of individual chapters in the dissertation reflect this evolution. In each context that we consider, we first describe the difficulties that students have with standard problems and then examine student understanding of the fundamental ideas that underlie these difficulties.

The study described here does not extend to student difficulties with energy and momentum in the context of relativistic collisions. This dissertation also does not address first-order relativistic corrections (in which relative speed is sufficiently small that $g \sim 1$ but the spatial separation between events is sufficiently large that $v\delta x/c^2$ is not negligible).

B. OVERVIEW OF DISSERTATION

The main body of the dissertation consists of four chapters that describe the research, curriculum development, and assessment of curriculum that form the bulk of this study. Chapter Two reports on student understanding of the concept of time in special relativity, with an emphasis on the relativity of simultaneity and the role of reference frames. The results of this research guided the development of instructional materials. The design and assessment of these materials are described in Chapter Three. Chapter Four discusses student understanding of spatial measurements, such as measurements of length and displacement. The design and assessment of instructional materials developed on the basis of this research are presented in Chapter Five.

The dissertation includes several appendices. The first of these describes a particular representation (the "event diagram") that is used in the curriculum developed in this study. This appendix includes a discussion of the relationship of the event diagram to better-known

representations in special relativity, such as Minkowski (spacetime) diagrams. The remaining appendices contain versions of the research tasks and the curriculum that are discussed in the body of the dissertation.

C. FOCUS OF THE RESEARCH

A major goal of the investigation was to determine the extent to which students, after instruction, are able to apply basic ideas of special relativity to simple physical situations. The context is one spatial dimension. The concepts probed are summarized below.

1. Events and reference frames

The construct of a *reference frame* is at the heart of relative motion. Most courses in special relativity begin with a discussion of a reference frame as a system of observers (or devices) by which the positions and times of events are determined. Understanding the concept of a reference frame forms the foundation for understanding any topic in special relativity. It provides the basis for the determination of all kinematical (and other physical) quantities and serves as the framework for relating measurements made by different observers. The concept of a reference frame presupposes an understanding of more basic measurement procedures. For clarity in the chapters that follow, we review the basic operational definitions associated with reference frames, *i.e.,* the determination of the position and time of an event and the conditions under which two events are treated as simultaneous.[4]

An *event* in special relativity is associated with a *single* location in space and a *single* instant in time. The *position of an event* is defined to be the coordinate label at the location of the event on a rigid ruler. The ruler is envisioned to extend indefinitely from some chosen origin.[5] The *time of an event* is most naturally defined as the reading on a clock located at the event's position at the instant at which the event occurs. The rulers and clocks used by any observer are at rest relative to the observer.

All observers in special relativity are assumed to be "intelligent observers" who use synchronized clocks. To determine the time of a distant event, an observer corrects for the travel

time of a signal originating at the event.[6,7] Inertial observers at rest relative to one another determine the same positions and times for events (and hence the same relative ordering of events). Such observers are said to be *in the same reference frame.*[8]

2. Simultaneity

The temporal separation between two events (1 and 2) in a certain reference frame (A) is defined as $\delta t_{12}^{(A)} = t_2^{(A)} - t_1^{(A)}$. That the time of an event is a concept that requires definition is itself a new idea to many students.[9] The definition cited in most texts rests on a primitive notion of simultaneous events: the event in question is simultaneous with the event of the nearby clock reading a certain time. Local simultaneity is a "primitive" concept in the sense that no alternative definition is usually considered.

Simultaneity for distant events, in contrast, must be carefully defined. Events are defined to be *simultaneous* in a given frame if their corresponding time readings are identical, according to the definition of the time of an event discussed above. A judgment of the simultaneity of widely separated events necessarily includes the idea of "spreading time over space,"[10] that is, synchronizing clocks at distant locations. Einstein's original article describing the special theory of relativity begins with these ideas. Einstein considered defining the time of an event as the time at which an observer *sees* the event, but rejected this definition since the time then depends on observer location.[11,12]

The special theory of relativity is based on Einstein's two postulates: first, that the laws of nature and the results of all experiments performed in a given frame of reference are independent of the translational motion of the system as a whole, and second, that the speed of light is finite and independent of the motion of its source.[13] These two postulates have an inevitable and unsettling implication. Two events at different locations that occur at the same time in a given frame are not simultaneous in any other frame. The relativity of simultaneity is among the key results of special relativity and one that is particularly difficult to grasp, as evidenced by the numerous "paradoxes" that arise from it. It is not reasonable to expect that students (even those facile with mathematical formalism) will master all the intricacies of the counterintuitive results that follow from the operational definition of simultaneity. In this study, we considered a

meaningful understanding of relativistic simultaneity to include the ability to identify relevant events, to determine the time at which an event occurs (by correcting for signal travel time), and to recognize that the time interval between two events is not invariant but depends on the reference frame. Other aspects of student ideas about time and reference frames are not discussed (*e.g.,* synchronization of clocks in relative motion, spatial measurements via a latticework of rods, differences between inertial and non-inertial frames).

3. Length and the spatial separation between events

The spatial separation between two events (1 and 2) in a certain reference frame (A) is defined as $\delta x_{12}^{(A)} = x_2^{(A)} - x_1^{(A)}$, *i.e.,* the signed distance between the positions at which the events occur. The displacement of a rigid object is defined operationally as the spatial separation between events occurring at the same point on the object. The length of an object is defined operationally as the magnitude of the spatial separation between two simultaneous events, one occurring at each end of the object. If the length measurement is conducted in a frame in which the object is not moving, the events need not be simultaneous for the spatial separation to equal the length of the object. Note that this measurement procedure for object length does not depend on previous knowledge of the velocity of the object.

The phenomenon of *length contraction* (or *Lorentz contraction*) is among the most striking and memorable relativistic effects. One common derivation of length contraction is based on the fact that the spatial separation between two events is a frame-dependent quantity, as expressed by the Lorentz transformation $\delta x_{12}' = g(\delta x_{12} - v\delta t_{12})$. Suppose events 1 and 2 occur (not necessarily at the same time) at either end of an object at rest in the primed frame. In the primed frame, the spatial separation between events 1 and 2 will equal the length of the object. If events 1 and 2 are simultaneous in the unprimed frame, then the spatial separation between events 1 and 2 will equal the length of the object in the unprimed frame. Using the Lorentz transformation with $\delta x_{12} = L$, $\delta x_{12}' = L'$, and $\delta t_{12} = 0$, we find that the length of the object is greater in the frame in which it is at rest by a factor of gamma.

To illustrate some questions in this dissertation, we use diagrams that we refer to as *event diagrams.* In an event diagram, the events of interest are shown at the locations and times at

which they occur. Later instants are drawn below earlier instants, and the location of each event is indicated in the appropriate picture (*i.e.,* at the appropriate instant). Event diagrams for different reference frames are displayed separately. An example is given below in Figure 1-1, which shows an event diagram illustrating the measurement of the length of an object by means of events, as described in the previous paragraph.
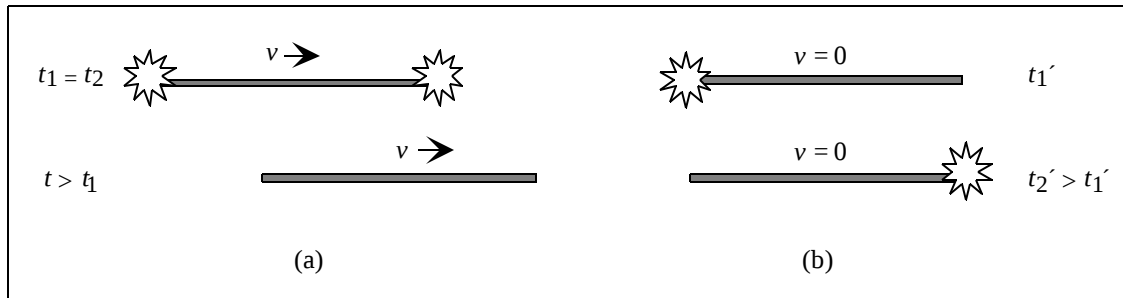
Figure 1-1:   Event diagrams representing events with a spatial separation equal to the length of an object. (a) In general, the length of an object (in this case, a rod) is defined as the spatial separation between two events (in this case, explosions) that occur simultaneously at the ends of the object. (b) In a frame in which the object is at rest, the spatial separation between the events is equal to the length of the object even if the events are not simultaneous.

D. REVIEW OF PREVIOUS RESEARCH

There is currently only a small body of research on student understanding of relativity, mostly in Galilean contexts (in particular, relative motion). Below we summarize relevant results from both Galilean and special relativity. Details of previous research relevant to the issues of individual chapters are reviewed within those chapters.

1. Related research on student understanding of Galilean relativity

In earlier investigations conducted among physics undergraduates in India by Panse, Ramadas, and Kumar have identified the belief that reference frames have limited physical extent.[14]   In their responses to multiple-choice questions, students claimed that objects can "emerge from a reference frame" by leaving the vicinity of the object associated with that frame,

as a cannonball may leave the deck of a boat. The authors further noted the apparent belief that the speed, size, or trajectory of an object in a given frame of reference is not unique but depends on the perception of the viewer. The multiple-choice questions were designed by limited use of interviews and open-ended questions. However, the reasoning behind specific choices by the students was not assessed. Results from this investigation that may be related to the results of Panse et al. are discussed in Chapter Two.

An investigation by Saltiel and Malgrange identified difficulties with relative motion among eleven-year-old-children and first- and fourth-year university students in France.[15] The investigation found little difference in performance among the different groups, all of which tended to identify motion as intrinsic to an object, not a quantity that is measured relative to a reference frame. Students tended to make a distinction between "real" motion, which has a dynamical cause, and "apparent" motion, which is an optical illusion. For example, a poster on an airport wall may appear to be moving for observers on a motorized walkway, but this motion does not have any physical reality. These results are relevant to the research presented in Chapter Four.

O'Brien-Pride has investigated student understanding of measurement procedures for speed with a small number of university students.[16] Her interviews were similar in form to some described in Chapter Four. While most of the students she interviewed were able to describe correct measurement procedures, they lacked facility with changes in reference frame (*e.g.,* measuring the speed of the ground from a moving train.)

2.   Related research on student understanding of special relativity

Villani and Pacca have demonstrated that the spontaneous reasoning of university students in relativistic contexts is similar to that observed by Saltiel and Malgrange in Galilean contexts.[17] Student difficulties with Galilean relativity, especially the belief in absolute motion, are identified as major obstacles to progress in the study of special relativity for both undergraduate and graduate students in physics. Chapter Four presents results related to this research.

A case study by Hewson with a single physics graduate student illustrated the effect of his belief that time is absolute on his understanding of special relativity.[18] Such "metaphysical beliefs" led him to classify certain relativistic effects (including length contraction) as distortions of perception. Posner et al. reported similar results in interviews with introductory students and their instructors.[19] These findings are consistent with those discussed in Chapter Two.

O'Brien-Pride, working with colleagues in the Physics Education Group at the University of Washington, conducted interviews and administered early versions of some of the research tasks described here in which university students appear to believe that the order of events depends on observer location.[20] Her preliminary results provided impetus for the investigation detailed in Chapter Two.

Few researchers have documented the effectiveness of particular instructional strategies designed to address known student difficulties with special relativity. O'Brien-Pride designed and tested a tutorial on the relativity of simultaneity that shares some features with tutorials described in Chapter Three.[21] She observed modest improvement on research tasks similar to those described in that chapter. Horowitz and Taylor describe computer simulation software ("RelLab") that illustrates relativistic phenomena.[22] In a classroom test with 40 high-school students, students used the software to construct and "run" classic relativity paradoxes. The software was used to supplement traditional instruction. In a post-test, the researchers observed good performance on one of the tasks shown by Villani and Pacca to be difficult for physics graduate students.

E. RESEARCH METHODS

The research methods that have been used in this study are similar to those described in other studies conducted by our group. We use a variety of methods in order to maximize our understanding of the nature and prevalence of student difficulties. The research has primarily been performed with university students who are currently enrolled in physics courses or have recently completed such courses.

1. **Individual student interviews**

In-depth knowledge of student understanding is obtained through individual student interviews. Each interview lasts for about one hour. One or more investigators meet with the student and pose a series of tasks about a particular physical situation. Students are asked to think aloud as they respond to the tasks and, in particular, to articulate the reasoning they are using to arrive at their responses. The interviews typically have a predetermined protocol, with a series of specific questions that are asked of all volunteers. However, the interviews are open-ended, in that the interviewer is free to ask additional questions to probe certain ideas more deeply. In some cases, the interviews include questions that we plan to incorporate into instructional materials. Student responses are analyzed to determine the effectiveness of the questions. The interviews are videotaped for later transcription and analysis. The students who participate in individual interviews volunteer to do so and tend to be from the top half of the class.

2. **Written questions**

We have designed written questions for use in a variety of contexts. *Pretest questions* are usually used to determine the prevalence and persistence of conceptual and reasoning difficulties that have been identified previously in the research (usually through interviews or other written questions). The questions are usually qualitative in nature. When particular difficulties are already known, the questions are designed so that the difficulties lead to specific incorrect responses. Pretest questions are often administered after traditional instruction and before research-based instruction, but may also be administered before there has been any instruction. *Examination questions* are posed after traditional or research-based instruction in order to assess the effectiveness of that instruction. In these cases, we work with the course instructor to ensure that the problems are representative of the material covered in the course.

3. **Informal methods**

Like many instructors, we have informal discussions with students in order to gain insight into their understanding of the subject matter. Questions posed by students during instruction or in office hours, responses given by students on homework assignments, and conversations among

students are rich sources for the initial stages of formulating research questions. These questions are then investigated more systematically by the formal methods described above.

F. DEVELOPMENT OF CURRICULUM

Work by the Physics Education Group and other researchers in identifying student conceptual difficulties has demonstrated that instruction in which students are passive learners often fails to address the conceptual difficulties with which many students enter the course and can contribute to the development of new misconceptions.[23] Results have shown that, for many qualitative problems, student performance is essentially the same before and after instruction that consists primarily of lecture and textbook exercises.[24] In contrast, instruction in which students are actively engaged in the construction of ideas can lead to significant improvements in conceptual understanding.[25] This dissertation describes examples of such instruction in detail in Chapters Three and Five. Below is an overview of the approach taken by the Physics Education Group in developing instructional materials.

Essential to the development of effective curriculum is systematic assessment with the population of students for whom it is intended. In this study, as in others carried out by our group, this assessment is primarily based on the analysis of the results of student responses to written problems. When possible, we compare the performance on these problems of students who have completed standard (lecture and textbook) instruction in a topic and of students who have also completed instruction using the curriculum that we have developed. In some cases, we also compare the performance of students before instruction to that after instruction. By analyzing student performance before and after use of the curriculum, we can determine to what extent the curriculum has helped students to develop the ability to answer conceptual questions. This analysis suggests that the curriculum either has successfully addressed the student difficulties or that it needs modification. We go through an iterative cycle of research, development of curriculum, testing of curriculum, further research, and further development of curriculum. Using this iterative process, the Physics Education Group has developed curricular materials that are designed to develop conceptual understanding, address student difficulties, and improve reasoning skills. Two primary curriculum development projects result from this work,

Physics by Inquiry[26] and *Tutorials in Introductory Physics.*[27] The curriculum described in this dissertation is part of the *Tutorials* project.[28]

The preliminary edition of the *Tutorials* is designed to supplement a traditional textbook in a large lecture-based introductory course.[29] The work described in this dissertation is part of an effort to extend the use of *Tutorials* to more advanced contexts. The curriculum is composed of a series of *tutorial sequences.* The instructional method used in many of these sequences can be characterized by the terms *elicit-confront-resolve.*[30] Each tutorial sequence includes a *pretest,* a *tutorial worksheet, tutorial homework,* and a *post-test.* The pretest is a short qualitative written problem that tests student understanding on the topic of the tutorial (see also research methods, above) and serves to elicit student difficulties by having students commit to a written response. The tutorial worksheets are a series of carefully structured written and experimental tasks that guide students through the reasoning necessary to develop a sound conceptual understanding of a topic. As they progress through the tutorial worksheets, students, working together in groups of four, typically confront the conceptual difficulties elicited by the pretest. They are then guided to resolve contradictions. Tutorial instructors do not lecture but rather engage students in semi-Socratic dialogues intended to guide them in answering their own questions. Tutorial homework helps students to extend, reflect upon, and generalize the concepts studied in the tutorial worksheets. In all cases, questions based on the tutorials are covered on course examinations.

G. STUDENT POPULATIONS

This investigation was conducted over a period of five years at the University of Washington and at three other large research universities. Fourteen instructors at the University of Washington and one faculty member at each of the other universities have cooperated with the Physics Education Group in this study.

Most of the research was conducted at the University of Washington in courses that include special relativity. The study has involved about 800 students from about 30 sections of various courses. The populations include: non-physics students (in the descriptive liberal arts physics course); introductory students (in the introductory calculus-based honors course and in the sophomore-level course on modern physics); advanced undergraduate students (in the junior-level

course on electricity and magnetism and in an elective course on relativity and gravitation); and students in our upper-division course for prospective high school physics teachers.[31] We also present results from physics graduate students at the University of Washington who participated in interviews and others who were given a written question on a graduate qualifying examination. In addition, the investigation includes students in the honors section of the calculus-based course at two of the other research universities (the Massachusetts Institute of Technology and Oregon State University) and advanced undergraduate students from the other collaborating university (The Ohio State University). We found that student performance from all three universities was similar. The results, therefore, have been combined.[32]

1. Non-physics students

At the University of Washington, special relativity is included in several courses for non-physics majors. This study includes results from the Liberal Arts Physics course, in which students spend about two weeks on special relativity. The text used in the quarter in which we were involved was *Physics: A world view* by Kirkpatrick and Wheeler.[33] We refer to the students from this course as "non-physics students."

2. Introductory students

The majority of students who engage in formal coursework in special relativity at the University of Washington first encounter this topic in either the honors section of the introductory calculus-based course or the sophomore modern physics course, which is the final course in the five-quarter introductory sequence. Only a small number of students take both courses. Each of these courses devotes about two weeks of instruction to special relativity; the texts are Resnick, Halliday, and Walker[34] and Tipler's *Modern Physics.*[35] Students who are taking (or have taken) one or the other of these courses, but not more advanced courses, will be referred to as "introductory students" in this dissertation.

3. Advanced undergraduate students

Special relativity is typically covered in some detail in two upper-division physics courses at the University of Washington: the junior electricity and magnetism course and a junior elective

in special and general relativity. Students in these courses have typic ally taken the sophomore modern physics course, and therefore have previously studied special relativity. We refer to these students as "advanced undergraduate students."

The electricity and magnetism course typically devotes about a week to relativistic kinematics, following Griffiths' text.[36] The elective relativity course spends four to five weeks exploring the same topics in greater depth and uses Taylor and Wheeler's *Spacetime Physics*.[37] In cases in which we combine results from these two groups of students, the relevant questions were administered after equivalent instruction.

4. Graduate students

Graduate students in physics at the University of Washington have typically had some instruction in special relativity during their undergraduate study. They receive additional instruction in special relativity as part of the graduate course in electrodynamics typically taken in the first year of graduate school. The course uses the text by Jackson.[38] Most of the graduate student participants in our study were first-year students who had already studied special relativity in the graduate course; a few were more advanced graduate students who had taken the course in the past. We refer to these students collectively as "graduate students."

NOTES TO CHAPTER ONE

[1] I. Newton, *Principia* (University of California Press, Berkeley, CA, 1971), Vol. I, p. 6. Translation by A. Motte; revised by F. Cajori.

[2] For an extensive bibliography, see the relevant sections in L.C. McDermott and E.F. Redish, "Resource Letter: PER-1: Physics Education Research," Am. J. Phys. **67**, 755 (1999).

[3] By 'standard problems,' we mean the type of problems usually found at the end of textbook chapters and on undergraduate physics examinations.

[4] That these are concepts that require definition is itself a new idea to many students. For insightful discussions of these definitions and the pedagogical concerns that they raise, see, for instance, P.W. Bridgman, *A sophisticate's primer of relativity* (Wesleyan University Press, Middletown, CT, 1962) and A.B. Arons, *A guide to introductory physics teaching* (Wiley, New York, NY, 1990).

[5] The definition of a global coordinate system breaks down in non-inertial frames and in general relativity. The need for an inertial frame to have finite extent in both space and time is a refinement not usually encountered in courses in special relativity.

[6] One method of synchronizing clocks in special relativity includes sending the reading on one clock to a clock at another location by means of some signal. The second clock is synchronized with the first by setting it to read the time sent from the first clock plus the signal travel time. The use of light signals for the synchronization of clocks is customary but not necessary. See, for instance, the first book in Ref. 9.

[7] Although it is possible to define the time of an event as the time at which an observer *sees* the event, the time then depends on observer location. Einstein, for instance, considered and rejected such a definition. See A. Einstein, "On the electrodynamics of moving bodies," in *The principle of relativity: A collection of original memoirs on the special and general theory of relativity* (Dover, New York, NY, 1952).

[8] Some authors define the term "observer" to indicate the full set of measuring devices and procedures that comprise a reference frame. For an example of this approach, see E.F. Taylor and J.A. Wheeler, *Spacetime Physics* (W.H. Freeman, New York, NY, 1992), p. 39.

[9] For insightful discussions of these definitions and the pedagogical concerns that they raise, see, for instance, P.W. Bridgman, *A sophisticate's primer of relativity* (Wesleyan University Press, Middletown, CT, 1962); A.B. Arons, *A guide to introductory physics teaching* (John Wiley & Sons, New York NY, 1990).

[10] P.W. Bridgman, ref. 9.

[11] A. Einstein, "On the electrodynamics of moving bodies," in *The principle of relativity: A collection of original memoirs on the special and general theory of relativity* (Dover, New York, NY, 1952).

[12] The observer-dependent definition was standard for synchronizing European train stations in Einstein's day. See P. Galison, "Einstein's clocks: The place of time," Critical Inquiry **26**, 355 (2000).

[13] J.D. Jackson, *Classical electrodynamics,* third edition (New York, NY, John Wiley & Sons Inc., 1999), pp. 517-8.

[14] S. Panse, J. Ramadas, and A. Kumar, "Alternative conceptions in Galilean relativity: frames of reference," Int. J. Sci. Educ. **16**, 63 (1994); J. Ramadas and A. Kumar, "Alternative conceptions in Galilean relativity: inertial and non-inertial observers," Int. J. Sci. Educ. **18**, 615 (1996).

[15] E. Saltiel and J.L. Malgrange, " 'Spontaneous' ways of reasoning in elementary kinematics," Eur. J. Phys. **1**, 73 (1980).

[16] T.E. O'Brien-Pride, "An investigation of student difficulties with two dimensions, two-body systems, and relativity in introductory mechanics," Ph.D. dissertation, Department of Physics, University of Washington, 1997.

[17] A. Villani and J.L.A. Pacca, "Students' spontaneous ideas about the speed of light," Int. J. Sci. Educ. **9**, 55 (1987); "Spontaneous reasoning of graduate students," Int. J. Sci. Educ. **12**, 589 (1990).

[18] P.W. Hewson, "A case study of conceptual change in special relativity: The influence of prior knowledge in learning," Int. J. Sci. Educ. **4**, 61 (1982).

[19] G. Posner, K. Strike, P. Hewson, and W. Gertzog, "Accommodation of a scientific conception: Toward a theory of conceptual change," Sci. Ed. **22**, 211 (1982).

[20] See ref. 16.

[21] See ref. 16.

[22] P. Horowitz and E.F. Taylor, " 'Relativity readiness' using the RelLab program," Phys. Teach. **32**, 81 (1994)

[23] An example of a difficulty that seems to be connected to instruction can be found in F.M. Goldberg and L.C. Mcdermott, "An investigation of student understanding of the real image formed by a converging lens or concave mirror," Am. J. Phys. **55**, 108 (1987). Students in an introductory optics course were shown a lens that was producing a real image of the filament of a light bulb on a screen. Many students predicted that covering the top half of the lens would remove half of the image, often citing the principal rays algorithm that is typically taught in the class.

[24] For evidence in support of this statement, see, for example, L.C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding," Am. J. Phys. **60,** 994 (1992); Printer's erratum to Part I, Am. J. Phys. **61**, 81 (1993), P.S. Shaffer and L.C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity, Part II: Design of instructional strategies," Am. J. Phys. **60**, 1003 (1992), and McDermott, L.C., "Millikan Lecture 1990: What we teach and what is learned—Closing the gap," Am. J. Phys., **59**, 301 (1991).

[25] See, for example, R. Hake, "Interactive engagement versus traditional methods: a six-thousand-student study of mechanics test data for introductory physics courses," Am. J. Phys. **66,** 64 (1998).

[26] L. C. McDermott and the Physics Education Group at the University of Washington, *Physics by Inquiry,* Vols. I and II, (New York, NY, John Wiley & Sons Inc., 1995).

[27] L.C. McDermott, P.S. Shaffer, and the Physics Education Group, *Tutorials in Introductory Physics,* Preliminary Edition, (Upper Saddle River, NJ, Prentice Hall, 1998).

[28] Related curriculum development is taking place as part of the *Physics by Inquiry* project.

[29] For a description of the modifications to the calculus-based introductory course at the University of Washington that led to the development of tutorials, see, for example, P.S. Shaffer, "Research as a guide for improving instruction in introductory physics," Ph.D. dissertation, Department of Physics, University

of Washington, R. R. Harrington, "An investigation of student understanding of electric concepts in the introductory university physics course," Department of Physics, University of Washington, P. Kraus, "Promoting active learning in lecture-based courses: demonstrations, tutorials, and interactive tutorial lectures," Department of Physics, University of Washington, Ph.D. dissertation, K. Wosilait, "Research as a guide for the development of tutorials to improve student understanding of geometrical and physical optics," Ph.D. dissertation, Department of Physics, University of Washington, as well as the papers in reference 24.

[30] See K. Wosilait, P.R.L. Heron, P.S. Shaffer, and L.C. McDermott, "Development and assessment of a research-based tutorial on light and shadow," Am. J. Phys. **66**, 906 (1998), K. Wosilait, P.R.L. Heron, P.S. Shaffer, and L.C. McDermott, "Addressing student difficulties in applying a wave model to the interference and diffraction of light," Phys. Educ. Res., Am. J. Phys. Suppl. **67**, S5 (July 1999), and L.C. McDermott, P.S. Shaffer and M.D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," Am. J. Phys. **62,** 46 (1994) for more description of this approach for addressing student difficulties.

[31] For a description of the course for high school teachers, see L.C. McDermott, "A perspective on teacher preparation in physics and other sciences: The need for special courses for teachers," Am. J. Phys. **58**, 734-742 (1990).

[32] The results from the various classes at the University of Washington were consistent within statistical fluctuations. The results from the other universities were within the same range. For the purposes of this investigation, the results from corresponding classes have been combined.

[33] L. Kirkpatrick and G. Wheeler, *Physics: A world view* (Saunders College Publications, Fort Woth, TX, 1998)

[34] R. Resnick, D. Halliday, and K.S. Krane, *Physics,* 4th edition (John Wiley & Sons, New York, NY, 1992)

[35] P.A. Tipler, *Modern Physics* (Worth Publishers, New York, NY, 1978)

[36] D.J. Griffiths, *Introduction to Electrodynamics* (Prentice-Hall, Englewood Cliffs, NJ, 1989)

[37] E.F. Taylor and J.A. Wheeler, *Spacetime Physics* (W.H. Freeman, New York, NY, 1992)

[38] See Ref. 13.

CHAPTER TWO:

STUDENT UNDERSTANDING OF TIME IN SPECIAL RELATIVITY:

SIMULTANEITY AND REFERENCE FRAMES

> *"What time is it, Casey?"*
> *"You mean right now?"*
> – Casey Stengel[1]

A. INTRODUCTION

This chapter reports on an investigation of student understanding of time in special relativity. The emphasis is on the relativity of simultaneity and the role of reference frames. We found that students have serious difficulties with determining the time at which an event occurs, recognizing the equivalence of observers at rest relative to one another, and applying the definition of simultaneity. In the following discussion we describe how we gradually obtained a detailed picture of student thinking by the design and successive refinement of a set of research tasks.

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B. REVIEW OF PREVIOUS RESEARCH

There is a small body of research on student understanding of reference frames that is relevant to the findings reported in this chapter. Studies reporting students' belief that reference frames have limited physical extent (in the sense that objects may enter and leave them) may be relevant to aspects of this research.[2] The results we will report are consistent with studies

reporting the importance of students' "metaphysical beliefs" (*e.g.,* time is absolute) to their understanding of special relativity.[3]

O'Brien-Pride has conducted a small number of interviews related to the Spacecraft and Seismologist questions described in section C.[4] Her results are suggestive of the results detailed in section E. O'Brien-Pride also gave preliminary versions of the location-specific Spacecraft and Seismologist questions to 48 introductory students. Their performance was consistent with that described below.

C. RESEARCH TASKS AND PRIOR INSTRUCTION

In this section we describe the questions we have used to probe student understanding of simultaneity and the measurement of the time of an event. We also discuss the student populations to which each question was posed, the prior instruction students had when answering these questions, and the format for administering each question (written, interview, *etc.*)

To obtain an increasingly deeper understanding of how students apply the concepts of simultaneity and reference frame, we used variants of three questions: the Spacecraft question (four versions), the Explosions question, and the Seismologist question. All involve two observers with a given relative motion. Students are told the time ordering of the events for one observer and asked about the time ordering of the events for the second observer. These questions and their solutions are described below.

Each research question was posed in at least two ways, with a different physical context and/or slight changes in wording. In general, we found that such changes had little effect on student performance. Therefore, in this paper we present only a representative description of each question. A complete set of the research questions for this chapter appears in Appendix B.

In order to minimize distraction, some student responses that originally referred to one context have been "standardized" to refer to the context used in this chapter. Any modifications to student quotes are explicitly noted where they appear.

1. Spacecraft question

a. *Description of the question*

Results from four versions of the Spacecraft question are discussed in this paper.   All involve two volcanoes, Mt. Rainier and Mt. Hood, that erupt simultaneously according to an observer at rest on the ground, midway between the volcanoes.[5]   A spacecraft moves at a given relativistic velocity from Mt. Rainier to Mt. Hood.[6]   Students are asked questions that probe their beliefs about the order of the eruptions in the moving frame.

b. *Correct response*

A correct answer to all versions can be obtained by qualitative or quantitative reasoning or from a spacetime diagram. The following is an example of a qualitative argument that we would have accepted as correct.[7] In the spacecraft frame, light from the two eruptions moves outward at the speed of light in spherical wavefronts from two points that are stationary. In that frame, the observer on the ground, who receives both signals simultaneously, is moving backward (*i.e.,* in the direction of an arrow pointing from the front of the spacecraft toward the rear). According to the spacecraft observer, the ground-based observer is closer to the center of the signal from Mt. Rainier at the instant that observer receives both signals. The spacecraft observer therefore concludes that Mt. Hood erupted first since its signal travels farther in order to reach the ground-based observer at the same time as the signal from Mt. Rainier.

A correct answer can also be obtained using the Lorentz transformation for time: $\delta t' = g(\delta t - v\delta x/c^2)$, $g = (1 - v^2/c^2)^{-1/2}$. In this context, $\delta t' = t_H' - t_R'$ and $\delta t = t_H - t_R$ are the elapsed times between the eruptions at Hood and Rainier in the spacecraft frame and the ground frame respectively, v is the velocity of the spacecraft relative to the ground, and $\delta x = x_H - x_R$ is the spatial coordinate separation between the eruptions in the ground frame.[8]   Taking the positive direction to be directed from Rainier to Hood, then $v > 0$ and $\delta x > 0$. Since $\delta t = 0$ (simultaneous eruptions in the ground frame), then $\delta t' < 0$.

c.   Versions of the question

At each stage of our study, we tried to determine whether student responses truly reflected their understanding of the material. For instance, we wanted to determine the extent to which specific difficulties are linguistic or conceptual and the extent to which mistaken beliefs are easily addressed or deeply held. To this end, we continually refined the research tasks. Results from earlier tasks guided us in designing new questions that would enable us to probe student thinking more thoroughly.

Four versions of the Spacecraft question will be presented in this chapter. As we will describe in sections D and E, student responses to the four versions were considerably different. Since each new version was motivated by student responses to an earlier version of the question, we will describe the different versions here only briefly along with a summary of their administration to different student populations. Detailed discussions of the different versions appear in sections D and E.

i.   Undirected version

We refer to the first version of the Spacecraft question as *undirected*. We were interested in finding out whether or not students would recognize, without prompting, that simultaneity is relative and, if not, the degree of prompting that is necessary for them to apply the relativity of simultaneity.

The students were asked to draw spacetime diagrams for both the ground and spacecraft frames. They were told to show the volcanoes, the spacecraft, and the eruption events. They were not asked explicitly whether the eruption events are simultaneous in the spacecraft frame. Rather, we inferred their ideas indirectly from their diagrams.

> Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes. A fast spaceship flies directly from Rainier toward Hood with constant speed $= 0.8\,c$.
>
> Sketch spacetime diagrams for the ground frame and for the spacecraft frame. Explain your reasoning.

Figure 2-1:   Undirected version of the Spacecraft question.

ii.   Directed version

In the *directed* version of the Spacecraft question, students are asked explicitly whether, in the reference frame of the spacecraft, Mt. Rainier erupts before, after, or at the same time as Mt. Hood. They are also asked to find the time between the eruptions. The directed version of the Spacecraft question is shown in Figure 2-2.

> Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame (the Earth), suddenly erupt at the same time as determined by observers on Earth.
>
> 1. What is the time interval between the two eruptions as determined by observers in a fast spacecraft (v=0.8c) flying directly from Rainier toward Hood? Show your work.
>
> 2. Which eruption occurs first according to the observers in the spacecraft? Explain.

Figure 2-2:   Directed version of the Spacecraft question.

iii.   Location-specific version

In the third *(location-specific)* version of the Spacecraft question, students are told that the spacecraft, which is flying from Mt. Rainier to Mt. Hood, is over Mt. Rainier at the instant Mt. Rainier erupts. The eruption events, which are simultaneous in the ground frame, are explicitly labeled as Event 1 (Mt. Rainier erupts) and Event 2 (Mt. Hood erupts). Students are to determine whether, in the reference frame of the spacecraft, Event 1 occurs before, after, or at the same time as Event 2. Figure 2-3 shows the location-specific version of the Spacecraft question.

Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes. A fast spacecraft flying with constant speed $v = 0.8c$ from Rainier towards Hood is directly over Mt. Rainier when it erupts.

Let Event 1 be "Mt. Rainier erupts," and Event 2 be "Mt. Hood erupts."

In the reference frame of the spacecraft, does Event 1 occur *before, after,* or *at the same time as* Event 2? Explain your reasoning.

Figure 2-3:   Location-specific version of the Spacecraft question.

iv.   Explicit version

In the *explicit* version of the Spacecraft question, students are told that "observers are intelligent observers, *i.e.,* they correct for signal travel time in order to determine the time of events in their reference frame. Each observer has clocks that are synchronized with those of all other observers in his or her reference frame." Figure 2-4 shows the explicit version of the Spacecraft question.

In this problem, all events and motions occur along a single line in space. Non-inertial effects on the surface of the Earth may be neglected.

Two volcanoes, Mt. Rainier and Mt. Hood, are 300 km apart in their rest frame. Each erupts suddenly in a burst of light. A seismologist at rest in a laboratory midway between the volcanoes receives the light signals from the volcanoes at the same time. The seismologist's assistant is at rest in a lab at the base of Mt. Rainier.

Define Event 1 to be "Mt. Rainier erupts," and Event 2 to be "Mt. Hood erupts."

A fast spacecraft flies past Mt. Rainier toward Mt. Hood with constant velocity $v = 0.8\,c$ relative to the ground ($g = 5/3$). At the instant Mt. Rainier erupts, the spacecraft is directly above it and so the spacecraft pilot receives the light from Mt. Rainier instantaneously.

All observers are *intelligent* observers, *i.e.,* they correct for signal travel time to determine the time of events in their reference frame. Each observer has synchronized clocks with all other observers in his or her reference frame.

For each intelligent observer below, does Event 1 occur *before, after,* or *at the same time as* Event 2? Explain.

- Seismologist

- Seismologist's assistant

- Spacecraft pilot

Figure 2-4:   Explicit version of the Spacecraft question.

d. *Administration of the question: student populations and prior instruction*

We have given the Spacecraft question as a written question before and after traditional instruction to several hundred students in over a dozen non-physics, introductory, and advanced undergraduate physics courses. The question has also appeared on the graduate qualifying exam for doctoral candidacy. In addition, we have conducted interviews with about 20 graduate and advanced undergraduate physics students.

In all cases in which the Spacecraft question was given after instruction, the relativity of simultaneity had been introduced either by means of the Lorentz transformations, as a consequence of length contraction, or by a discussion of a paradox such as Einstein's train paradox.[9] The train paradox approach was favored in less advanced classes, presumably because it requires a minimum of mathematical sophistication; it also has the advantage of proceeding

solely from basic principles, in particular, causality, the invariance of the speed of light, and the isotropy of free space. The discussions of the train paradox conducted in class or appearing in the textbook relied on the reception of light signals by certain observers. Almost invariably, for any particular physical situation that is discussed, the observers of interest are equidistant from the events in question, so that the order of reception of signals is the same as the order in which the events occur.

In some introductory courses and in all advanced undergraduate and graduate courses, the Lorentz transformations were emphasized as one important means of determining (or verifying) the relativity of simultaneity. Students were introduced to the idea of event coordinates and expected to apply the Lorentz transformations in standard textbook problems.

2. Explosions question

The Explosions question is the converse of the Spacecraft question. Students are told that two events occur at different times in a given frame and are asked if there is another frame in which the events are simultaneous.

a. Description of question

In the Explosions question, an explosion occurs at each end of a landing strip with proper length of 3000 m. In the frame of an engineer at rest on the strip, the explosion at the right end occurs a time $c\,dt = 1200$ m after the explosion on the left end. (In some variations, students were given a time of $dt = (1200 \text{ m})/c = 4\ \mu s$. The conversion seemed to present no difficulty.) Students are asked whether there is a frame in which the explosions are simultaneous, and if so, to determine the relative velocity of the frame.

> Two harmless explosions occur at the ends of a landing strip whose proper length is 3000 m. In the reference frame of the landing strip engineer (at rest on the strip), the first explosion occurs at the left end of the strip, and the second explosion occurs at the right end of the strip a time δt = 1200 m later.
>
> Is there a reference frame in which the two explosions occur at the same instant? If so, determine the magnitude and direction of the velocity of this frame relative to the landing strip. If not, explain why not.

Figure 2-5:   The Explosions question.

b. Correct response

A correct answer can be found through use of the Lorentz transformations.[10] The spatial separation between the explosions (*δx*) is 3000 m and the time separation (*c δt*) is 1200 m. Thus the time duration between the explosions *(c $\delta t'$)* is zero in a frame that moves from left to right with speed 0.4*c*.

$$c\delta t' = \boldsymbol{g}(c\delta t - v\delta x/c)$$
$$0 = \boldsymbol{g}(1200 \text{ m} - v(3000 \text{ m})/c) \qquad \text{(positive direction to the right)}$$
$$v = 1200 \text{ m} / (3000 \text{ m}/c) = 0.4c$$

c. Administration of the question: student populations and prior instruction

The Explosions question has been given to about 200 students in introductory and advanced undergraduate physics courses. The question was given on an examination after instruction in special relativity, as described in relation to the Spacecraft question (page 25). We have also asked the Explosions question during interviews with advanced undergraduate and graduate students and on the graduate qualifying exam for doctoral candidacy.

3. Seismologist question

a. Description of the question

The Seismologist question probes student understanding of reference frames and simultaneity within a single reference frame. The context is similar to that of the Spacecraft question: two volcanoes, Mt. Rainier and Mt. Hood, suddenly erupt and a seismologist at rest

midway between them sees the eruptions at the same instant. The Seismologist question differs from the Spacecraft question in that the second observer (the "assistant") is not moving, but remains at rest relative to the ground at the base of Mt. Rainier. Students are asked whether Mt. Rainier erupts before, after, or at the same instant as Mt. Hood in the reference frame of the assistant.

Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes. The seismologist's assistant is at rest in another laboratory at the base of Mt. Rainier.

In the reference frame of the seismologist's assistant, does Rainier erupt *before, after,* or *at the same time as* Hood? Explain your reasoning.

Figure 2-6:   The Seismologist question.

b. Correct response

To answer the Seismologist question correctly, students must be able to apply the definition of simultaneity and understand the role of a reference frame in establishing a common time coordinate for observers at rest relative to one another. That is, they must understand and be able to apply the idea of an intelligent observer, who corrects for signal travel time as necessary to determine the time of an event. Since the seismologist and the assistant are intelligent observers in the same reference frame, they obtain the same answer for the order of the eruptions. Since the seismologist is equidistant from the mountains, the signal travel times are the same. Therefore, the eruptions occurred at the same time in the frame of the seismologist and the assistant.

c. Modifications to the question

In the course of our efforts to better understand students' approach to this basic material, we have asked versions of the Seismologists question in which one or more of the following modifications were made:

- Students were asked about the reception events (the arrival of the light at each observer's location) in addition to the emission events (the eruptions).

- Instead of stating that "the volcanoes erupt at the same instant in the seismologist's reference frame," the question stated that "the seismologist receives the light from the two eruptions at the same instant."

- Instead of stating that "the volcanoes erupt at the same instant in the seismologist's reference frame," the question stated that "the seismologist concludes that the volcanoes erupted simultaneously," and asked students to decide "whether the assistant will conclude that Mt. Rainier erupts before, after, or at the same time as Mt. Hood."

- The observers were at different locations than those discussed above.

- The relevant signals were sound signals instead of light signals.

Results from those modified versions of the question are consistent with the results from the original version, indicating that students' responses were not greatly influenced by the wording of the question. In the discussion following, we combine results from various versions.

d. Administration of the question: student populations and prior instruction

We have given the Seismologist question as a written question before and after traditional instruction to several hundred students in dozens of non-physics, introductory, and advanced undergraduate physics courses. The question has also appeared on the graduate qualifying exam for doctoral candidacy, and we have conducted interviews with about 30 graduate and advanced undergraduate physics students.

In all courses in which we asked the Seismologist question, there was explicit instruction in basic definitions and procedures relevant to simultaneity and the measurement of the time of an event. Instructors typically outlined the construction of a reference frame by describing an infinite lattice of rods and clocks, discussed the assumption of correcting for signal travel time, and presented procedures for clock synchronization. These discussions tended to be brief, and instructors normally assumed that students already understood or could readily assimilate technical terms such as *event* and *intelligent observer.*

4. Commentary

At each stage of our study, we tried to determine whether student responses truly reflected their understanding of the material. For instance, we wanted to determine the extent to which specific difficulties are linguistic or conceptual and the extent to which mistaken beliefs are easily addressed or deeply held. To this end, we continually refined the research tasks. Results from earlier tasks guided us in designing new questions that would enable us to probe student thinking more thoroughly.

D. PRELIMINARY INVESTIGATION OF STUDENT UNDERSTANDING OF THE CONCEPTS OF SIMULTANEITY AND REFERENCE FRAMES

Our preliminary investigation of student understanding of special relativity was based on two versions of the Spacecraft question: the *undirected* and the *directed* versions. The two versions and the results from each are described below.

1. Failure to recognize spontaneously that simultaneity is relative

We refer to the first version of the Spacecraft question, as *undirected*. We were interested in finding out whether or not students would recognize, without prompting, that simultaneity is relative and, if not, the degree of prompting that is necessary for them to apply the relativity of simultaneity.

Spacecraft Question: Undirected version

We administered the *undirected* version of the Spacecraft question as an interview task to 7 graduate students and later to 20 advanced undergraduate students enrolled in a course in special and general relativity. All the graduate students had had undergraduate instruction in special relativity and were studying relativistic kinematics, dynamics, and electromagnetism in their graduate-level electricity and magnetism course at the time of the interviews. The undergraduates had completed instruction on the relativity of simultaneity.   All had worked with spacetime diagrams in their current or previous courses.

All the graduate students correctly drew the worldlines of each object in the spacetime diagram for each frame. However, only one recognized spontaneously the relativity of simultaneity in this context. All the others indicated on their spacetime diagrams and in their verbal explanations that the two eruptions had identical vertical (time) coordinates in the ground frame *and* identical time coordinates in the spacecraft frame. (See Figure 2-7 for examples of correct and incorrect spacetime diagrams.)
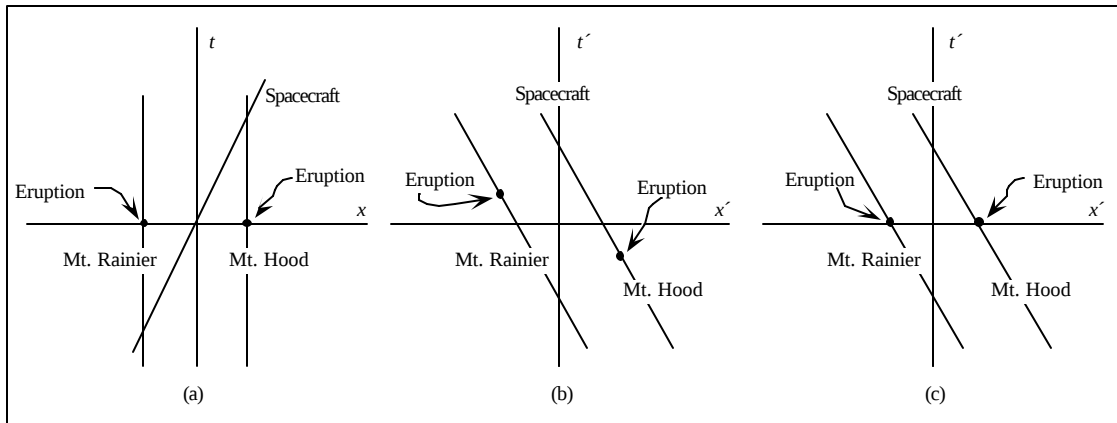
Figure 2-7:   Spacetime diagrams for the first *(undirected)* version of the Spacecraft question.   (a) Correct diagram for the ground frame.   (b) Correct diagram for the spacecraft frame. (c) Typical incorrect diagram for the spacecraft frame drawn by students.

The results from the undergraduate students were similar. All drew spacetime diagrams that included the correct features except that about 85% denoted the eruption events as simultaneous in both frames. Some of the difficulty may have been related to a lack of facility in relating spacetime diagrams for two frames. Nonetheless, the majority of the students failed to recognize spontaneously that simultaneity is relative and to draw their diagrams appropriately. These results are summarized in Table 2-1.

Table 2-1:   Results of the undirected Spacecraft question.

	Written question	
	After instruction	
	Graduate students Sp97 (N=7)	Advanced undergrads Wi97 (N=20)
Correct (Rainier erupts after Hood)	15% (1)	5% (1)
Mountains erupt simultaneously	85% (6)	85% (17)
Rainier erupts before Hood	0	10% (2)

The relativity of simultaneity is arguably the central result of relativistic kinematics and a key consequence of the Lorentz transformations. The fact that advanced students do not apply this idea spontaneously is a matter of concern. On the other hand, the fact that the time order of events is not the same in all frames is among the most counterintuitive ideas in special relativity. We wondered whether students might apply the simultaneity of relativity if prompted explicitly to do so.

2.   Failure to apply spontaneously the formalism of a reference frame in determining the time of an event

We decided to develop a new version of the Spacecraft question that would be more directive than the first. This version was written in collaboration with the course instructor. The terminology was identical to that used in class.

Spacecraft Question: Directed version

We gave the *directed* version of the Spacecraft question to 49 students on an examination in an introductory honors calculus-based physics class. The students had completed the study of the relevant material. All students answered that the eruptions are not simultaneous in the spacecraft frame. (Very few students had done so on the undirected version.) However, only

about 45% gave the correct time ordering (Hood erupts first in the spacecraft frame) with correct reasoning.[11] About 25% of the students gave the correct time order of events but used incomplete reasoning to support their answers. About 25% of the class gave the reverse time ordering. These results are summarized in Table 2-2.

Table 2-2: Results of the directed version of the Spacecraft question.

	Written question
	After instruction
	Introductory physics students Au98 (N=49)
Correct answer with correct reasoning	45% (22)
Correct answer with incomplete reasoning (including perception reasoning)	25% (13)
Reverse time ordering	25% (12)
Other	5% (2)

Essentially all of the students who gave a correct answer with incomplete reasoning answered in a similar way. The responses below are typical.

"Mt. Hood erupted first because the spacecraft is moving towards it, so the wavefront of the eruption of Mt. Hood will reach the craft first."
(introductory student)

"Hood first, because the spacecraft will encounter those wavefronts first."
(introductory student)

We categorize the reasoning given by these students as incomplete since these students describe only the order in which the signals from the distant events reach the spacecraft. They make no explicit mention of the relative velocity or the relativity of simultaneity. The sequence

in which the signals are received does not provide enough information to determine the time ordering of the eruption events. Different choices of observer location result in different reception orders and some students might have obtained the correct answer by a fortuitous choice of observer location (*e.g.,* half-way between the two volcanoes). We started to suspect the presence of incorrect ideas when we realized that most of the remaining students (about 25%) seemed to have made a different assumption about the spacecraft location. These students obtained the opposite answer for the time ordering of the events but gave explanations similar to those illustrated above. The following responses were typical.

> "The observer will witness Mt. Rainier erupting first because they are directly over Rainier when the explosion happens, so the light from the explosion has less distance to travel than the light from Mt. Hood's explosion." (introductory student)

> "Mt. Rainier would erupt first because the spacecraft is closer to Mt. Rainier and would therefore receive the wavefront from Mt. Rainier first. If the craft were flying from Hood to Rainier, Mt. Hood would erupt first, because the spacecraft would be closer to Mt. Hood and would receive its wavefront first." (introductory student)

Several students regarded both the velocity and position of the spacecraft as important in the time ordering of the eruption events. However, like the students above, they focused on the reception of the light signals by the observer. They did not treat the relative motion as the determining feature of a reference frame but as a factor that complicates the calculation of the time at which the observer receives the signals. One student claimed necessary information was missing from the problem statement.

> "It would depend on where the spacecraft was when the first explosion occurs. If it is close enough to Hood that the distance between the ship and Hood plus the distance the ship travels while the light is en route, then it sees Hood explode first." (introductory student)

The students quoted above all failed to treat the spacecraft observer as representative of a class of observers, all moving with the same velocity. They seemed to interpret the time of an event as an observer- (not frame-) dependent quantity. These results suggest that students, on their own, fail to apply the formalism of reference frames (*i.e.,* a system of clocks and rods) in defining the time of an event.

3. Commentary

The versions of the Spacecraft question used in the preliminary investigation are similar to many end-of-chapter questions on relativistic simultaneity. Students are told that two events are simultaneous in one frame *(S)* and asked about the time order of the events in another frame *(S′)* that moves relative to the first with a given velocity.

In the context of the Spacecraft question, we found that many students fail to apply spontaneously the relativity of simultaneity. When prompted to think explicitly about the order of the events, essentially all students state that the events are not simultaneous in the spacecraft frame. However, most reason incorrectly. They tend to focus on the relative *position* of the spacecraft and volcanoes and fail to recognize that the relative *velocity* determines the time order of the eruptions in the spacecraft frame.

The results from the undirected and directed versions of the Spacecraft question suggested the presence of serious conceptual and reasoning difficulties with basic concepts in special relativity. We considered, however, the possibility that student responses did not reflect what students actually thought. We needed to probe more deeply into the nature of their conceptions of time, simultaneity, and reference frames.

E. DETAILED INVESTIGATION OF STUDENT UNDERSTANDING OF THE CONCEPTS OF TIME, SIMULTANEITY, AND REFERENCE FRAMES

This section is divided into three inter-related and inter-dependent parts. Part 1deals primarily with a prevalent and persistent student interpretation of simultaneity that is observer-dependent. Many students believe that the time order of distant events is determined by the time

order in which signals from the events are perceived by an observer. Part 2 presents evidence that many students have a deeply held underlying belief that simultaneity is absolute. Part 2 also describes how students often attempt to reconcile these two contradictory beliefs with each other and with what they have been taught about the relativity of simultaneity. Student interpretations and beliefs about simultaneity described in Parts 1 and 2 have direct implications on student understanding of the role of an observer in a given frame. Part 3 is devoted to an exploration of student beliefs about the concept of a reference frame.

1. Belief that events are simultaneous if an observer receives signals from the events at the same instant

We decided that the Spacecraft question would be a better probe of student thinking about simultaneity if two changes were made: (1) specifying not only the velocity but also a location for the moving observer and (2) rewording the question to describe the eruptions as spacetime events. By choosing the observer location appropriately, we sought to distinguish between students who obtained a correct answer for correct reasons and students who thought the observer's location affects the time ordering of the events. By describing the eruptions as events, we tried to make clear to students that the time interval of interest is not that between the reception of the light signals by the moving observer, but rather that between the emission of the signals by the volcanoes.

Spacecraft Question: Location-specific version

In the third *(location-specific)* version of the Spacecraft question, students are told that the spacecraft, which is flying from Mt. Rainier to Mt. Hood, is over Mt. Rainier at the instant Mt. Rainier erupts. The eruption events, which are simultaneous in the ground frame, are explicitly labeled as Event 1 (Mt. Rainier erupts) and Event 2 (Mt. Hood erupts). Students are to determine whether, in the reference frame of the spacecraft, Event 1 occurs before, after, or at the same time as Event 2.

Table 2-4, Table 2-5, and Table 2-6 summarize the results of the location-specific Spacecraft question when given as an ungraded written question to non-physics students,

introductory students, and advanced undergraduate students. In most cases, the question was posed after lecture instruction on the relativity of simultaneity. As can be seen from the tables, prior instruction had little effect on student performance.[12]

The question was also given as an interview task to advanced undergraduates and physics graduate students. Table 2-7 summarizes these results.

Table 2-4:   Results of the location-specific version of the Spacecraft question given to non-physics students.

	Written question	
	Non-physics students	
	Before instruction Au98 (N=23)[13]	After instruction Au98 (N=16)
Correct answers (Hood erupts first) with correct reasoning *or* incomplete reasoning	15% (3)	15% (2)
Simultaneous eruptions (reasoning consistent with being based on absolute simultaneity)	45% (10)	30% (5)
Rainier erupts first (reasoning consistent with being based on the times at which signals are received by the observer)	35% (8)	45% (7)
Other (*e.g.,* student stated not enough information given)	10% (2)	15% (2)

Table 2-5:   Results of the location-specific version of the Spacecraft question given to introductory students.[14]

	Written question	
	Introductory students	
	Before instruction Sp98, Au99 (N=67)	After instruction Sp97, Au98, Sp99[15] (N=73)
Correct answers (Hood erupts first) with correct reasoning *or* incomplete reasoning	5% (3)	10% (8)
Simultaneous eruptions (reasoning consistent with being based on absolute simultaneity)	20% (12)	5% (5)
Rainier erupts first (reasoning consistent with being based on the times at which signals are received by the observer)	65% (46)	75% (55)
Other (*e.g.*, student stated not enough information given)	10% (6)	5% (5)

Table 2-6: Results of the location-specific version of the Spacecraft question, given to advanced undergraduate students.

	Written question	
	Advanced undergraduate students	
	Before instruction Wi98 (N=20)	After instruction Wi98, Au98, Au99, Sp00[16], Au00 (N=93)
Correct answers (Hood erupts first) with correct reasoning *or* incomplete reasoning	15% (3)	25% (24)
Simultaneous eruptions (reasoning consistent with being based on absolute simultaneity)	25% (5)	20% (19)
Rainier erupts first (reasoning consistent with being based on the times at which signals are received by the observer)	45% (9)	40% (39)
Other (*e.g.,* student stated not enough information given)	15% (3)	10% (11)

Table 2-7:   Results of the location-specific version of the Spacecraft question, given to advanced undergraduate and graduate students as an interview task.

	Interview task
	Advanced undergrad. & grad. students Sp99 (N=11)
Correct answers (Hood erupts first) with correct reasoning *or* incomplete reasoning	25% (3)
Simultaneous eruptions (reasoning consistent with being based on absolute simultaneity)	0
Rainier erupts first (reasoning consistent with being based on the times at which signals are received by the observer)	55% (6)
Other (*e.g.,* student stated not enough information given)	20% (2)

In every student group, less than 25% of the students gave a correct response (ignoring reasoning). Only a few students who gave the correct order used incomplete reasoning similar to that in the preliminary investigation *(e.g.,* "Hood erupts first since the spacecraft is flying towards Hood."). The majority of students answered incorrectly. Analysis of student responses revealed two related modes of incorrect reasoning.

a. Tendency to associate the time of an event with the time at which an observer receives a signal from the event

Despite having been told explicitly that the events of interest are Event 1 (Mt. Rainier erupts) and Event 2 (Mt. Hood erupts), most students attributed the time of each eruption to the time at which a given observer *sees* the eruption. Both the advanced students in the interviews and the introductory students on the written problems made this error. The following statements are typical.

"The spacecraft is near Rainier, so he gets the signal about the same time Rainier
erupts. So the spacecraft pilot would say Rainier erupts before Hood." (graduate
student)

"Mt. Rainier erupts first because the light from Mt. Hood takes time to reach the
spaceship." (introductory student)

Some students included the motion of the spacecraft relative to the ground as a factor
complicating the timing of an observer's reception of relevant signals. The following
conversation with a graduate student exemplifies the issue:

S: The distance between the two mountains is d, and this is c times t, this
distance would be traveled by light, c times t. He would see event 2 at the
time t, at the time $t = d/c$ later. And now we have to figure out which
distance Mt. Hood travels in this time t, because he's traveling with v. And
then I would draw it like, this is Mt. Hood, and then he would see it explode.

I: So is this a picture of…?

S: The eruption of Mt. Hood. The signal traveled in that time from Hood to
here, but Mt. Hood is moving too, so the spacecraft would move a little bit
more in that direction, and then the space craft pilot will see this guy [Hood]
erupt.

*b. Tendency to regard the observer as dependent only on his or her personal sensory
experiences*

The failure to distinguish between the time of an event and the time at which an observer
sees that event occurring did not seem to be a superficial error but seemed to have deep roots.
Many students failed to recognize that an observer is not isolated but has access to information
provided by other observers in his or her frame.

"For example if he looks at [the volcano, it] looks peaceful. There is nothing
going on. So he would say it hasn't erupted. I would say, to state that something

happened you have to have any evidence, and he hasn't got any evidence that
something happened. So it hasn't happened." (graduate student)

c. Commentary

The third, *location-specific* version of the Spacecraft question provided insights into student thinking about simultaneity that the first two versions had not. The responses to the written questions and interviews show that many students very strongly associate the time of an event with the time at which an observer receives a signal from the event. Whether or not distant events are simultaneous is, therefore, judged by many students only on the basis of the time order of the received signals. The difficulties seem to be intimately tied to their ideas of reference frames.

The tendency of students to interpret simultaneity in terms of signal reception had, thus far, prevented us from determining whether or not the students recognized that the events themselves are not simultaneous in all frames. We thought, however, that the misidentification of the reception events with the emission events might be easy to correct with upper-level undergraduate students and graduate students. We wondered whether students would apply the relativity of simultaneity properly if, during interviews, it were pointed out to them that the reception events are not the ones to consider.

2. Belief that simultaneity is absolute

As described earlier, we found that many students fail to apply spontaneously the idea of relativity of simultaneity after instruction. On the other hand, we also found that many students appear to believe in a type of relativity of simultaneity that we would term excessive. Students often claim that observers at different locations determine different time orderings for events based on the reception of signals from the events. We now present evidence that these apparently contradictory beliefs (that simultaneity is absolute, and that simultaneity is "excessively" relative) often coexist harmoniously. Our results suggest that many students believe that simultaneity is relative *only* in the limited sense that signals from events arrive at different observers at different times – and that fundamentally, simultaneity is absolute.

Spacecraft Question: Explicit version

We wanted to ensure that students were not hindered by semantic misinterpretations of technical terms such as "intelligent observer" and "reference frame." In an effort to remove possible ambiguity from the task and to probe even more explicitly than before, we designed a fourth version of the Spacecraft question. We refer to it as the *explicit* Spacecraft version because it makes explicit the correction for signal travel time employed by intelligent observers. We gave the question as an interview task so that we could carefully observe and correct, as necessary, the way in which students interpreted the question. We probed both qualitative and quantitative reasoning. The question also appeared on the graduate qualifying exam for doctoral candidacy.

In the *explicit* version of the Spacecraft question, students are told that "observers are intelligent observers, *i.e.,* they correct for signal travel time in order to determine the time of events in their reference frame. Each observer has clocks that are synchronized with those of all other observers in his or her reference frame." In the course of the interview, students were reminded repeatedly to consider all observers as making corrections for signal travel time. When students used technical terms such as "reference frame," the interviewer probed their understanding of the term.   If a student's interpretation differed from the conventional interpretation, the interviewer attempted to correct the student, and asked the student to reconsider his or her response in light of the accepted interpretation. (We did not offer such corrections to students responding to the question on the qualifying exam.)

a. Tendency to regard the relativity of simultaneity as an artifact of signal travel time

During the interview, many students seemed to resist thinking about simultaneity in terms of emission, rather than reception, of the signals. As the interviews progressed, we realized that part of the difficulty was that they believed strongly that the two events were simultaneous in every reference frame. Many seemed to treat the non-simultaneity of the reception of the signals as a way of reconciling this belief with what they thought they had learned about the relativity of simultaneity.

As shown in Table 2-8, four of the seven advanced undergraduate and graduate students who responded to the explicit Spacecraft question clearly articulated the idea that the order of

events in the spacecraft frame is determined by the order in which the signals from the events arrive at the spacecraft. The pattern of responses to the question administered on the qualifying exam is nearly identical. This similarity suggests that students take the interview tasks seriously, that their answers represent their best understanding of the subject matter, and that the interview sample is fairly representative of the population of graduate students as a whole.

Table 2-8: Results of the explicit version of the Spacecraft question, given to advanced undergraduate and graduate students as an interview task and to graduate students on the qualifying examination for doctoral candidacy.

	Interview task	Qualifying exam
	Advanced undergrad. & grad. students Sp99 (N=7)	Graduate students Au00 (N=23)
Correct answers (Hood erupts first) with correct reasoning *or* incomplete reasoning	40% (3)	30% (7)
Simultaneous eruptions (reasoning consistent with being based on absolute simultaneity)	0	10% (2)
Rainier erupts first (reasoning consistent with being based on the times at which signals are received by the observer)	55% (4)	60% (14)
Other (*e.g.,* student stated not enough information given)	0	0

Students' explanations were similar to those quoted previously in response to the other versions of the Spacecraft question. When reminded to consider the spacecraft observer as making corrections for the signal travel time, all of these interview subjects claimed that after making such corrections, the intelligent observer in the spacecraft would determine the eruptions to have been simultaneous.

"Using her correction, assuming she's intelligent…I mean if she measured the effect relativistically, she would measure them happening at the same time if she subtracted the time she calculated." (graduate student)

"If we are in relative motion we will measure different distances and so on but if we are all intelligent observers we will all figure out that the events were simultaneous in our rods-and-clocks reference frame." (graduate student)

An advanced undergraduate reached a similar conclusion after some clarification:

I: Can you tell which order the eruptions occur in, in the spacecraft frame? Considered separately from the time of the light hit[ting] the spacecraft.

S: I'm not really sure how to do that. It would seem to me that just logically, it doesn't matter, it would still go Rainier, Hood.

I: You're speculating, if I can paraphrase you, that after the spacecraft [observer] made corrections for the fact that it took the Hood signal some time to get to him, after he made those corrections he would wind up concluding that Rainier went first?

S: If he did everything right then he would have appropriately come up with the amount of time the signal would have traveled, the distance between the two mountains would probably also have been different to him, and so he'd have to account for that too, but once he made all of those accounting things then he would have to say that yes, they went up at the same time.

I: Oh, at the same time. I thought you said Rainier went first.

S: No, no, no, he would see Rainier go up first, but he would eventually after doing all of the math would agree with [two observers at rest on the ground], that they went up at the same time. (advanced undergraduate student)

These students appeared to believe that events simultaneous in the ground frame would be simultaneous according to an observer in any reference frame who made appropriate corrections for signal travel time. In the following example a graduate student compared the spacecraft observer to an observer on the ground under the spacecraft, at Mt. Rainier:

> "There is no real difference between the spacecraft and the [observer on the
> ground under the spacecraft]. Because I said the signals reach [that observer] at
> different times, but he can determine at which times the signals were emitted.
> …So I can do the same thing on the spacecraft: I might see the signals at
> different times but I can figure out that they happened at the same time."
> (graduate student)

Such responses indicated students' belief that simultaneity is absolute. Yet students, in their discussion of relativistic effects, appeared to have heard the idea of the relativity of simultaneity. How could these incompatible ideas coexist so openly? The resolution to this apparent contradiction came from in-depth probing.

S: There are really two separate kinds of reference frames. There is the kind of
   reference frames with all those rods and clocks extending to infinity, like in [the
   textbook]. But in practice, nothing happens except right where you are. So
   really, your reference frame is something you carry around with you …

I: There is this thing about simultaneity being relative, about events that are
   simultaneous in my reference frame not necessarily being simultaneous in
   another reference frame. Which kind of reference frame does that refer to?

S: Relativity of simultaneity is this local thing. It's not the rods and clocks thing,
   because if we are intelligent, we correct for that. It's this thing that if I see them
   at different times, they occurred at different times in my reference frame.
   (graduate student)

Responses such as these seemed to indicate that the belief that simultaneity is absolute is deeply held.[17] Any *appearance* to the contrary is only that – a visual appearance due to

differences in the reception of signals from the events.[18] This belief was typical among students who asserted that the order of events in the spacecraft frame is the order in which signals arrive at the spacecraft.

> I: This thing about events that are simultaneous in one reference frame, not being simultaneous in another reference frame? Do you have a sense of where that comes from?

> S: Light has a finite speed, so it's going to take some time for the information to travel from point A to point B wherever the observer is. This is a pretty good example actually. One observer is right between the mountains and he sees them at the same time, the other observer is not and so he sees them at different times. (graduate student)

b. Tendency to regard the Lorentz transformation for time as correcting for signal travel time

In deriving the relativity of simultaneity, many instructors invoke the Lorentz transformation for time. We found that conceptual difficulties with reference frames and the time of an event can prevent students from interpreting appropriately the terms in the Lorentz transformations.   In particular, many students appeared to believe that the Lorentz transformations constitute a correction for signal travel time:

> S: In the reference frame of the spacecraft, does Event 1 occur before, after, or at the same time as Event 2? … Before. Even though the spacecraft is traveling very fast, I would say that it's right next to Mt. Rainier so it's going to see Mt. Rainier go off, and even though it's traveling towards Mt. Hood and the light from Mt. Hood is traveling towards it, it will still take some amount of time for the information of Mt. Hood exploding to reach it.

> I: Okay, so it'll see Mt. Rainier first. In the spacecraft's reference frame, after he makes any corrections for signal travel time that might be appropriate –

S: Are we including Lorentz transformations in that?
         (advanced undergraduate student)

The "desynchronization term" in the Lorentz transformation for time ($-\,v\boldsymbol{d}x/c^2$) presented particular difficulty for students.[19] Several cited that term in support of the idea that the time of an event is influenced by the position of an observer relative to the event:

> "[This term] is the correction for the travel time of the light. The time I have to
> wait in my frame to *see* one event and then the next one." (graduate student)

The student above went on to express his confusion about the fact that the term is *proportional* to the speed of the spacecraft. He thought it should be inversely proportional since the travel time for the signal from Hood is reduced as the speed of the spacecraft in the ground frame is increased.

> Other students had difficulty with the δt term in the Lorentz transformation for time, interpreting that term as equal to the signal travel time from Hood to the spacecraft:

S: [Student has written $t' = \boldsymbol{g}\,(t - vx/c^2)$.] *X* in the ground [frame] would be this
         300 km, we know *v,* and *c,* but we don't know *t.*

I: Could you say what *t* is in words? What does it mean, it's the *t* of what?

S: For the eruptions. This is the time which goes by while light travels from
         Hood to the spacecraft. So that the spacecraft receives the signal from
         Hood…It's a time in the ground frame. The time between Hood erupts and
         the space pilot sees the signal.

c.   *Tendency to treat simultaneity as independent of relative motion*

Some students stated explicitly that relativity of simultaneity is not directly related to relative motion. Even graduate students expressed this idea. As in the preliminary investigation, some students appeared to believe that relative motion does play a role in the timing of events in

the spacecraft frame – but only to the extent that it influences the reception of signals by the spacecraft.

d.   Commentary

We have found that students often incorporate the relativity of simultaneity into their own conceptual framework in a way that allows them to continue to believe in absolute simultaneity. They do so by treating the time of an event as the instant at which that event is *seen* to occur by an observer and attributing the relativity of simultaneity to signal travel time. Such incorrect beliefs can insulate students from gaining an understanding of the relativity of simultaneity as a consequence of the invariance of the speed of light. Instructors and textbooks often admonish students to distinguish between the corrections of a finite signal travel time and the inevitability of the relativity of simultaneity.[20] Apparently, such admonitions are insufficient.

3.   Belief that every observer constitutes a distinct reference frame

The Spacecraft question discussed thus far had originally been designed to probe student understanding of simultaneity. We found, however, that student conceptions of simultaneity and reference frames are strongly intertwined. We therefore developed other questions to probe student beliefs about simultaneity, reference frames, and the role of observers. Two of these, the Explosions question and the Seismologist question, are described in Section C of this chapter *(Research tasks and prior instruction).*

Explosions question

In the Explosions question, students are given the time interval between two non-simultaneous events in one frame and asked whether there is a second frame in which the events are simultaneous. As in the first two versions of the Spacecraft question, no mention is made of a specific observer in the second frame. However, students often raised the issue of the observer location spontaneously.

a. Tendency to treat observers at the same location as being in the same reference frame, independent of relative motion

The following student correctly determined the relative speed of the frame in which the two explosions are simultaneous. [The student uses the rule developed in class that clocks in moving frames that follow (or "chase") other clocks read earlier times.] However, the student places an additional constraint on the solution by specifying a location for the observer such that the observer would see the explosions simultaneously.

> "If we travel at a speed in which [one] side is the chasing side, it will be ahead by
> a certain time. Now if we set it to be $c\boldsymbol{d}t = 1200$ m ahead, *and we stand where*
> *the engineer is standing,* we'll see the explosions at the same time. So, you must
> be traveling at 0.4c, and must be at the point where the engineer is standing."
> (introductory student – italics added for emphasis)

The student seems to believe that the explosions are simultaneous only for one observer in the 'moving' frame (the observer who sees them simultaneously). Another introductory student answered similarly.

> "There is no 'frame' that you will always see them [the two explosions] at the
> same instant, but there is a position. We can be [a certain number of] meters
> from the right [end] and see them at the same time." (introductory student)

The student seems to have interpreted the question "Is there a frame in which the events are simultaneous?" to mean "Is there an observer who sees the events at the same time?" This student apparently believes that a set of observers at rest relative to one another would not agree that the explosions are simultaneous since such observers would not all receive light from the two explosions at the same instant. The student was unable to apply the idea of a reference frame as a system for measuring the time of events.

Seismologist question

In the Seismologist question, students are asked about the relative ordering of two events for a seismologist and an assistant at rest relative to one another. The question was designed to

probe whether students would incorrectly treat simultaneity as relative, even for two observers in the same reference frame.

The question has been given to introductory and advanced students both as an examination question and as an ungraded written question. It has also been given during interviews to advanced undergraduates and graduate students. Relatively few students at any level answered correctly about the time order of events in the frame of the assistant. As shown in Table 2-11 and Table 2-12, even advanced students had significant difficulties. As for the Spacecraft question, interview responses are nearly identical to responses on the qualifying exam.

Table 2-8:   Results of the Seismologist question given to non-physics students.

	Written question	
	Non-physics students	
	During instruction Au98 (N=40)	After instruction Au98 (N=26)
Correct answers (simultaneous eruptions) regardless of reasoning	15% (6)	10% (3)
Rainier erupts first	65% (25)	75% (20)
Other (*e.g.*, Hood erupts first, student stated not enough information given)	20% (9)	10% (3)

Table 2-9:   Results of the Seismologist question given to introductory students.[21]

	Written question	
	Introductory students	
	Before instruction Sp98, Au99 (N=88)	After instruction Sp97, Au98, Sp99[22] (N=79)
Correct answers (simultaneous eruptions) regardless of reasoning	20% (19)	30% (25)
Rainier erupts first	65% (57)	60% (49)
Other (*e.g.,* Hood erupts first, student stated not enough information given)	15% (15)	5% (5)

Table 2-11:   Results of the Seismologist question given to advanced undergraduate students.

	Written question	
	Advanced undergrad students	
	Before instruction Wi97, Wi98 (N=48)	After instruction Au98, Au99, Sp00[23], Au00 (N=90)
Correct answers (simultaneous eruptions) regardless of reasoning	40% (20)	25% (24)
Rainier erupts first	55% (26)	60% (55)
Other (*e.g.,* Hood erupts first, student stated not enough information given)	<5% (2)	10% (11)

Table 2-12: Results of the Seismologist question given to advanced undergraduate and graduate students as an interview task and to graduate students on the qualifying exam for doctoral candidacy.

	Interview task	Qualifying exam
	Advanced undergrad. and grad. students Sp99, Sp00 (N=17)	Graduate students Au00 (N=23)
Correct answers (simultaneous eruptions) regardless of reasoning	60% (10)	65% (15)
Rainier erupts first	40% (7)	35% (8)
Other (*e.g.,* Hood erupts first, student stated not enough information given)	0	0

b. Tendency to treat observers at rest relative to one another as being in separate reference frames

Essentially all of the students who answered the Seismologist question incorrectly stated that Mt. Rainier erupts first in the frame of the assistant. Some of these students were explicit about their interpretation of the term "reference frame."

"Assuming the assistant is his reference frame, Rainier will erupt first because he will see its light first, and until he sees its light, effectively it hasn't erupted yet." (introductory student)[24]

"The light needs to travel 1 second before entering his [the assistant's] frame." (introductory student)[25]

"The volcano will erupt into the assistant's reference frame some time after the eruption occurs at Rainier." (introductory student)[26]

"Reference frames, they're dependent on position, so that's why [the seismologist] and [the assistant] measure two different things. I think of them being in different reference frames simply because some people would say 'Well, if they're in the same reference frame everything should happen the same.' And it doesn't happen the same, because they measure two different times. That's why I am tempted to say although they're both at rest with respect to each other, they're in different reference frames." (graduate student)

"… 'in the reference frame of the assistant' means you're sitting there and waiting for events to happen, and you record them when you see them, and that's when you mark them down so that's when they happen."
(graduate student)

In the view of the student above, "the reference frame of the assistant" consists of a single observer (the assistant). Many students treated a reference frame as being local to the position of an observer.[27]

In one version of the Seismologist question, students are asked explicitly about both (1) the order in which the light signals from the eruptions reach the assistant and (2) the order of the eruption events in the reference frame of the assistant. In their responses, some students stated explicitly that the questions were identical.

"If by his reference frame, you mean, 'When did he see it?' it would be before."
(advanced undergraduate student)[28]

[Student has circled (1) and (2) and written:] "Don't these mean the same thing?" (introductory student)[29]

Other students' responses did not make clear whether they were distinguishing between the emission and reception events:

"If one wave of light reaches them first, that would be the one they see flashing first." (introductory student)

"Rainier erupts first because that light hits him sooner." (introductory student)

"The events seem to occur when the light from the eruption reaches the assistant." (introductory student)

"I am just working off the idea that whichever eruption is closer to the observer is perceived first." (introductory student) [30]

"She [the assistant] perceives that its later arrival means that the event occurred later." (introductory student) [31]

Despite the fact that both observers are in the same frame, some students referred explicitly to the relativity of simultaneity or to the lack of synchronization of clocks.

"Mt. Rainier erupts first in the assistant's frame according to the relativity of simultaneity." (introductory student) [32]

"Events are simultaneous in reference frame of seismologist for an observer located [a certain distance] away from Mt. Rainier (in dir of Hood). No other such reference frame exists because of the relativity of simultaneity." (introductory student)

"According to the assistant Mt. Rainier erupted first because the two clocks (eruptions) are unsynchronized according to him and the light from Rainier is closer." (advanced undergraduate student)

The belief that each observer constitutes a separate reference frame was common and seems to be quite strongly held. The dialogue below, provides an example.

S: The assistant is at Mt. Rainier, and Mt. Rainier erupts, and Mt. Hood erupts at the same time. But he can't see it the same time, because if you look at this mountain you always see in the past.

I: So if he made measurements of [times and distances], and he knew the speed of the signal and so on, what would he conclude about the eruption times? …

S: For the assistant Mt. Rainier would erupt before Mt. Hood. This is what I would say.

I: In the assistant's reference frame Rainier would erupt first?

S: Yeah, definitely.

I: And what if the assistant made measurements, and had assistants of his own, whatever measurements he needed to make to reach a conclusion about the timing of the eruptions? After all those measurements, the assistant would say, "In my reference frame, Mt. Rainier erupted first?"

S: Yes. (graduate student)

Unlike some of the introductory students, nearly all of the advanced students distinguished clearly between emission events (the eruptions) and reception events (the arrival of the light from an eruption at a particular observer). They also recognized that the travel time of light is relevant. However, many of the students indicated a belief that such corrections were not appropriate for observers to make in attempting to determine the time of an event in their reference frames. The students expressed the view that "reference frame" describes what an observer perceives at a particular location. One student went so far as to express the belief that the time ordering of events in an observer's frame depends on which signals that observer is able to detect:

"Within normal human ability to comprehend time, I would say that the eruptions are going to be at the same time. But if he's blind, he's going to hear Rainier for sure go off before Hood. And so he's going to say that Rainier went off before Hood because it's going to take much longer for the sound from Hood to get there." (graduate student)

The student quoted above was also asked to sketch pictures of the explosions as they happen in the seismologists' reference frame. She responded, "Which seismologist? The one at

the base of the mountain, or the one in the middle?" At the interviewer's cautious response of "Both," she sketched two sets of pictures – one for the seismologist in the middle, and one for the assistant at Mt. Rainier. (See Figure 2-8.) She explicitly indicated that the eruptions were simultaneous according to one observer but not the other.
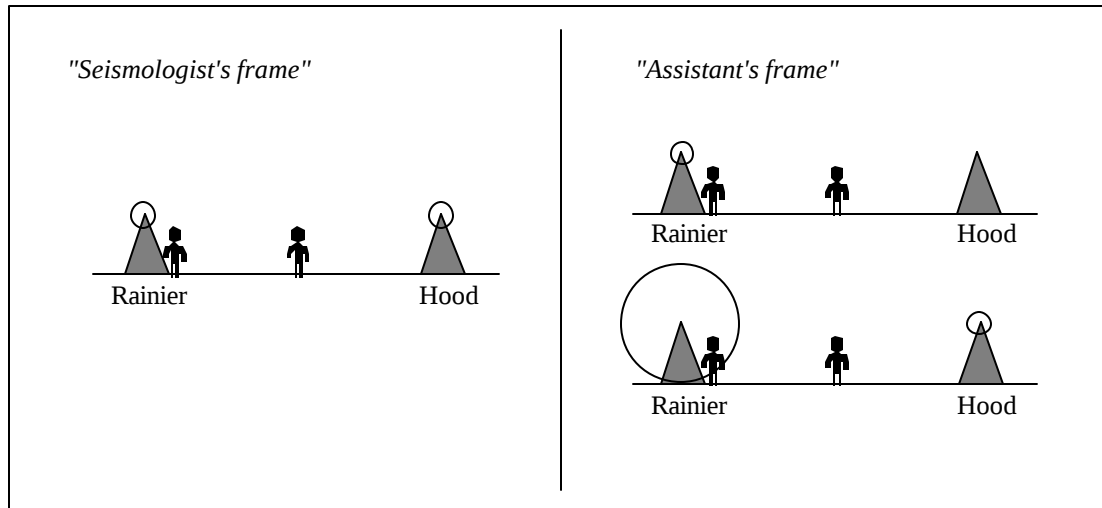
Figure 2-8:   Student response to the Seismologist question. The student has indicated that the eruptions are simultaneous in the reference frame of the seismologist (in the middle), but that in the reference frame of the assistant (at Mt. Rainier), Mt. Rainier erupts first.

Some students argued that observers at rest disagree on the time of an event even when the signal from that event explicitly contains the time of that event. The following exchange is illustrative.

Students who believed that a "reference frame" consists of a single observer at a particular location typically also believed that observers at different locations (but at rest relative to one another) were "in different reference frames" in the sense that they reached different conclusions about the times of events. Below is an example:

I: Let's say … you are in the middle, with your fancy Rolex [watch], and I'm at Mt. Rainier, with my fancy Rolex. And you determine that at exactly noon, you took your vitamins. And suppose I wanted to figure out what time you took your vitamins.

S: If you're looking at me through a telescope you will look at my clock and it will say 12, but you will look at your clock and it will say 12 plus ?*t.*

I: So in my reference frame, did you take your vitamins at 12? Or at 12 plus ?*t?*

S: Twelve plus ?*t.* (advanced undergraduate student)

c. Commentary

When students remarked that the assistant can only know what happens at his own location, they were indicating that they had not advanced beyond the most rudimentary understanding of the concept of a reference frame. They had not recognized that intelligent observers at rest with respect to one another can communicate about the spacetime coordinate events at each location. Instead, these students seemed to think that each observer is restricted to the information that he or she can obtain directly.

F. SUMMARY

This investigation has identified widespread difficulties that students have with the definition of the time of an event and the role of intelligent observers. After instruction, more than 2/3 of physics undergraduates and 1/3 of graduate students in physics are unable to apply the construct of a reference frame in determining whether or not two events are simultaneous. Many students interpret the phrase "relativity of simultaneity" as implying that the simultaneity of events is determined by an observer on the basis of the *reception* of light signals. They often attribute the relativity of simultaneity to the difference in signal travel time for different observers. In this way, they reconcile statements of the relativity of simultaneity with a belief in absolute simultaneity and fail to confront the startling ideas of special relativity.

Experienced instructors know that students often have trouble relating measurements made by observers in different reference frames. It is not surprising that students, even at advanced levels, do not fully understand the implications of the invariance of the speed of light. What is surprising is that most students apparently fail to recognize even the basic issues that are being addressed.   Students at all levels have significant difficulties with the ideas that form the

foundations of the concept of a reference frame. In particular, many students do not think of a reference frame as a system of observers that determine the same time for any given event. Such difficulties appear to impede not only their understanding of the relativity of simultaneity, but also their ability to apply correctly the Lorentz transformations.

Special relativity offers instructors an opportunity to channel student interest in modern physics into a challenging intellectual experience. For most people, the implications of special relativity are in strong conflict with their intuition. For students to recognize the conflict and appreciate its resolution, they need to have a functional understanding of some very basic concepts. Formulating an appropriate measurement procedure for the time of an event involves recognizing the inherently local nature of measurement, applying a well-defined measurement procedure in a given reference frame, and understanding the relationship between measurements made by different observers. These ideas are crucial in contexts ranging from the rolling of a steel ball on a level track to the motion of objects in the vicinity of massive stars. This investigation documents prevalent modes of reasoning with these fundamental concepts as a first step toward making special relativity meaningful to students.

NOTES TO CHAPTER TWO

[1] As quoted in E.F. Taylor and J.A. Wheeler, *Exploring Black Holes* (Addison Wesley Longman, San Francisco, 2000), p. 4-2.

[2] S. Panse, J. Ramadas, and A. Kumar, "Alternative conceptions in Galilean relativity: frames of reference," Int. J. Sci. Educ. **16**, 63 (1994); J. Ramadas and A. Kumar, "Alternative conceptions in Galilean relativity: inertial and non-inertial observers," Int. J. Sci. Educ. **18**, 615 (1996).

[3] P.W. Hewson, "A case study of conceptual change in special relativity: The influence of prior knowledge in learning," Int. J. Sci. Educ. **4**, 61 (1982); G. Posner, K. Strike, P. Hewson, and W. Gertzog, "Accommodation of a scientific conception: Toward a theory of conceptual change," Sci. Ed. **22**, 211 (1982).

[4] T.E. O'Brien-Pride, "An investigation of student difficulties with two dimensions, two-body systems, and relativity in introductory mechanics," Ph.D. dissertation, Department of Physics, University of Washington, 1997.

[5] The fact that the curved surface of a gravitating, rotating Earth is not an inertial frame did not elicit student concern. In one version of the question, students were told to neglect non-inertial effects. In another, the context was set in deep space. Neither statement seemed to change student responses.

[6] In keeping with standard practice in one-dimensional problems on special relativity, students were told in all questions that all motions were to be considered as occurring along a single line in space. No student seemed to have had difficulty neglecting the vertical dimension.

[7] The prototype of such a qualitative analysis is that of the classic train paradox often used to develop the relativity of simultaneity. See, for instance, P.A. Tipler and R.A. Llewellyn, *Modern Physics* (W.H. Freeman, New York, NY, 1999).

[8] We use the notation d instead of $?t$, *etc.,* to try to minimize confusion between *the difference between two quantities* and *a change in a quantity.* We are grateful to Eric Mazur for a discussion on this point.

[9] See, for example, P.A. Tipler, *Modern Physics* (Worth Publishers, New York, NY, 1978), pp. 15-17.

[10] Since the events are separated by a spacelike interval ($c^2 \delta t^2 - \delta x^2 < 0$), students can predict the *existence* of a frame in which the events are simultaneous without use of the Lorentz transformations.

[11] We have evidence that the students in this class performed as well as they did on the directed version of the *Spacecraft* question as a result of special instruction that they had received. On the basis of the research described in this paper, we have been developing instructional materials to address the specific difficulties that we have identified. This class had used preliminary versions of these instructional materials. Those materials are described in Chapter Three of this dissertation; results presented there are consistent with those shown here. As is demonstrated in section E, other classes after standard instruction did not do as well on similar questions.

[12] This finding is consistent with our experience that the study of advanced material does not necessarily deepen conceptual understanding. See, for example, S. Vokos, P.S. Shaffer, B.S. Ambrose, and L.C. McDermott, "Student understanding of the wave nature of matter: Diffraction and interference of particles," Phys. Educ. Res., Am. J. Phys. Suppl. **68**, S42-S51 (July 2000); B.S. Ambrose, P.S. Shaffer,

R.N. Steinberg, and L.C. McDermott, "An investigation of student understanding of single-slit diffraction and double-slit interference," Am. J. Phys. **67**, 146-155 (1999); K. Wosilait, P.R.L. Heron, P.S. Shaffer, and L.C. McDermott, "Development of a research-based tutorial on light and shadow," *ibid.* **66**, 906-913 (1999); T. O'Brien Pride, S. Vokos, and L.C. McDermott, "The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems," *ibid.* **66**, 147-157 (1998); L.C. McDermott, P.S. Shaffer, and M.D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," *ibid.* **62**, 46-55 (1994).

[13] These students had received research-based instruction on reference frames in Galilean relativity in which they had developed a definition for the time of an event. We have evidence that this special instruction may have been responsible for the low percentage of students answering that Rainier erupts first. The students had had no instruction in special relativity.

[14] See also data presented in ref. 4, p. 195.

[15] Data from Oregon State University.

[16] Data from The Ohio State University.

[17] The belief in absolute simultaneity may be related to the strong belief of students in a preferred reference frame that has been documented in the context of Galilean relativity. See E. Saltiel and J.L. Malgrange, "'Spontaneous' ways of reasoning in elementary kinematics," Eur. J. Phys. **1**, 73 (1980); A. Villani and J.L.A. Pacca, "Students' spontaneous ideas about the speed of light," Int. J. Sci. Educ. **9**, 55 (1987); "Spontaneous reasoning of graduate students," Int. J. Sci. Educ. **12**, 589 (1990); T.E. O'Brien Pride, "An investigation of student difficulties with two dimensions, two-body systems, and relativity in introductory mechanics," Ph.D. dissertation, Department of Physics, University of Washington, 1999 (unpublished).

[18] A related difficulty is described in the papers in Ref. 2. The authors document a student belief that the relative lengths of objects "in an observer's frame" are determined by that observer's visual perception of the objects.

[19] This term is related to the amount of time that two specific synchronized clocks in the spacecraft are measured to be out of synchronization by observers in the ground frame.

[20] See, for instance, D. Griffiths, *Introduction to Electrodynamics* (Prentice-Hall, Englewood Cliffs, NJ 1989), p.452. This widely used text states explicitly that the relativity of simultaneity is "a genuine discrepancy between measurements made by competent observers in relative motion, not a simple mistake arising from a failure to account for the travel time of light signals."

[21] See also data presented in ref. 4, p. 195.

[22] Data from Oregon State University.

[23] Data from The Ohio State University.

[24] This quotation has been standardized to the given context for the sake of clarity. The student was originally responding to a question identical to the *Events and reference frames* pretest described in Chapter 3, which involves sparks, not volcanic eruptions.

[25] See note 24.

[26] See note 24.

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[27] The belief that each observer constitutes a distinct reference frame may be related to results described in the papers in Ref. 2, in which the authors document (in Galilean contexts) a widespread belief that the extent of an observer's reference frame is the extent of the physical object on which that observer is located (*e.g.,* the deck of a boat).

[28] See note 24.

[29] See note 24.

[30] See note 24.

[31] See note 24.

[32] See note 24.

CHAPTER THREE:

ADDRESSING STUDENT DIFFICULTIES WITH TIME IN SPECIAL RELATIVITY:

SIMULTANEITY AND REFERENCE FRAMES

Nothing puzzles me more than time and space;
and yet nothing troubles me less, as I never think about them.

– Charles Lamb[1]

A. INTRODUCTION AND OVERVIEW

In our investigation of student understanding of time in Chapter Two, we found that many introductory and advanced students who have studied special relativity do not have a functional understanding of basic topics such as simultaneity and reference frames. Issues of simultaneity are essential to the study of special relativity, and addressing these student difficulties is a high priority.

In this chapter, we describe a series of tutorials that we have developed to address some specific difficulties with simultaneity. We begin by describing exercises in which students articulate appropriate procedures for the measurement of the time of an event and the definition of simultaneity within a single reference frame. We then describe materials in which students develop the relativity of simultaneity from basic principles and practice its application in a variety of contexts. We present results from examination questions designed to assess student understanding of simultaneity in special relativity.

We have used the two-part series of tutorials described below in courses for non-physics students, introductory students, and advanced undergraduate students. The materials we describe are designed for use as a supplement to lecture instruction; the tutorials are not a stand-alone

curriculum, but assume that students are introduced to certain ideas (*e.g.,* the invariance of the speed of light) in other parts of the course. The series takes about two hours of class time to complete.

This series of tutorials is in fact integrated with another series that we will describe in Chapter Five. We discuss here only those aspects of the instructional sequence that pertain to the development of student understanding of time. The complete sequence of integrated exercises is in Appendix C.

B. ADDRESSING THE BELIEF THAT EVERY OBSERVER CONSTITUTES A DISTINCT REFERENCE FRAME

It is our experience that the definition of simultaneity and the construction of a reference frame in Galilean contexts can provide an essential foundation on which students build their understanding of more advanced concepts. We have designed a sequence of exercises to help motivate and justify the construction of a reference frame as a system of observers and measurement devices by which the time of an event may be determined. The sequence is part of a tutorial titled *Events and reference frames,* and is designed to guide students to develop appropriate measurement procedures for event coordinates while addressing the specific difficulties with the construction of a reference frame that have been identified by research.

1. Tutorial sequence: *Events and reference frames*

a. Eliciting the belief that the time order of events depends on the time order in which an observer receives signals from the events

In order to elicit students' incorrect ideas about reference frames, we ask a pretest question similar to the Seismologists question described in Chapter Two. In this question, shown in Figure 3-1, two physics students, Alan and Beth, have measured their exact relative distances from points X and Y. Sparks jump at points X and Y; when each spark jumps, it emits a flash of light that expands outward in a spherically symmetric pattern. Alan, who is equidistant from points X and Y, receives the wavefront from each spark at the same instant. Students are asked to answer,

for each observer, (i) whether he or she receives the wavefront from the spark that jumped at X before, after, or at the same time as the spark that jumped at Y and (ii) whether, in his or her reference frame, the spark that jumped at point X jumped before, after, or at the same time as the spark that jumped at point Y.

Two physics students, Alan and Beth, are shown in the diagram at right. Alan and Beth have measured their exact relative distances from points *X* and *Y*.

Sparks jump at the points marked *X* and *Y*. When each spark jumps, it emits a flash of light that expands outward in a spherically symmetric pattern. Alan, who is equidistant from points *X* and *Y*, receives the wavefront from each spark at the same instant.

Beth   Alan

X   *Y*

Diagram not to scale.

Answer each of the following questions for the observers listed.

(i)   Does he or she receive the wavefront from the spark that jumped at point *X before, after,* or *at exactly the same time as* the wavefront from the spark that jumped at point *Y?*

(ii) In his or her reference frame, does the spark that jumped at point *X* jump *before, after,* or *at exactly the same time as* the spark that jumped at point *Y?*

Explain your reasoning in each case.

° Alan   ° Beth
(i) Same time (given).   (i)

(ii)   (ii)
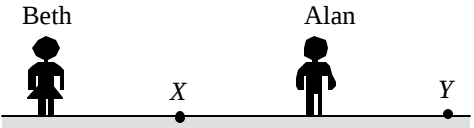
Figure 3-1:   *Events and reference frames* pretest.

Correct responses would indicate that although Beth receives the wavefront from spark X before the wavefront from spark Y, the sparks jump at the same time in her frame as they do in Alan's frame. As we have seen in Chapter Two, this question is very effective in eliciting the belief that the order of events in an observer's reference frame is the order in which that observer receives signals from the events. Only 20-35% of introductory or advanced undergraduate students answer correctly after relevant standard instruction. Asking parts (i) and (ii) together is

particularly effective in highlighting students' ideas about the relationship between an observer's reception of signals and that same observer's conclusion about event ordering.

b.   Guiding students in the appropriate determination of the time of an event

The instructional sequence for *Events and reference frames* begins with an exercise designed to guide students to formulate appropriate procedures for the measurement of the time of an event. In the relevant exercise, shown in Figure 3-2, an observer, Alan, wishes to know the exact time at which a beeper beeps but is constrained to his present location some distance from the beeper. Alan is equipped with accurate meter sticks, clocks, and assistants who can help him as necessary. The tutorial asks students to describe a set of measurements by which Alan can determine the exact time at which the beeper beeps in two cases: (i) using his knowledge of the speed of sound in air and (ii) without knowing or measuring the speed of sound first. In this way students articulate for themselves at least two appropriate measurement procedures for the time of a distant event: they may note the time of arrival of the sound of the beep, measure the distance from Alan to the beeper, and correct for the signal travel time, or they may place an assistant at the beeper and have the assistant mark the time at which it beeps. This exercise introduces students to the local nature of measurement and motivates the idea of a system of observers situated everywhere in spacetime that may be used to record the positions and times of events. The exercise builds on student understanding of the finite nature of signal travel time which, as we observed during the investigation discussed in Chapter Two, generally appears to be good.

A physics student named Alan and a beeper are arranged as shown at right. The beeper is about to emit a beep, and Alan wants to determine the exact time at which it does so. However, he is a long way from the beeper and is unable to travel to it.

Alan is equipped with accurate meter sticks and clocks, and there are a number of other physics students available to assist him if necessary.

1. Describe a set of measurements by which Alan can determine the time at which the beep is emitted using his knowledge of the speed of sound in air.

2. Describe a method by which Alan can determine the time at which the beep is emitted *without* knowing or measuring the speed of sound first. (*Hint:* Alan's assistants are free to stand at any location.)

Figure 3-2: Tutorial exercise to develop a measurement procedure for the time of an event.

The tutorial exercise is in some ways similar to the pretest question, which many students answer incorrectly, as we have seen in Chapter Two. However, students are much more successful on this question in the classroom than on the written pretest question. We attribute the difference in performance to at least two factors: the emphasis on the development of a measurement procedure (rather than on the result of an implied measurement) and the suggestion that the speed of sound may be relevant to the determination of the time of the beep.

c. Guiding students in the appropriate construction of a reference frame

In the next tutorial exercise, students are asked to generalize their measurement procedure for the time of an event by imagining an arrangement of observers and equipment with which one may record the position and time of a single arbitrary event. Once students have devised and articulated such a system of observers, we define the term reference frame to be such an arrangement. The technical term "intelligent observer" is also defined at this time. (See Figure 3-3.)

<div style="border: 1px solid black; padding: 10px;">

A fugitive from justice is at large in Seattle. His identity and exact whereabouts are unknown. A reporter has reason to believe that the fugitive will soon confess to the crime, and wishes to record as exactly as possible the time and place of the confession.

1. Describe an arrangement of observers and equipment with which the reporter may record the position and time of the confession.

An observer's *reference frame* is an arrangement of assistants and equipment with which the observer may record the position and time of anything that occurs.

2. Jusitfy the claim that the reporter's arrangement of observers and equipment is her *reference frame.*

An *intelligent observer* is equipped with measuring devices (such as meter sticks, clocks, and assistants) and is able to use them to make correct and accurate determinations of where and when something occurs. All observers in the study of relativity are intelligent observers.

</div>

Figure 3-3: Tutorial excerpt defining the terms *reference frame* and *intelligent observer.*

d. Addressing the belief that events are simultaneous if an observer receives signals from the events at the same instant

After the term "reference frame" is defined, students are immediately asked to apply the definition. The context used is one that is known to elicit the belief that the time order of events in an observer's reference frame is the order in which signals from the events are perceived by the observer. A horn is placed between Alan and the beeper (see Figure 3-4). The beeper beeps once and the horn honks once; Alan hears the two sounds at the same instant in time. Students are asked to describe a method by which Alan can measure the time separation between the emission of the beep and the emission of the honk in his reference frame without knowing or measuring the speed of sound first. Finally, students are asked whether the beeper beeps before, after, or at the same time as the horn honks in Alan's reference frame. The correct answer (that in order for the signals to reach Alan simultaneously, the farther event must have occurred first) guides students to recognize that when we speak of events being simultaneous, we are not speaking of signals generated by those events arriving at a certain observer simultaneously, but of a comparison of the measured times of those two events. In the classroom, students typically recognize that the correct answer is the only one consistent with the definition of reference frame.

67

A horn is located between Alan and the beeper. The beeper beeps once and the horn honks once. Alan hears the two sounds at the same instant in time.

Alan

beeper

horn
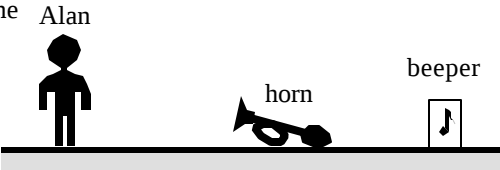
Diagram not to scale.

1. Describe a method by which Alan can measure the time separation between the emission of the beep and the emission of the honk *in his reference frame* *without* knowing or measuring the speed of sound first.

2. In Alan's reference frame, is the beep emitted *before, after,* or *at the same instant as* the honk is emitted? Explain.

Figure 3-4:   Tutorial exercise to develop an appropriate definition of simultaneity.

2. Assessing student understanding after *Events and reference frames* tutorial sequence

We assess student understanding of reference frames after the *Events and reference frames* tutorial sequence with a question in which the time order of events in an observer's reference frame is different from the order in which signals from the events are perceived by the observer. The context is special relativity, not Galilean relativity as in the tutorial.

a. Description of question

The *Events and reference frames* post-test is shown in Figure 3-5. In this question, two spaceships, A and B, pass very close to each other with relativistic speed. The figure shows the two ships in the reference frame of ship A. Two observers, Alan and Andy, are at rest in ship A at the locations indicated; two other observers, Beth and Becky, are at rest in ship B. At the instant shown, two sparks jump between the spaceships and make "char marks" on both ships. One spark marks an X, and the other marks an O. When each spark jumps, it emits a flash of light that expands outward in a spherically symmetric pattern.   The sparks jump at the same time in the reference frame of ship A. Students are asked to answer, for each observer, (i) whether he or she receives the wavefront from the spark that marked the X before, after, or at the same time as the spark that marked the O and (ii) whether, in his or her reference frame, the spark that marked the X jumped before, after, or at the same time as the spark that marked the O. Only

questions regarding the observers on ship A are relevant post-tests of the *Events and reference frames* tutorial; the questions about the observers on ship B serve as a pretest for the *Relativistic kinematics* tutorial sequence.

Two spaceships, A and B, pass very close to each other with relativistic relative speed. Alan is at rest in the front of spaceship A and Beth is at rest in the front of spaceship B. Andy and Becky are at rest in the backs of spaceships A and B respectively.

The diagram shows the two spaceships in Alan's frame. At the instant shown, two sparks jump between the spaceships and make char marks on both ships. One spark marks an **X** and the other marks an **O**. When each spark jumps, it emits a flash of light that expands outward in a spherically symmetric pattern.

The sparks jump at the same instant in the reference frame of ship A.

Answer each of the following questions for each observer.

(i)   Does he or she receive the wavefront from the spark that marks the **X** *before, after,* or *at exactly the same time as* the wavefront from the spark that marks the **O**?

(ii)   In his or her frame, does the spark that marks the **X** jump *before, after,* or *at exactly the same time as* the spark that marks the **O**?

Briefly explain your reasoning for each case.

Figure 3-5:   *Events and reference frames* post-test.

b. Correct response

To answer part (i) correctly, students should recognize that since the sparks jump at the same time in the reference frame of ship A, each observer receives the wavefront from the spark that jumps closer to him first. Andy receives the wavefront from the spark that marked the X first, and Alan receives the wavefront from the spark that marked the O first. To answer part (ii) correctly, students should recognize that the reference frame of ship A is equivalent to the reference frame of any observer at rest relative to the ship.   Hence, since the sparks jump

simultaneously in the ship frame, the sparks jump simultaneously in Alan's and Andy's reference frames. (Correct responses regarding the order of events in Beth and Becky's reference frame are discussed in section C.1.a below, where this question appears as the pretest to the *Relativistic kinematics* tutorial sequence.)

c. Administration of question

We have given the question described above to assess student understanding of the construction of a reference frame for observers at rest relative to one another. The post-test has been given in courses for introductory, advanced undergraduate, and non-physics students. In some cases, the question was given after traditional instruction in reference frames and served as a pretest of the *Events and reference frames* tutorial. In other cases, the question was given after students had completed the *Events and reference frames* tutorial and served as a post-test of that tutorial. Student performance after only traditional instruction was reported in detail in Chapter Two. We repeat these post-traditional-instruction results below for comparison but not the details of the administration.

When used as a post-test, the question described above was part of the pretest for the next tutorial (*Relativistic kinematics,* described below). Although the question was not part of an examination, we have found that the results of questions given as ungraded quizzes and as exams are typically the same.

d. Student performance

Nearly all students answer part (i) correctly for each observer, typically using correct but incomplete reasoning that the wavefront from the spark that jumps closer to a certain observer arrives at that observer first. (Such reasoning is incomplete because the answer also depends on the fact that the sparks jump simultaneously in the frame of interest).

The performance of students on part (ii) of this question after the *Events and reference frames* tutorial is improved over their performance without tutorial instruction, as shown in Table 3-1, Table 3-2, and Table 3-3; introductory and advanced undergraduate students perform at a level comparable to that of physics graduate students (see Table 2-10). However, many students

continue to identify the event order in an observer's frame as the order in which signals from the events arrive at that observer, consistent with the incorrect belief that a reference frame consists of an observer at a single location.

Table 3-1:   Introductory student performance before and after *Events and reference frames* (ERF) tutorial instruction.

	Written question	
	Introductory students	
	Traditional instruction Sp97, Au98, Sp99[2] (N=79)	ERF tutorial instruction Au99 (N=70)
Correct answers (simultaneous events) regardless of reasoning	30% (25)	50% (34)
Closer event occurs first	60% (49)	45% (31)
Other	5% (5)	5% (5)

Table 3-2:   Advanced undergraduate student performance before and after *Events and reference frames* (ERF) tutorial instruction.

	Written question	
	Advanced undergraduate students	
	Traditional instruction Au98, Au99, Sp00[3], Au00 (N=90)	ERF tutorial instruction Wi97, Wi98, Au98, Au99, Au00 (N=108)
Correct answers (simultaneous events) regardless of reasoning	25% (24)	70% (74)
Closer event occurs first	60% (55)	30% (31)
Other	10% (11)	<5% (3)

Table 3-3:   Non-physics student performance before and after *Events and reference frames* (ERF) tutorial instruction.

	Written question	
	Non-physics students	
	Traditional instruction Au98 (N=26)	ERF tutorial instruction Au98 (N=16)
Correct answers (simultaneous events) regardless of reasoning	10% (3)	25% (4)
Closer event occurs first	75% (20)	45% (7)
Other	10% (3)	30% (5)

e. Commentary

Correct application of the definition of a reference frame seems to be very difficult for students. Apparently, a single effort to address known student difficulties with reference frames is not sufficient for students to perform well on basic questions. Therefore, we revisit these ideas in later tutorials as described below.

C. ADDRESSING THE BELIEF THAT SIMULTANEITY IS ABSOLUTE

Applications of the relativity of simultaneity typically dominate the study of special relativity. As we have seen in Chapter Two, student difficulties with the relativity of simultaneity are serious and prevalent. Addressing these difficulties was perhaps the highest priority of the tutorials we designed for special relativity. In this section we describe an integrated sequence of two tutorials that we refer to together as the *Relativistic kinematics* tutorial sequence (abbreviated REK).   In the *Relativistic kinematics* tutorial sequence, students develop the relativity of simultaneity from basic principles and apply it in a variety of contexts. We present evidence from written questions that student understanding of the relativity of simultaneity is improved after the REK tutorial sequence.

We also present evidence that student understanding of the definition of simultaneity within a single reference frame is enhanced by use of the *Relativistic kinematics* tutorial sequence. We have seen (in part B above) that while some students apparently change their beliefs about the nature of simultaneity by working through the *Events and reference frames* tutorial, many students do not. The REK sequence seems to provide students with an experience of the relativity of simultaneity that is in sharp contrast to the "relativity" they apply in response to the *Events and reference frames* pretest (and post-test), and improves their performance on written questions on this topic.

1. Tutorial sequence: *Relativistic kinematics*

The specific tutorial sequence described in this chapter focuses on addressing student difficulties with simultaneity and the measurement of the time of an event. The sequence also addresses other difficulties, notably difficulties with spatial measurements.   However, in this

chapter, we discuss only the aspects of the tutorials relevant to student difficulties with simultaneity. The specific difficulties and strategies used to address them are given below. Other aspects of these tutorials are discussed in Chapter 5.

a. Eliciting incorrect beliefs about simultaneity

The pretest for the *Relativistic kinematics* tutorial is the same as the post-test for the *Events and reference frames* tutorial, except that students are asked about observers in the frame in which the time ordering of events is not given (the frame of ship B). In order to distinguish students who reason correctly from those who base their answers on the idea that signal reception order is identical to event order in a particular observer's frame, we analyze student responses for an observer who receives signals from the events in the opposite order from that in which they occur in that observer's frame (*e.g.,* Becky in Figure 3-5, page 68).

To answer correctly for Becky's reference frame, students may deduce that an observer equidistant from the char marks on ship B receives the wavefront from the spark that marks the X before the wavefront from the spark that marks the O. (In the frame of ship A, that observer is moving towards the location of the X spark and hence sees that wavefront first; the fact that that observer also sees that wavefront first in the frame of ship B is a consequence of causality. Section c below includes a more detailed discussion of this issue.) Since that observer is equidistant from the two event locations in her own frame, the correction for the signal travel time is the same for each event, and so the X spark must have jumped before the O spark in her reference frame, which is equivalent to Becky's. (The Lorentz transformations may also be used to obtain this result.)

Student performance on this question is very poor. As reported for the equivalent location-specific Spacecraft question in Chapter Two, after traditional instruction only about 10% of introductory students and 25% of advanced students give correct responses with either correct or incomplete reasoning. Many students (40-75%) state that the O spark jumps first since the light from the O spark arrives first at the observer's location. Other students (5-30%) answer consistent with the idea that simultaneity is absolute.

b. Guiding students to apply the invariance of the speed of light and the isotropy of free space

Key to understanding the pretest question described above is the observation that the speed of light is the same in all inertial reference frames. Most students are able to state this result and recognize that the speed of light is the same in any direction within a single reference frame. The REK tutorial helps students to apply this result to analyze a situation similar to that of the pretest. We have found during instruction that few students have the ability to use their knowledge of the invariance of the speed of light to analyze a relativistic scenario. The following instructional sequence begins with an abstract context and then provides students with practice applying the invariance of the speed of light and the isotropy of free space in increasingly more difficult physical scenarios.
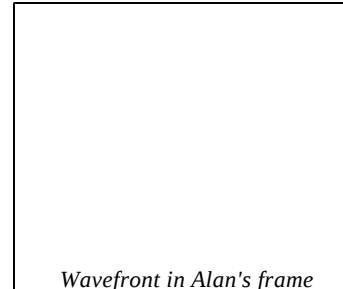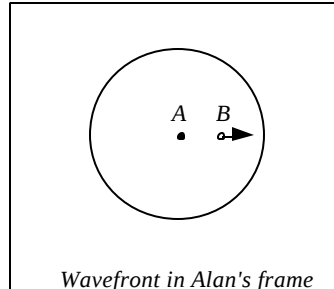
i.   Abstract context

The *Relativistic kinematics* instructional sequence begins with an exercise that asks students to apply the fact that the speed of light is invariant and isotropic in a relatively abstract context. In the excerpt shown in Figure 3-6, Alan and Beth move past each other; at the instant they pass, a spark jumps between them, emitting a flash of light. Students are shown a picture of Alan, Beth, and the wavefront of the flash of light a short time after the spark jumps in Alan's frame. They are asked to identify the features of the diagram that illustrate the fact that Alan observes the speed of light to be the same in all directions. They are asked to sketch Alan, Beth, and the wavefront of the flash of light a short time later in Alan's reference frame.

Next, students are asked to sketch Alan, Beth, and the wavefront a short time after the spark jumps in Beth's frame. Students should recognize that since Beth also observes the speed of light to be the same in all directions, Beth is at the center of a circular wavefront in her frame, and Alan moves relative to her. Correct diagrams are shown in Figure 3-7.

Two physics students, Alan and Beth, move past each other. At the instant that they pass, a spark jumps between them. The spark emits a flash of light that travels outward in a spherically symmetric pattern.

The first diagram at right represents the wavefront of the flash of light a short time after the spark jumps in Alan's frame.
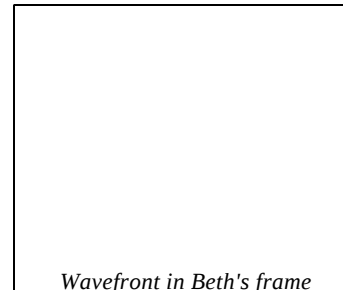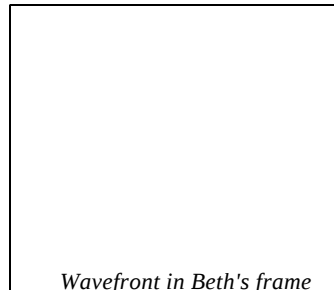
Wavefront in Alan's frame   *Wavefront in Alan's frame*

A. Explain how this is consistent with the fact that Alan observes the speed of light to be the same in all directions.

   In the second diagram above, sketch the wavefront at a later time in Alan's frame. Include Alan's and Beth's positions in your sketch.

B. Sketch the wavefront a short time after the spark jumps in Beth's frame and at a later time in Beth's frame. Include Beth's and Alan's positions in your sketches.

Wavefront in Beth's frame   *Wavefront in Beth's frame*

   Is your sketch consistent with the fact that Beth observes the speed of light to be the same in all directions? If not, modify your diagram so that it is consistent with this observation.

Figure 3-6:   Tutorial excerpt asking students to apply the isotropy of free space and the invariance of the speed of light.

Wavefront in Alan's frame   *Wavefront in Alan's frame*

Wavefront in Beth's frame   *Wavefront in Beth's frame*
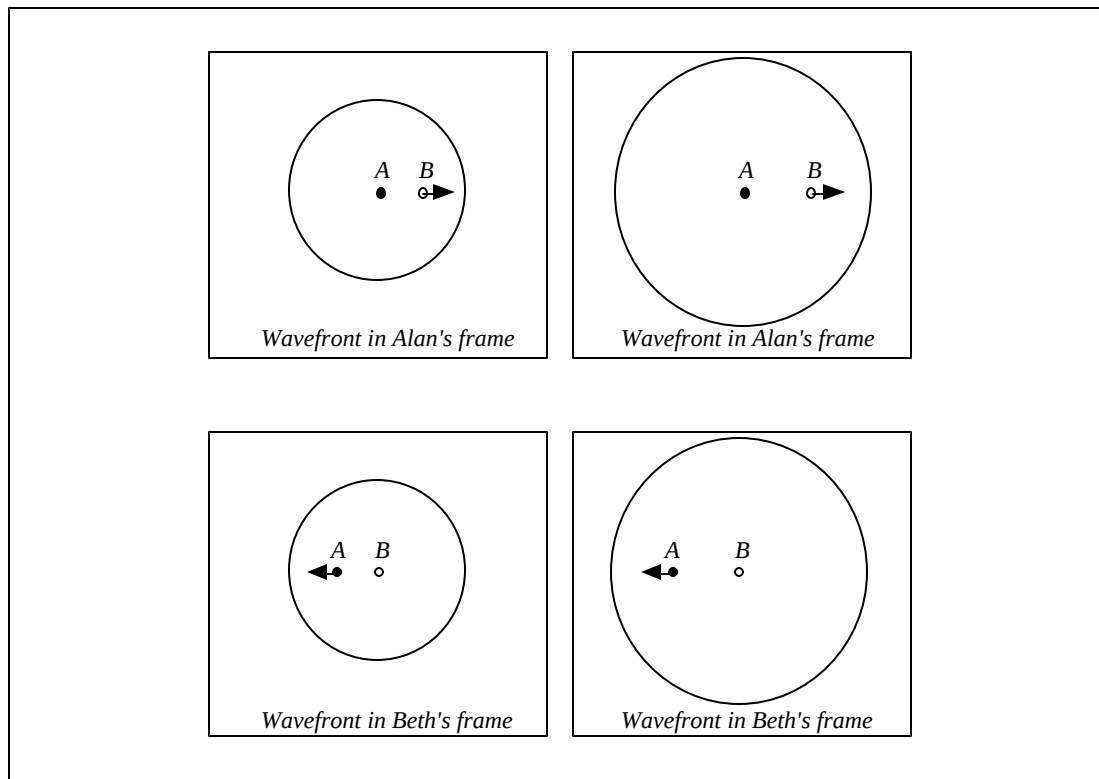
Figure 3-7:   Correct sketches for the tutorial exercise of Figure 3-6.

In the classroom, we observe that the tutorial exercise shown in Figure 3-6 is not difficult for students to answer correctly. The exercise lays important groundwork for the next exercise, described below.

ii.   Physical context
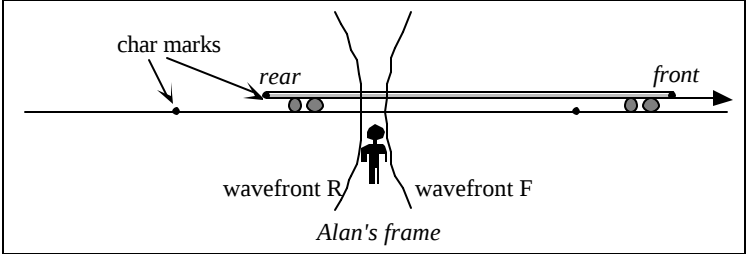
In the next part of the instructional sequence, students begin to analyze a modified version of Einstein's classic train paradox. The analysis begins with a description of the sequence of events in the ground frame. The exercise states that sparks jump simultaneously in the ground frame at either end of a long train that moves with relativistic speed relative to the ground. The sparks leave char marks on the ground and on the train. (The char marks are not part of the classic statement of the paradox.)

As shown in the excerpt in Figure 3-8, students are asked to draw a diagram for the ground frame showing the spherical wavefronts of the light from each spark some time after the sparks

jump. They are told that Beth is on the train at its midpoint. They are then asked whether, in Alan's reference frame, Beth receives the wavefront from the front spark (wavefront F) before, after, or at the same time as the wavefront from the rear spark (wavefront R).

A spark jumps between the front end of a train and the track (spark F), and another spark jumps between the rear end of the train and the track (spark R). When each spark jumps, it emits a flash of light (wavefronts F and R). Each spark also leaves a char mark on both the train and the track.
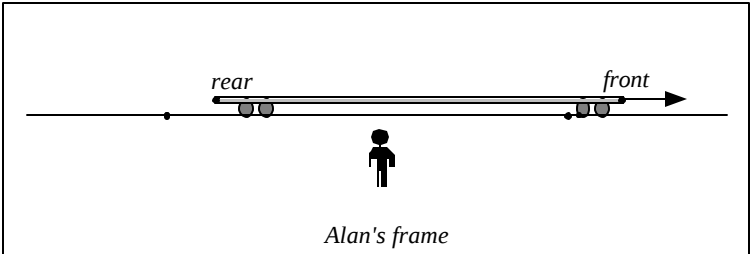
Alan is equidistant from the char marks on the track. Wavefronts F and R hit him at the same instant. The diagram at right represents this instant in Alan's frame.

The diagram below represents an instant a short time after the spark jumps between the front of the train and the track in Alan's frame (before he receives either wavefront).

Complete the diagram by sketching the *entire* wavefronts at this instant (*i.e.,* complete circles). Where are the wavefronts centered?

Beth stands at rest relative to the train, exactly halfway between the front and rear of the train.

In Alan's frame, does wavefront F hit Beth *before, after,* or *at the same instant as* wavefront R? Explain.

Figure 3-8:   Tutorial excerpt describing the physical scenario for the train paradox.

In order to sketch a correct diagram, students should note that the wavefronts from the front and rear sparks are the same size in the ground frame (since the sparks jumped simultaneously) and that they are circles centered on the char marks on the ground (since the propagation of light is isotropic). Beth receives wavefront F before wavefront R since in Alan's frame she is moving towards the center of the front wavefront. A correct diagram for the situation in Alan's frame is shown in Figure 3-9.
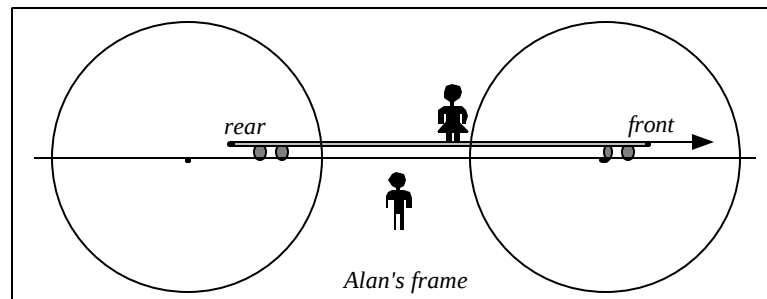
Figure 3-9:   Train paradox: Correct diagram for the ground frame.

c. Addressing difficulties with the consequences of causality

The instructional sequence continues with students now determining the order of events in the train frame. Issues of causality are inherent in this determination, and the instructional sequence is designed to assist students in using causality to find the time ordering of the events according to Beth.

In the train frame, the train is at rest, and the wavefronts from the sparks are spheres centered on the ends of the train. Correct analysis of the situation in the train frame requires recognition that causality requires that in the train frame as in the ground frame, Beth receives wavefront F before wavefront R. Since the front wavefront reaches Beth's location first in her frame and in her frame she is equidistant from the event locations, the front spark must jump first in her frame.

Many treatments of the train paradox devote little attention to the transition from the ground frame to the train frame. Our interactions with students as they work through the tutorials in this sequence, however, indicate that the above sequence of reasoning is highly nontrivial for students. Many students fail to recognize that events with a possible causal relationship in one frame must have a possible causal relationship in all frames. In particular, they fail to recognize that events that occur at the same location in one frame in a certain time order must occur in that same time order in all reference frames.

In order to assist students in applying the idea of causality, we have introduced into the tutorial a device that makes the issue of causality more immediate to the students. The device is a

cassette tape player that sits at Beth's feet and operates as follows: When wavefront F hits the tape player, it plays music at top volume. When wavefront R hits the tape player, it is silenced. If both wavefronts hit the tape player at the same instant, it remains silent. As shown in the excerpt in Figure 3-10, students are asked whether the tape player plays (i) in Alan's frame and (ii) in Beth's frame. The analysis of the scenario in *Alan's* frame indicates that Beth receives wavefront F before wavefront R in that frame, and therefore the tape player plays in Alan's frame. Correct analysis of the situation in the train frame requires recognition that the tape player plays in the train frame as well.

A cassette tape player sits at Beth's feet. In Alan's frame, when wavefront F hits the tape player, it plays the opening chords of Beethoven's Fifth Symphony at top volume; when wavefront R hits the tape player, it is silenced. If both wavefronts hit the tape player at the same instant in Alan's frame, it remains silent.

Does the tape player play the opening chords of the symphony:

　　° in Alan's frame?

　　° in Beth's frame?

Check that your responses are consistent with your answers to the following questions.

1.　　Later in the day, Beth ejects the cassette from the tape player. She descends from the train, and she and Alan examine the cassette together. Will the cassette have wound at all from its starting position?

2.　　Will Beth hear the opening chords of the symphony?

Figure 3-10:　Tutorial excerpt regarding the tape player on the train.

In the classroom, we observe that this fact is very difficult for students to accept; the majority of students are quite ready to ignore requirements of causality in order to retain their incorrect belief that simultaneity is absolute. Although some students realize that if the music plays in the ground frame, it must do so in any frame, many claim that the music plays in the ground frame but not the train frame. They argue vigorously that since Beth is equidistant from the ends of the train, she receives the two wavefronts at the same time. The absoluteness of simultaneity is implicit in this argument.

The subsequent questions in the tutorial address this difficulty by asking whether Beth will hear the music (assuming her ear is next to the tape player) and whether Beth will later observe the tape to have wound at all from its starting position. Presented with such concrete physical applications of causality, students begin to recognize the need to discard their belief in the absoluteness of simultaneity. The transition is wrenching for most students. The following conversation between three students was recorded in the classroom.[4]

S1:   We just figured out that the tape player plays in Alan's frame.

S2:   But it can't. In Beth's frame they hit her at the same time. So she won't hear it.

S3:   But look down here, it's asking if she hears it and if the tape will have wound from its starting position. If the tape is going to play, that's it, it's going to play.

S2:   But it can't play for Beth! She's in the middle! They hit her at the same time!

S1:   But we just figured out that it plays!

S2:   Right! And then a black hole opens up! And God steps out! and he points his finger and says [shouting] "YOU CAN'T DO THAT!"

Other students are tempted to dismiss the consequences of causality and bring in poorly-understood ideas from quantum mechanics to support the idea that the cassette tape player somehow both plays and does not play.[5] ["I" indicates the instructor.]

S1: Wait, so Alan hears it and Beth doesn't?

S2: That's so cool!

S1: That's one awesome tape player!

I: But when you take the tape out, when you stop the train and you look at the tape, has it been wound or has it not been wound?

S1: This is what you were telling us last week! That in some universe Sara was
wearing purple and in another one she was wearing blue or something,

S2: Oh oh oh, parallel universes! Yeah!

Eventually, most students are able to recognize that they can retain faith in the consequences of causality if they are willing to give up their belief in the absolute nature of simultaneity. Students acquiesce to the idea that wavefront F hits Beth first in her frame as well as in Alan's, and conclude that the front spark must have jumped first in Beth's frame, since Beth is equidistant from the ends of the train (the locations of the sparks in her reference frame). Students illustrate their conclusions with a diagram of the wavefronts in Beth's frame, shown in Figure 3-11, in which the wavefronts are centered on the ends of the train and the front wavefront is larger.

rear   *front*

Beth's frame
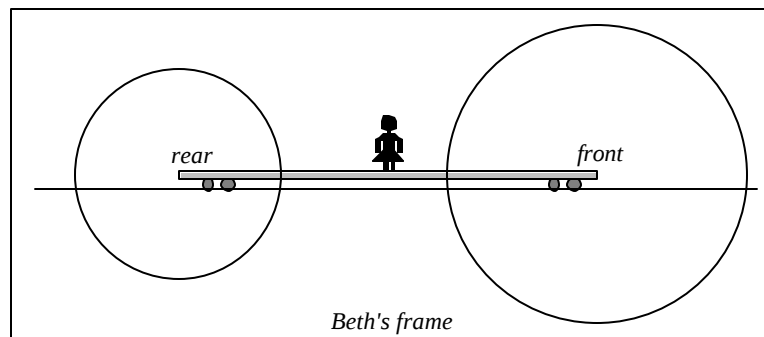
Figure 3-11:   Train paradox: Correct diagram for the train frame.

We have observed with interest that difficulties with the consequences of causality rarely arise in traditional treatments of the relativity of simultaneity. We believe that they rarely arise because students rarely reach the level of sophistication required to consider them. For example, students who believe that simultaneity is a matter of signal perception accept immediately that Beth records the events in a different time order than Alan does; causality is irrelevant to their analysis. We think that one of the great successes of the described instructional sequence is its capacity to bring students to the intellectual heart of special relativity.[6]

d. Addressing the belief that a reference frame consists of a single observer

We have seen in Chapter Two that before completing the tutorial sequence described above, many students believe that the simultaneity of events is independent of reference frame. Many believe that any appearance to the contrary is an artifact of observers at different positions receiving signals from the events at different times. Student understanding of the relativity of simultaneity may be profoundly affected by having worked through the exercises described above. It is crucial that they reexamine their earlier conclusions about the meaning of reference frame in light of their new understanding.

In order to provide an opportunity for this reexamination, the tutorial describes an additional observer, Becky, at rest on the rear of the train and asks whether, in Becky's frame, the front spark jumps before, after, or at the same time as the rear spark. Students should be able to articulate that even though Becky sees wavefront R first, wavefront F jumps first in her frame as it does in Beth's.

e. Reinforcing the relativity of simultaneity in new contexts

The ideas developed in the REK tutorial sequence are unfamiliar and counterintuitive to most students. The tutorial helps them to deepen their understanding by applying these concepts in a variety of other situations.[7]

i.   Relativity of simultaneity as a consequence of Lorentz contraction of length

Students are typically introduced to Lorentz contraction of length by means of lectures and quantitative problems before they participate in the *Relativistic kinematics* tutorial sequence. They appear to have little difficulty accepting the idea that the length of an object is longest in the frame in which it is at rest (although, as we shall see in Chapter 4, they have substantial difficulty in applying length contraction appropriately).   The following exercise makes use of length contraction to reinforce the conclusion that simultaneity is relative.

The exercise shown in Figure 3-12 describes two ships (A and B) in relative motion. In the reference frame of ship A, ship B (which is moving) has the same length as ship A. Alan and Andy are at the front and back of ship A; Beth and Becky are at the front and back of ship B.

Students are asked to draw event diagrams for the three events "Alan is next to Beth," "Alan is next to Becky," and "Andy is next to Beth" in both the frame of ship A and the frame of ship B. In the frame of ship A, the ships have the same length, and the second and third events occur simultaneously. In the frame of ship B, ship B is longer than ship A, and the second and third events are not simultaneous. Correct event diagrams for frames A and B are shown in Figure 3-13. (For a detailed discussion of event diagrams, see Appendix A.)
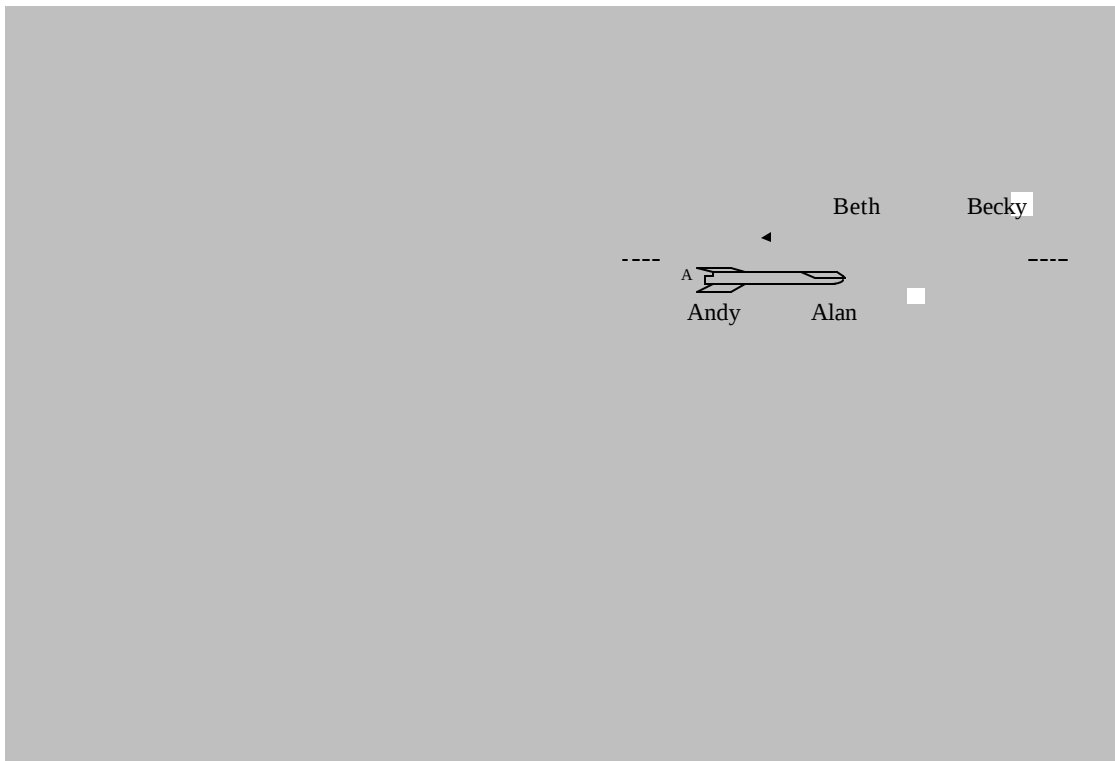
Figure 3-12:   Tutorial exercise to reinforce understanding of the relativity of simultaneity.
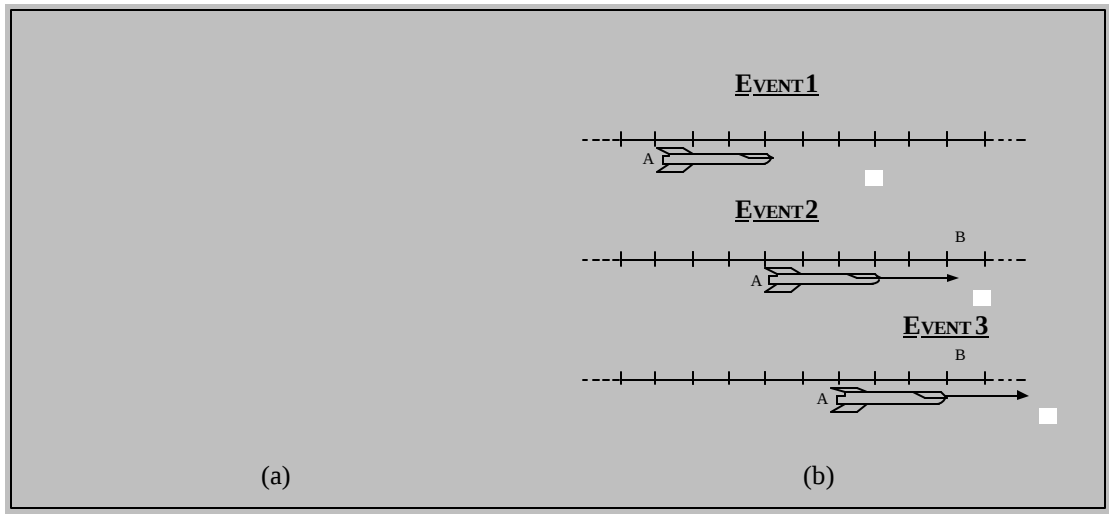
Figure 3-13:   Correct event diagrams for the tutorial exercise shown in Figure 3-12.
(a) Frame of ship A. (b) Frame of ship B.

Tutorial exercises such as the one described above use event diagrams to support and illustrate the analysis of relativistic scenarios. We have found that event diagrams are particularly useful for helping students to relate event coordinates to the physical context in which they occur. Additional exercises help students to relate their conclusions to other representations, including algebraic representations (particularly Lorentz transformations and the invariance of the spacetime interval) and spacetime diagrams.

ii.   Relativity of simultaneity as the resolution of a classic paradox

In the homework associated with the *Relativistic kinematics* tutorial sequence, students analyze a variation of a classic paradox in which a moving object of rest length greater than the contracted length of a container fits within the container (see Figure 3-14). Students are required to illustrate their analysis with event diagrams, and to show that their conclusions lead to consistent physical outcomes in both reference frames of interest. A correct analysis of the paradox requires application of the relativity of simultaneity.

A train moves with constant velocity down a straight track that passes through a tunnel. When the train is at rest with respect to the tunnel, the train is exactly the same length as the tunnel. However, the train in this case is moving relative to the tunnel at nearly the speed of light.

The engineer on the train says: "The tunnel is Lorentz-contracted and is shorter than the train; therefore at no time can the train be wholly within the tunnel."
The keeper of the tunnel says: "The train is Lorentz-contracted and is shorter than the tunnel; therefore, there will be a time at which the train is wholly within the tunnel."

They are both infuriated by their failure to reach an agreement.

A. The engineer decides to settle the issue by placing rockets on the front and rear of the train, equipped with timing devices such that the rockets will be launched simultaneously, in a vertical direction, when the midpoint of the train passes the midpoint of the tunnel. (The engineer synchronizes these timing devices while the train is approaching the tunnel.)

1. Sketch event diagrams for the train frame and the tunnel frame, showing the train and the tunnel at the instant(s) associated with the following events. Show qualitatively correct relative lengths of objects and relative times of events.
   ° The midpoint of the train is at the midpoint of the tunnel
   ° The front rocket fires
   ° The rear rocket fires

2. Do the rockets fire inside or outside the tunnel in the train frame? in the tunnel frame?

Figure 3-14:   Tutorial homework excerpt: Analysis of a variation of a classic paradox.

2. **Assessing student understanding of the relativity of simultaneity after *Relativistic kinematics***

We have given a variety of examination questions to assess student understanding of the relativity of simultaneity after the *Relativistic kinematics* tutorial sequence. We report student performance on questions in which the time order of events in an observer's reference frame is different from the order in which signals from the events are perceived by the observer. Student performance on questions regarding observers in motion relative to the ground frame (about which information is given) is much improved over performance after traditional instruction by lecture, textbook, and practice in solving typical problems in relativity.

a. Description of question

A typical *Relativistic kinematics* post-test is the location-specific version of the Spacecraft question described in Chapter Two, reproduced in Figure 3-15. Two volcanoes, Mt. Rainier and Mt. Hood, erupt at the same time in the reference frame of a seismologist in a laboratory exactly midway between the two mountains, at rest relative to the ground. A very fast spacecraft moves at a given relativistic speed relative to the ground as it flies past Mt. Rainier towards Mt. Hood, and is over Mt. Rainier at the instant it erupts. (The question is identical to the location-specific Spacecraft question described in Chapter Two, so termed because the location of the moving observer is specified.) Students are asked whether Mt. Rainier erupts before, after, or at exactly the same time as Mt. Hood in the spacecraft frame, and to explain their reasoning.

Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes. A fast spaceship flying with constant speed $d = 0.8\,c$ from Rainier toward Hood is directly over Mt. Rainier when it erupts.

In the reference frame of the spaceship, does Rainier erupt *before, after,* or *at exactly the same time as* Hood? Explain your reasoning.

Figure 3-15: *Relativistic kinematics* post-test.

b. Correct response

As in the examples discussed earlier in this and other chapters, the event that is farthest ahead of the spaceship (Mt. Hood's eruption) occurs first in the spaceship frame. To obtain this result, students may use qualitative reasoning such as that described in section 1.c (page 78) or another method.

c. Administration of question

We have given the question described above to introductory and advanced undergraduate students. In some cases, the question has been given after traditional instruction in the relativity of simultaneity; in other cases, the question was given after students had completed the *Relativistic kinematics* (REK) tutorial sequence. Student performance on the question given after traditional instruction was reported in detail in Chapter Two; we reproduce these post-traditional-

instruction results below for comparison but do not discuss the details of the administration. When the question was given after tutorial instruction, it appeared as an examination question.

d. Student performance

Table 3-4 summarizes student performance on the post-test after traditional instruction and after the *Relativistic kinematics* tutorial sequence. Both introductory and advanced students benefited from tutorial instruction. After tutorial instruction, introductory students performed at a level comparable to that of advanced students. Both introductory and advanced students did substantially better than graduate students who had not had tutorial instruction.[8]

Table 3-4: Introductory student performance before and after *Relativistic kinematics* tutorial instruction. Graduate student performance without tutorial instruction is included for comparison.

	Written question		
	Traditional instruction		REK tutorial instruction
	Intro students Sp97, Au98, Sp99[9] (N=73)	Graduate students (qualifying exam) Au00[10] (N=23)	Intro students Sp97, Sp98, Au99, Au00 (N=173)
Correct answers with correct reasoning *or* incomplete reasoning	10% (8)	30% (7)	50% (89)
Simultaneous eruptions (reasoning consistent with being based on absolute simultaneity)	5% (5)	10% (2)	<5% (2)
Rainier erupts first (reasoning consistent with being based on the times at which signals are received by the observer)	75% (55)	60% (14)	40% (70)

Table 3-5: Advanced undergraduate student performance before and after *Relativistic kinematics* tutorial instruction. Graduate student performance without tutorial instruction is included for comparison.

	Written question		
	Traditional instruction		REK tutorial instruction
	Adv, students Wi98, Au98, Au99, Sp00[11], Au00 (N=93)	Graduate students (qualifying exam) Au00[12] (N=23)	Adv students Wi98, Au99, Au00 (N=70)
Correct answers with correct reasoning *or* incomplete reasoning	15% (15)	30% (7)	55% (38)
Simultaneous eruptions (reasoning consistent with being based on absolute simultaneity)	20% (19)	10% (2)	10% (8)
Rainier erupts first (reasoning consistent with being based on the times at which signals are received by the observer)	40% (39)	60% (14)	35% (24)

3. Assessing student understanding of reference frames after *Relativistic kinematics*

We have given examination questions to assess student understanding of observers at rest relative to one another after the *Relativistic kinematics* tutorial sequence. We discuss student performance on questions in which the time order of events in an observer's reference frame is different from the order in which signals from the events are perceived by the observer.

a. Description of question

We have given the *Events and reference frames* post-test, described in detail on page 67, to assess student understanding of reference frames after the *Relativistic kinematics* sequence. In the *Events and reference frames* post-test, two volcanoes, Mt. Rainier and Mt. Hood, erupt at the same time in the reference frame of a seismologist in a laboratory exactly midway between the two mountains, at rest relative to the ground. The seismologist's assistant is at rest at the base of Mt. Rainier; we ask whether Mt. Rainier erupts before, after, or at exactly the same time as Mt. Hood in the assistant's frame.

b. Correct response

As described in detail on page 68, Andy receives the wavefront from the spark that marked the X first, Alan receives the wavefront from the spark that marked the O first, and the sparks jump simultaneously in Alan's and Andy's reference frames.

c. Administration of question

We have given the *Events and reference frames* post-test after traditional instruction, after the *Events and reference frames* (ERF) tutorial sequence, and after the *Relativistic kinematics* (REK) tutorial sequence. Student performance on the questions given after traditional instruction is reported in detail in Chapter Two. Student performance on the questions given after completion of the *Events and reference frames* tutorial sequence is reported above (section B.2.d, beginning on page 69). We reproduce these results below for comparison but do not discuss the details of the administration. The question has been given as a post-test of the tutorial sequence to non-physics students, introductory students, and advanced undergraduate students.

d. Student performance

We have shown above that student performance on the *Events and reference frames* post-test is somewhat improved after the *Events and reference frames* tutorial sequence. After students have completed the *Relativistic kinematics* tutorial sequence as well, their performance on the *Events and reference frames* post-test is very good. Table 3-6, Table 3-7, and Table 3-8 summarize these results. It is our belief that the *Relativistic kinematics* tutorial sequence provides

an important opportunity for students to confront their incorrect ideas about simultaneity, and that this confrontation and its resolution help many students to improve their understanding of the meaning of simultaneity within a single frame.

Table 3-6:   Introductory student performance on the *Events and reference frames* post-test after various levels of instruction.

	Written question			
	Introductory students			
	No instruction Sp98, Au99 (N=88)	Traditional instruction Sp97, Au98, Sp99[13] (N=79)	ERF tutorial instruction only Au99 (N=70)	ERF and REK instruction Sp97, Sp98, Au98, Au99, Au00 (N=197)
Correct answers (simultaneous events) regardless of reasoning	20% (19)	30% (25)	50% (34)	85% (167)
Closer event occurs first	65% (57)	60% (49)	45% (31)	15% (28)
Other	15% (15)	5% (5)	5% (5)	0

Table 3-7:   Advanced undergraduate student performance on the *Events and reference frames* post-test after various levels of instruction.

	Written question			
	Advanced undergraduate students			
	No instruction Wi97, Wi98 (N=48)	Traditional instruction Au98, Au99, Sp00[14], Au00 (N=90)	ERF tutorial instruction only Wi97, Wi98, Au98, Au99, Au00 (N=108)	ERF and REK instruction Wi98, Sp99, Au99, Au00 (N=98)
Correct answers (simultaneous events) regardless of reasoning	40% (20)	25% (24)	70% (74)	85% (82)
Closer event occurs first	55% (26)	60% (55)	30% (31)	15% (14)
Other	<5% (2)	10% (11)	<5% (3)	<5% (2)

Table 3-8:   Non-physics student performance on the *Events and reference frames* post-test after various levels of instruction.

	Written question		
	Non-physics students		
	Traditional instruction Au98 (N=26)	ERF tutorial instruction only Au98 (N=16)	ERF and REK instruction[15] Au98 (N=28)
Correct answers (simultaneous events) regardless of reasoning	10% (3)	25% (4)	50% (14)
Closer event occurs first	75% (20)	45% (7)	35% (10)
Other	10% (3)	30% (5)	15% (4)

e. Effect of repeated administration of the same question

In a few cases, the *Events and reference frames* post-test question was given, in effect, more than once to the same students. Although we did not administer exactly the same question twice in the same course, we did sometimes ask questions that were different only in context. For example, in Autumn of 1998, we gave the *Events and reference frames* pretest described on page 62 after traditional instruction. We then gave the (very similar) question described on page 90 on the final examination for the course, after tutorial instruction.

We do not believe that student performance on this question improves simply as a result of practice. When the question is given as an ungraded quiz during class time (as pretest questions typically are), they are not graded or returned to students with feedback. Although students are welcome to discuss the question with one another or with the course instructor, there is not usually any formal treatment of the question.

We have had few opportunities to test whether any of the improvement we have observed is due only to students' additional practice responding to questions of this type. However, in one advanced undergraduate physics course, we gave nearly identical questions as ungraded quizzes during class time on two successive days. As shown in Table 3-9, the results are essentially identical, suggesting that the changes in performance discussed above are effects of instruction, not practice.

Table 3-9:   Results of repeated administration of the *Events and reference frames* pretest on successive days.

	Written question	
	Advanced undergraduate students Wi98 (N=21)	
	Day 1	Day 2
Correct answers (simultaneous events) regardless of reasoning	65% (14)	70% (15)
Closer event occurs first	30% (6)	30% (6)
Other	5% (1)	0

D. SUMMARY

Guided by the research described in Chapter Two, we developed a two-part series of tutorials to address students' difficulties with reference frames, simultaneity, and the relativity of simultaneity. The series focuses first on the development of the concept of a reference frame, measurement procedures for the time of an event, and simultaneity. The second part develops the concept of the relativity of simultaneity from basic principles. We have found that students who have completed this series are more likely to answer correctly basic questions about simultaneity, both for observers at rest relative to one another and for those in motion relative to one another.

Although difficulties with reference frame and the measurement of the time of an event are persistent, it appears that it is possible to address them successfully with a few hours of tutorial instruction. After completion of the first tutorial in the series, student performance is comparable to that of graduate students without special instruction. After completion of the second tutorial in the series, student performance is excellent.

Unsurprisingly, difficulties with the relativity of simultaneity are harder to address; only about half of the students are able to answer our most challenging questions correctly after tutorial instruction. However, student performance improves substantially with completion of the

tutorial sequence. Even introductory students who have completed the sequence significantly outperform graduate students who have not completed the sequence. In this sense, the tutorial sequence as a whole is successful in addressing student difficulties with simultaneity and reference frames.

NOTES TO CHAPTER THREE

[1] C. Lamb, Letter to Thomas Manning, 2 January 1806, quoted in *A Dictionary of Scientific Quotations,* edited by Alan L. Mackay (Institute of Physics Publishing, Bristol and Philadelphia, 1991), p. 145.

[2] Data from Oregon State University.

[3] Data from The Ohio State University.

[4] The conversation is between three students in a course for prospective K-12 science teachers. The course used a *Physics by Inquiry* adaptation of the tutorial sequence described in this chapter. S1 and S3 are advanced undergraduate physics students; S2 is a first-year graduate student in physics.

[5] This conversation was recorded in a modern physics course in a California high school that served as a pilot site for the tutorials.

[6] For a theoretical discussion of the circumstances under which encounters with new ideas produce dissatisfaction with an existing conception, see G. Posner, K. Strike, P. Hewson, and W. Gertzog, "Accommodation of a scientific conception: Toward a theory of conceptual change," Sci. Ed. **22**, 211 (1982).

[7] The author is grateful to T.E. O'Brien-Pride, B.S. Ambrose, and C. Richardson for their pioneering work in the development of the exercises described in this section. See T.E. O'Brien-Pride, "An investigation of student difficulties with two dimensions, two-body systems, and relativity in introductory mechanics," Ph.D. dissertation, Department of Physics, University of Washington, 1997.

[8] The graduate student data is for the *explicit* version of the Spacecraft question, which is similar but not identical to the tutorial post-test (the *location-specific* version). See Chapter Two for a detailed discussion of each version of the Spacecraft question.

[9] Data from Oregon State University.

[10] See note 8.

[11] Data from The Ohio State University.

[12] See note 8.

[13] Data from Oregon State University.

[14] Data from The Ohio State University.

[15] These students had completed a version of the *Relativistic kinematics* tutorial sequence that did not include the reinforcements described in part C.1.e (due to external constraints). There is evidence that student performance on the *Events and reference frames* post-test is improved by completion of the reinforcing exercises.

CHAPTER FOUR:

STUDENT UNDERSTANDING OF SPATIAL MEASUREMENTS IN SPECIAL RELATIVITY

> *What time does this place get to New York?*
> – Barbara Stanwyck, during trans-Atlantic
> crossing on the steamship *Queen Mary*[1]

A. INTRODUCTION

In this chapter, we present results from an investigation into student performance on questions related to spatial measurements in special relativity. We began the with standard quantitative questions to probe student ability to apply the Lorentz transformations appropriately in the context of length and displacement. We observe that students frequently do not apply the Lorentz transformations to answer quantitative questions. Instead, they often apply the formula for length contraction, even in situations where the spatial quantity in question does not correspond to the length of an object. This error appears to have its basis in the belief that the distance between the locations at which two events occur is identical to the distance between two objects associated with those events. Students equate these quantities even in contexts in which the events are not simultaneous and the associated objects are moving.

Students' incorrect beliefs about object length suggested the presence of underlying difficulties with reference frames and, in particular, the determination of the position of an event. We examined these difficulties in greater depth through the use of specially designed questions. We found that students often do not spontaneously apply the formalism of a reference frame when measuring spatial quantities. Instead, they tend to ascribe the position of an event to the position of an object associated with the event. This association is correct as long as the object is

not moving in the reference frame of interest. Many students, however, appear to be unaware of the conditions under which the location of an object uniquely specifies the location of an event. Among their incorrect claims is that the location of an event can move with time or be at rest.

B. REVIEW OF PREVIOUS RESEARCH

There is only a small body of research on student understanding of relativity that is relevant to the findings reported in this chapter. Saltiel and Malgrange have found in Galilean contexts that students at all levels tend to identify motion as intrinsic to an object, not a quantity that is measured relative to a reference frame;[2] results presented in section E.2.b.ii may be related to this finding. Villani and Pacca have identified similar issues in relativistic contexts and have also observed that student difficulties with Galilean relativity present a major obstacle to the study of special relativity.[3] Section E.1.a of this chapter adds further weight to this observation.

O'Brien-Pride has conducted interviews similar in form to the Measurement interviews described in section C.3.[4] However, the emphasis in her interviews was on measurements of speed, rather than length or the position of an event. Students in her interviews were largely able to articulate correct procedures for measuring the speed of a moving train in the reference frame of the station platform, but had difficulty describing an analogous procedure for measuring the speed of the platform in the train frame.

C. RESEARCH TASKS AND PRIOR INSTRUCTION

This investigation into student understanding of spatial measurements in special relativity has made use of three research tasks which we call the Eruptions question, the Ratios question, and the Measurement question. The contexts for some of these questions are similar to the contexts for some of the questions described in Chapters Two and Three. All involve two reference frames in relative motion. Several versions of each question were used in the course of the study. In the Eruptions and Ratios questions, students are given the spatial separation between two events in one frame and asked about the spatial separation of the events in a second frame moving with given speed relative to the first. In the Measurement question, students are

asked to describe measurement procedures for spatial quantities. All three tasks probe student ability to interpret spatial quantities, *i.e.,* to relate them to the physical context in which they appear. The questions and their solutions are described below. A brief discussion of prior instruction is also provided.

1. Eruptions question

a. *Description of the question*

Results from three versions of the Eruptions question are discussed in this chapter. All involve two volcanoes, Mt. Rainier and Mt. Hood, that erupt at given times (either simultaneously or nonsimultaneously) in the reference frame of the ground. A spacecraft moves at relativistic velocity from Mt. Rainier to Mt. Hood; the distance between the volcanoes is given. Each version either contains *implicit* information about the order of events in the spacecraft frame or requests the order of events in that frame *explicitly.* One version is in a *nonrelativistic* context in which the order of events is not frame-dependent. In all three versions, students are asked first about the time order of the eruptions in the frame of the spacecraft. Second, students are asked to calculate the spatial separation between the eruptions in the spacecraft frame.

b. *Correct response*

In each of the versions administered, the events are (by construction) not simultaneous in the reference frame of the spacecraft. Appropriate methods by which to determine the order of the events in the spacecraft frame are detailed in Chapter Two. A brief description of how students could determine the time order of the events in the spacecraft frame is given following each version of the question. A general description of how to calculate the spatial separation between the eruptions is given below.

Students could calculate the spatial separation between the events in the spacecraft frame by application of the Lorentz transformation $\delta x' = g(\delta x - v\delta t)$ (which reduces to the Galilean transformation $\delta x' = \delta x - v\delta t$ in the nonrelativistic limit). In what follows, the unprimed frame is the ground frame, the primed frame is the frame of the spacecraft, and the positive direction is from Rainier to Hood. In each version of the question, the spatial separation δx and time duration

δt between the events in the ground frame and the relative velocity v of the two frames of interest are given. The calculation of $\delta x'$ is a straightforward application of the relevant transformation.

In each version, the spatial separation between the events is greater than the distance between the volcanoes in the spacecraft frame. This is the case because the volcanoes move away from the location of the first eruption in the time between the eruptions. An event diagram for the spacecraft frame appears in Figure 4-1. (Event diagrams were not required of the students.)
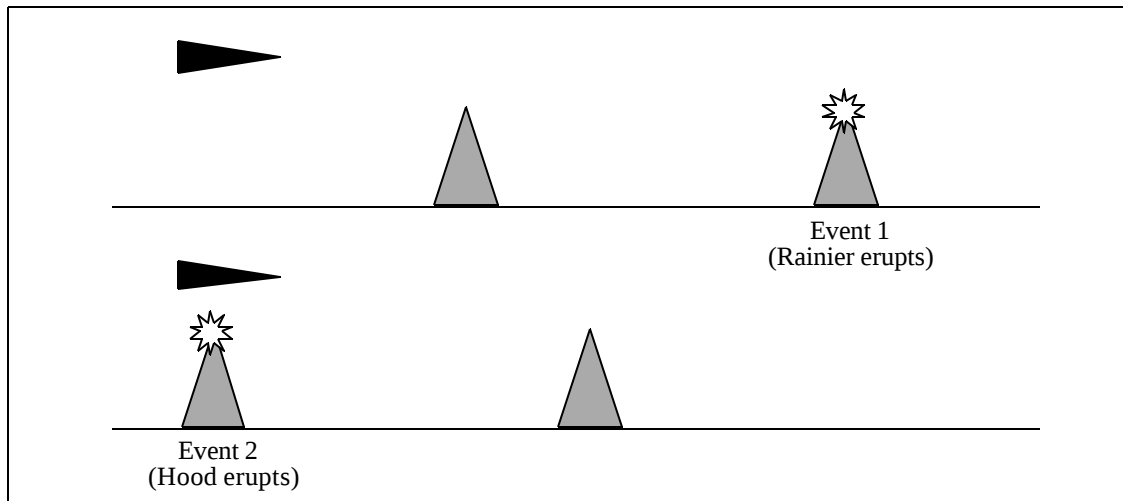
Figure 4-1:   Event diagram for the Eruptions question.

c.   Versions of the question

i.   Implicit version

In the *implicit* version of the Eruptions question, students are told that Mt. Hood erupts a given amount of time before Mt. Rainier in the ground frame. They are asked (i) whether there is a reference frame in which the eruptions are simultaneous; if so, they are to determine the velocity of that frame relative to the ground. Next, students are asked (ii) to determine the spatial separation between the eruptions in the frame of the spacecraft. The implicit version of the Eruptions question is shown in Figure 4-2.

Two volcanoes, Mt. Rainier and Mt. Hood, suddenly erupt on the same day. The volcanoes are 300 km apart in their rest frame. In the frame of a seismologist at rest on the ground, Mt. Hood erupts first; Mt. Rainier erupts at a time $c^2 t = 120$ km later.

1. Is there a frame in which the eruptions occur simultaneously? If so, determine the magnitude and direction of the velocity of this frame relative to the ground. If not, explain why not.

2. A spacecraft flies with constant speed past Rainier towards Hood at v = 0.8c. Determine the spatial separation between the two eruptions in the reference frame of the spacecraft. Show your work.

Figure 4-2:   Implicit version of the Eruptions question.

A correct answer to part (i) may be obtained by observing that the events have a spacelike separation, so there is a frame in which the events are simultaneous. The speed of that frame relative to the ground may be obtained by application of the Lorentz transformations. Note that $\delta x = x_H - x_R$ is positive, but $\delta t = t_H - t_R$ is negative for the situation described.

$$c\delta t'' = 0 = g(c\delta t - V\delta x/c)$$
$$= g(-120 \text{ km} - V(300 \text{ km})/c)$$
$$V = -0.4c$$

Determination of the frame in which the events are simultaneous (the "double-primed" frame) also establishes the simultaneity (or lack thereof) of the events in the spacecraft frame. Since the events are simultaneous in a frame moving with speed $0.4c$ relative to the ground, they are not simultaneous in the frame of the spacecraft, which moves at $0.8c$ relative to the ground. Student responses to this version of the question thus contain *implicit* information about their determination of the simultaneity of events in the spacecraft frame. This information will be useful in interpreting student responses to part (ii) of this version of the question, discussed in section D below.

A correct answer to part (ii) may also be obtained through use of the Lorentz transformations:

$$\delta x' = g(\delta x - v\delta t)$$
$$= 5/3\ (300\ \text{km} - (0.8c)\ (-120\ \text{km}))$$
$$= 660\ \text{km}$$

ii.   Explicit version

The *explicit* version of the Eruptions question is similar to the implicit version except that students are requested to state explicitly the order of events in the spacecraft frame. In addition, the eruptions are simultaneous in the ground frame in this version.

Two volcanoes, Mt. Rainier and Mt. Hood, erupt simultaneously in the reference frame of a seismologist at rest midway between them. The volcanoes are 300 km apart in their rest frame. A spacecraft flying with constant speed v =0.8c past Rainier towards Hood is directly over Rainier when it erupts.

1. In the reference frame of the spacecraft, does Rainier erupt *before, after,* or *at the same time as* Hood? Explain your reasoning.

2. Determine the spatial separation between the two eruptions in the reference frame of the space craft. Show your work.

Figure 4-3:   Explicit version of the Eruptions question.

As in the first version, the spatial separation between the eruptions in the spacecraft frame may be calculated with the Lorentz transformations.

$$\delta x' = g(\delta x - v\delta t)$$
$$= 5/3\ (300\ \text{km})$$
$$= 500\ \text{km}$$

iii.   Nonrelativistic version

The *nonrelativistic* version of the Eruptions question is similar to the other versions except that the spacecraft is replaced by an airplane that passes over the volcanoes at nonrelativistic speed relative to the ground. Students are asked to calculate the distance between the locations of

the eruptions in the reference frame of the airplane. The nonrelativistic version of the Eruptions question is shown in Figure 4-4.

Two volcanoes, Mt. Rainier and Mt. Hood, suddenly erupt on the same day. The volcanoes are 300 km apart in their rest frame. In the frame of a seismologist at rest on the ground, Mt. Hood erupts first; Mt. Rainier erupts exactly 100 minutes (6000 seconds) later.

An airplane flies with a constant speed of 50 m/s past Rainier towards Hood. Determine the spatial separation between the two eruptions in the reference frame of the airplane. Show your work.

Figure 4-4:   Nonrelativistic version of the Eruptions question.

Since the airplane moves at nonrelativistic speed relative to the ground, a correct answer to this version of the Eruptions question may be obtained by application of the Galilean transformation for spatial separations.

$$\delta x' = \delta x - v\delta t$$
$$= 300 \text{ km} - (50 \text{ m/s})(-6000 \text{ s}))$$
$$= 400 \text{ km}$$

d. Administration of the question: student populations and prior instruction

We have administered the Eruptions question as a written question to about 150 students in four introductory and advanced undergraduate physics courses. We have also conducted interviews with 16 physics graduate students.

In all cases, the question was administered after traditional instruction in the Lorentz transformations. Both introductory and advanced courses prepared students for application of the Lorentz transformations with an introduction to the technical definition of events and event coordinates. In introductory courses, the Lorentz transformations were either postulated or were introduced by analogy to spatial rotations; in advanced courses, the transformations were derived from basic principles including linearity and the invariance of the spacetime interval. Students at all levels were required to perform applications of the Lorentz transformations in the context of quantitative problems such as those found in Resnick, Halliday, and Krane[5] (introductory course) or Griffiths[6] (advanced course). Although most textbooks discuss procedures for determining

event coordinates,[7] very few include explicit discussion of measurement procedures for spatial quantities such as length or displacement.[8]

Both introductory and advanced courses emphasized length contraction as a striking consequence of Einstein's postulates and a means for resolution of some classic paradoxes. Length contraction was derived either by a qualitative argument (such as that in Taylor and Wheeler[9]) or by a quantitative argument proceeding from the Lorentz transformations (such as that outlined in the introduction to this dissertation).

2. Ratios question

a. *Description of the question*

Results from three versions of the *Ratios* question are discussed in this chapter. All involve two spaceships (A and B) that pass one another with given relative speed. Alan is at rest in the front of spacecraft A and Beth is at rest in the front of spacecraft B. Andy and Becky are at rest in the back of spacecraft A and B respectively. (See Figure 4-5.) Events 1, 2, and 3 are defined to be "Alan and Beth are adjacent," "Andy and Beth are adjacent," and "Alan and Becky are adjacent," respectively. In two versions, students are asked to determine numerical values for the ratios (i) $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$ and (ii) $\delta x_{13}^{(A)}/\delta x_{13}^{(B)}$. In the third version they were asked to find the quantities (i) $\delta x_{12}^{(B)}$ and (ii) $\delta x_{13}^{(A)}$. The setup for each version is identical and is shown in Figure 4-5.
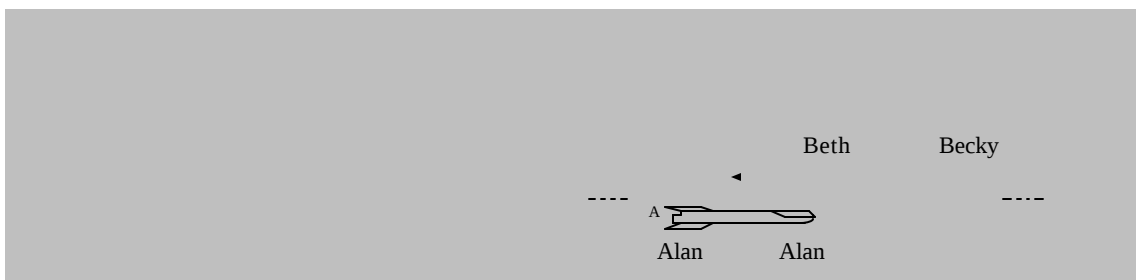
Figure 4-5:   Setup for the Ratios question.

b. Correct response

Unlike the relativistic versions of the Explosions question, the Ratios question does not depend on student understanding of the relativity of simultaneity. In all three versions, $\delta x_{12}^{(B)}$ is zero and the ratio $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$ is equal to zero. Events 1 and 2 both occur at Beth's location, so the quantity δx_{12} is equal to Beth's displacement. The quantity $\delta x_{12}^{(A)}$ is Beth's displacement in frame A (which is not zero). Beth is at rest in frame B, so the quantity $\delta x_{12}^{(B)}$ is zero. Thus, the ratio is equal to zero as well.

Similarly, events 1 and 3 both occur at Alan's location, and the quantity δx_{13} is equal to Alan's displacement. Alan is at rest in frame A, so the quantity $\delta x_{13}^{(A)}$ is zero. The quantity $\delta x_{13}^{(B)}$ is Alan's displacement in frame B, which is not zero. The value of the numerator and thus the ratio $\delta x_{13}^{(A)}/\delta x_{13}^{(B)}$ is zero.

Event diagrams for the Ratios question are shown in Figure 4-6.
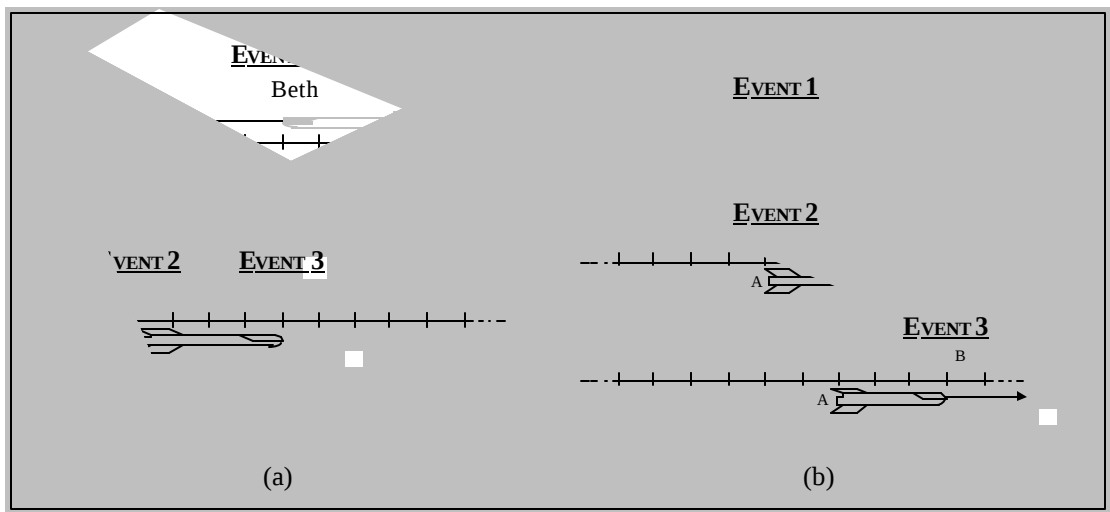
Figure 4-6: Event diagrams for the Ratios question. (Lengths are contracted as for the relativistic version of the question.)

c. Versions of the question

The three versions of the question differ in the relative speed of the ships and in the quantities that students are requested to calculate.

i.   Relativistic version

In the relativistic version of the Ratios question, the relative speed of the two spaceships is relativistic and students are asked to determine numerical values for the ratios (i) $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$ and (ii) $\delta x_{13}^{(A)}/\delta x_{13}^{(B)}$.

In Alan's frame, the speed of spaceship B is $0.6c$ ($g = 1.25$) and spaceships A and B each have length 120 c-ns. (One light-nanosecond, abbreviated c-ns, is the distance light travels in one nanosecond.)

Determine numerical values for the following ratios, in which, for example, $\delta x_{23}^{(B)} = x_3^{(B)} - x_2^{(B)} =$ the spatial separation between events 2 and 3 in Beth's frame.

1. $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$ 2. $\delta x_{13}^{(A)}/\delta x_{13}^{(B)}$

Figure 4-7:   Relativistic version of the Ratios question.

ii.   Nonrelativistic version

The nonrelativistic version of the Ratios question is similar to the relativistic version except that the relative speed of ships A and B is nonrelativistic.

In Alan's frame, the speed of spaceship B is 25 m/s and spaceships A and B each have length 12 m.

Determine numerical values for the following ratios, in which, for example, $\delta x_{23}^{(B)} = x_3^{(B)} - x_2^{(B)} =$ the spatial separation between events 2 and 3 in Beth's frame.

1. $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$ 2. $\delta x_{13}^{(A)}/\delta x_{13}^{(B)}$

Figure 4-8:   Nonrelativistic version of the Ratios question.

iii.   Numerator version

The numerator version of the Ratios question is identical to the nonrelativistic version except that instead of being asked to calculate the value of the ratios $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$ and $\delta x_{13}^{(A)}/\delta x_{13}^{(B)}$, students are asked to determine the values of the numerators only ($\delta x_{12}^{(B)}$ and $\delta x_{13}^{(A)}$). The motivation for the development of this version is discussed below (section E.1.a).

In Alan's frame, the speed of spaceship B is 25 m/s and spaceships A and B each have length 12 m.

Determine numerical values for the following quantities,
in which, for example, $\delta x_{23}{}^{(B)} = x_3{}^{(B)} - x_2{}^{(B)} =$ the spatial separation between events 2 and 3 in Beth's frame.

1. $\delta x_{12}{}^{(B)}$ 2. $\delta x_{13}{}^{(A)}$

Figure 4-9:   Numerator version of the Ratios question.

d. Administration of the question: student populations and prior instruction

We have administered the Ratios question as a written question after instruction to about one hundred students in six introductory, and advanced undergraduate physics courses. We have also conducted interviews with about a dozen physics graduate students.

The instruction that students had received was the same as that described in reference to the Eruptions question (section 1.d, page 103). Note, however, that understanding of the Lorentz transformations (or any result of special relativity) is not necessary for a correct response to the Ratios question. Instead, the question relies on student understanding of basic Galilean relativity. All students had had such instruction in the context of the introductory course.

3. Measurement question

a. Description of the question

In the Measurement question, a spacecraft moves at constant speed relative to a long, straight landing strip. Students are asked how a person on the landing strip might measure (i) the speed and (ii) the length of the passing spacecraft. Students were required to describe any equipment or assistance that that person might need. In cases in which students described highly technical equipment (*e.g.,* a radar gun for the measurement of speed), they were asked to outline the operation of the equipment, or to explain how one might perform the measurement using less sophisticated devices such as meter sticks and clocks. If students' proposed measurement procedures included simultaneous actions at different locations, students were asked to be specific about how such actions might be carried out.

In practice, questions about measurement of speed and length were often interrelated and arose naturally together. Students who proposed, for example, that one might measure the length of the spacecraft using prior knowledge of its speed were asked how they might measure the speed of the spacecraft (and vice versa). Interviewers also asked students whether their measurement procedures were equally valid for relativistic and nonrelativistic relative speeds and whether a person on the spacecraft could use the same procedures for measuring the speed and length of the landing strip as did the observer on the strip for the spacecraft.

b. Correct response

Many correct procedures for measuring the speed of the spacecraft and for measuring the length of the spacecraft are possible. The procedures described below are independent of one another. Each is equally applicable in any frame, and transparently related to events and event coordinates. Both procedures are valid at relativistic as well as nonrelativistic speeds.

An observer on the landing strip could measure the speed of the spacecraft by measuring the displacement of any point on the spacecraft in a given amount of time. The observer might prepare to record the displacement by arranging assistants with synchronized clocks along the landing strip, equally spaced at known distances as measured by a ruler laid out on the landing strip – that is, by establishing a reference frame. Any two assistants may record the time at which the specified point (X) on the spacecraft passes his or her location. The two events (1 and 2) – "Point X on the spacecraft is next to first assistant" and "Point X on the spacecraft is next to second assistant" – occur at times t_1 and t_2 on the assistants' clocks, and at locations x_1 and x_2 on the landing strip ruler. The ratio of the distance between those two assistants to the time between the two events, $(x_2 - x_1)/(t_2 - t_1)$, is the speed of the spacecraft in the landing strip frame.

To measure the length of the spacecraft in the landing strip frame, the team of observers at rest on the landing strip might make prior arrangements so that each records the times at which the front and rear of the ship passes each of their locations. A certain observer records the passing of the front of the ship at time t_0 (event 3). If the landing strip is long enough, there will be an observer who recorded the passing of the rear of the ship at the same time t_0 (event 4). The

distance between these two observers measured on the landing strip ruler, $|x_4 - x_3|$, is the length of the spacecraft in the landing strip frame.

c. Administration of the question: student populations and prior instruction

The Measurement question has been administered as an interview task to nine physics graduate students. Relevant prior instruction includes a basic knowledge of events and kinematic quantities such as velocity.

D. PRELIMINARY INVESTIGATION OF STUDENT DIFFICULTIES WITH SPATIAL MEASUREMENTS: INDISCRIMINATE APPLICATION OF LENGTH CONTRACTION

In the preliminary part of our investigation, we used standard quantitative questions to probe students' ability to apply the Lorentz transformations appropriately in the context of spatial quantities. The implicit and explicit versions of the Eruptions question are examples of such questions.

Students at all levels seemed to have similar difficulties with both versions of the Eruptions question. In part (ii), in which they are asked to calculate the spatial separation between the eruptions in the spacecraft frame, only a few of the students in each class used the Lorentz transformations. Most of the students who applied the Lorentz transformations did so correctly. However, the most common answer, in all classes, was to answer based on the formula for length contraction. For the implicit version, students often wrote

$$\delta x^{\text{(spacecraft)}} \quad = \delta x^{\text{(ground)}}/\boldsymbol{g}$$
$$= (1000 \text{ c-ns})/(5/3)$$
$$= 600 \text{ c-ns};$$

the corresponding approach for the explicit version is

$$\delta x^{(\text{spacecraft})} = \delta x^{(\text{ground})}/\boldsymbol{g}$$
$$= (300 \text{ km})/(5/3)$$
$$= 180 \text{ km}.$$

Other incorrect approaches were relatively rare. The results are summarized in Table 4-1.

Table 4-1:   Results of part (ii) of the Eruptions question, implicit and explicit versions. The second row is a subset of the first.

	Written question		Interview task
	Introductory students Au98, Sp99, Au99 (N=127)	Advanced undergraduate students Sp98 (N=34)	Graduate students Sp98, Sp99 (N=16)
Correct approach (Lorentz transformations)	25% (30)	30% (10)	20% (3)
Correct answer	*15% (21)*	*25% (8)*	*20% (3)*
Approach based on length contraction	60% (78)	60% (20)	80% (13)
Other incorrect approach	15% (19)	10% (4)	0

The algebraic expression for length contraction that most students employed is consistent with the Lorentz transformations only in the special case that the events are simultaneous in the spacecraft frame. In that case only, the spatial separation between the eruptions in the spacecraft frame would be equal to the distance between the volcanoes in that frame and length contraction would appropriately relate the two spatial separations.

It might be questioned whether the students recognized that the two events were not simultaneous in the spacecraft frame. However, the great majority of students (90%) either implied or stated explicitly in their responses to part (i) that the events in question were *not* simultaneous in the spacecraft frame. Table 4-2 shows the results from part (ii) of the Eruptions question for the students who indicated correctly in part (i) that the eruptions were not

simultaneous in the spacecraft frame. The performance of this subset of students is virtually identical to that of the entire group (shown in Table 4-1). This result suggests that the difficulty students had with the Eruptions question is not attributable to a belief that the eruptions are simultaneous in the spacecraft frame.

Table 4-2: Results of part (ii) of the Eruptions question, implicit and explicit versions. The second row is a subset of the first. Includes only students who imply or state in part (i) that the eruptions are not simultaneous in the spacecraft frame.

	Written question		Interview task
	Introductory students Au98, Sp99, Au99 (N´=114)	Advanced undergraduate students Sp98 (N´=25)	Graduate students Sp98, Sp99 (N´=16)
Correct approach (Lorentz transformations)	25% (29)	35% (9)	20% (3)
Correct answer	*15% (20)*	*30% (7)*	*20% (3)*
Approach based on length contraction	60% (66)	50% (12)	80% (13)
Other incorrect approach	15% (19)	15% (4)	0

It is interesting to note that the graduate students' performance on this task was about the same as that of the less advanced students. Most undergraduate students who applied the Lorentz transformations appeared to do so without explicitly considering the relationship between the individual terms and the physical context. The graduate students were more likely to identify mathematical expressions with physical quantities. In the process of doing so, they tended to associate incorrectly the spatial separation between the events with the distance between the volcanoes and hence used length contraction to relate spatial separations. We will discuss their reasoning in greater depth later in this chapter.

Students also tended to apply length contraction inappropriately in the context of the Ratios question. Table 4-3 and Table 4-4 indicate that the majority of students at all levels are unable to

answer the relativistic version of the Ratios question correctly. Most state that the ratios are equal to the factor g or its reciprocal. Student performance seems to be slightly better in part (i), in which the numerator of the ratio is a quantity measured in the frame of ship A (about which information is given directly).

Table 4-3:   Results of the relativistic version of the Ratios question, part (i), in which students determine the value of the ratio $\delta x_{13}^{(A)}/\delta x_{13}^{(B)}$.

	Written question	
	Introductory students Sp98, Au98, Sp99[10], Au99 (N=51)	Advanced undergraduate students Au98, Sp99 (N=19)
Correct (zero)	20% (9)	45% (9)
Incorrect: g or 1/g	60% (31)	55% (10)
Other incorrect	20% (11)	0

Table 4-4:   Results of the relativistic version of the Ratios question, part (ii), in which students determine the value of the ratio $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$.

	Written question		Interview task
	Introductory students Sp98, Au98, Sp99, Au99 (N=65)	Advanced undergraduate students Au98, Sp99 (N=26)	Graduate students Sp98, Sp99 (N=12)
Correct (zero)	10% (5)	40% (10)	0
Incorrect: g or 1/g	75% (49)	60% (16)	100% (12)
Other incorrect	15% (11)	0	0

Commentary

The versions of the Eruptions question used in the preliminary investigation are similar to many end-of-chapter questions on relativistic simultaneity. Students are given the spatial separation and time duration between two events in one frame *(S)* and asked about the spatial

separation between the events in another frame *(S′)* that moves relative to the first with a given velocity. The Ratios question, while less standard, is also simpler in that it relies only on ideas of relative motion, not the results of special relativity.

In the context of these two questions, we found that many students fail to apply spontaneously the Lorentz transformations. Instead, they apply the formula for length contraction in contexts in which it is not valid. The errors do not seem to result from difficulties with the relativity of simultaneity. The results from the Eruptions and Ratios questions suggest the presence of underlying difficulties with basic concepts in special relativity. To understand better the nature of these difficulties, we needed to probe more deeply into the nature of students' conceptions of events and reference frames.

E. DETAILED INVESTIGATION OF STUDENT UNDERSTANDING OF THE CONCEPTS OF SPATIAL MEASUREMENTS

1. Difficulty interpreting the spatial separation between events

a. Tendency to associate the distance between two co-moving objects with the spatial separation between nonsimultaneous events involving those objects

As illustrated in the previous section, many students inappropriately apply the formula for length contraction in response to the Eruptions and Ratios questions. In order to probe more deeply into student reasoning, we carefully analyzed students' interpretations of the spatial separation between two events. We also asked an additional question in order to understand whether students' incorrect ideas were limited to relativistic scenarios.

In responding to the Eruptions question, most students explained their reasoning only briefly. However, their explanations plainly state their belief that the spatial separation between the eruptions is identical to the distance between the mountains.

> "The distance between the mountains is the distance between the eruptions, in
> space. The eruptions occur at the mountains!" (graduate student)

"?z would correspond to this distance [indicates on a diagram the distance between the mountains as measured in the ground frame], ?z' would correspond to the distance that the spacecraft pilot measures between the two mountains." (graduate student)

"The explosions are separated by 600 c-ns [$\delta x/g$] by the observation of the pilot because that is the distance between the mountains as seen by him." (introductory student)

The difficulties of the students did not seem to result from the particular phrasing of the question. For most, the phrases "distance between the eruptions," "how far apart in space the eruptions are," and "spatial separation between the eruptions" appeared to be equivalent. The following excerpt from an interview with a graduate student is illustrative.

S: In the rest frame of the ship [the spatial separation] would appear to be smaller…You will see the ground contracted, and I think if it's going to be contracted then you divide the distance that you're looking at by gamma. And you would get 180 km [$\delta x/g$] for the distance between the mountains.

I: [Is] the distance between the mountains…how far apart in space the eruptions are in the spacecraft frame?

S: How far the eruptions are apart in space… I'm assuming the distance apart in space is the distance between the mountains. I guess that's what it is.

I: Other ways to put this would be 'the distance between the location at which the first eruption occurs in the spacecraft frame and the location at which the second eruption occurs in the spacecraft frame.'

S: Yeah, I would say that the eruption occurs at each mountain, and then the distance between the mountains is the distance in space between the eruptions. And measured from the spacecraft that distance would be

shortened by this factor to 180 km. That's what I would guess is the
distance. (graduate student)

Some students recognized that the identification of spatial separation with object length is problematic in some cases. In one interview, a graduate student considered the spatial separation between the events "the spacecraft is over Mt. Rainier" and "the spacecraft is over Mt. Hood" (which is zero in the spacecraft frame). Although he recognized that an inconsistency existed, he was unable to resolve the contradiction during the interview.

"If Rainier erupted and then some time later Hood was immediately underneath
me and it erupted, then they would be both at the same place. And I know that
they're not at the same place cause I know that they're moving. So the spatial
separation between the mountains I wouldn't compute as zero. My interpretation
or my understanding of what spatial separation means is breaking down.
Because in the rest frame of the ship, yeah, they would occur at the same – the
spatial separation in that frame is zero. But I can calculate the distance between
the mountains and say that the two events occurred here and here." (graduate
student)

Another student, considering a similar situation, tried to resolve the contradiction by insisting that the spacecraft pilot record positions using the ground coordinate system.

I: Suppose the spacecraft pilot, he's got his ruler and his coordinate system
laid out, and zero is right under his foot. And positive numbers go in the
front and negative numbers go in the back. So at the time that he is right
above Mt Rainier, what will he record the position of Mt Rainier to be?

S: If he records this as zero, then he's going to say that this distance [to Mt.
Hood] is negative 180 km.

I: What would he record for the position of Mt. Hood when it's right under
him?

> S: He's going to record zero [when Rainier is beneath him] because it's
>   right underneath him. And then he's going to record zero down here
>   [when Hood is beneath him]. But it hasn't just been zero because he's
>   been moving relative to – I mean the things that marking as zero, he
>   can't mark two different points as the origin. (graduate student)

Such responses indicate the strength of students' tendency to associate the spatial separation between events with the distance between the mountains. (This belief may be intertwined with a failure to recognize that displacement is a frame-dependent quantity and/or difficulty visualizing object motion in different reference frames. We discuss these possibilities further in section 2.)

Student responses to the Ratios question are also consistent with this tendency. Although students' explanations of the Ratios question are typically extremely brief, consisting only of a citation of the formula for length contraction, most identify spatial separation with length either implicitly or explicitly.

> "I want to say that the ratio is either gamma or one over gamma. Because I
> would expect the distance to be shortened…and it would be shortened by
> gamma." (graduate student)

> "The ratio between these two is just going to be gamma. Because we have
> $L' = L/g$" (graduate student)

In interviews, a few students did not explain their answers in terms of contraction of the length of objects, but instead claimed that spatial separations are inherently proportional:[11]

> "I use the measurement in Alan's frame to find the spatial difference in Beth's
> frame, and I use the relation $\delta x_{12}^{(A)} = \delta x_{12}^{(B)}/g$" (advanced undergraduate student)

These students appear to believe that spatial separations are proportional regardless of whether they are equal to the length of an object. In the context of written questions, these two different explanations are not easily distinguishable from one another, and we assume that either may be present in student responses to the Ratios question.

We have presented evidence that, in relativistic contexts, many students' responses are consistent with the belief that the spatial separation between events is identically equal to the length of an object whose ends are located at the events. Even students who answered correctly regarding the relativity of simultaneity appeared to hold this belief. However, as illustrated in Chapter Two, difficulties with the relativity of simultaneity are difficult to pinpoint without the aid of specifically targeted questions. Students may state that simultaneity is relative without a genuine understanding of what the term implies.

In order to probe whether difficulties with the relativity of simultaneity are responsible for difficulties with spatial measurements, we have asked questions similar to those described above in entirely nonrelativistic contexts. The nonrelativistic version of the Ratios question is one example. As shown in Table 4-5, the majority of students at all levels are unable to answer the nonrelativistic version of the Ratios question correctly. Most state that the ratio is equal to one. This response is consistent with identification of the spatial separation between events with the length of the spacecraft.

Table 4-5: Results of parts (i) and (ii) of the nonrelativistic version of the Ratios question.

	Written question		
	Introductory students Sp98 (N=18)	Advanced undergraduate students Au99, Au00 (N=38)	
	Part (i): $\delta x_{13}^{(B)}$		Part (ii): $\delta x_{12}^{(A)}$
Correct (zero)	15% (3)	35% (14)	40% (16)
Incorrect: one	85% (15)	45% (18)	40% (15)
Other incorrect	0	15% (6)	20% (7)

We were further concerned that posing the Ratios question in terms of a quotient might inadvertently trigger a length contraction response. For this reason, we administered a version of the "Ratios" question that did not contain any ratios, but instead asked students for the values of the numerators only ($\delta x_{12}^{(B)}$ and $\delta x_{13}^{(A)}$). As shown in Table 4-6, the results of this question are

similar to the results of the other versions, suggesting that students' beliefs are larrgely independent of the format of this question.

Table 4-6:   Results of the numerator version of the Ratios question.

	Written question	
	Advanced undergraduate students Au98 (N=21)	
	$\delta x_{12}^{(B)}$	$\delta x_{13}^{(A)}$
Correct (zero)	40% (8)	40% (8)
Incorrect: length of ship	60% (13)	55% (12)
Other incorrect	0	5% (1)

b. Tendency to reject coordinate transformations in favor of length transformations

We found that students consistently treated the spatial separation between two events as being identically equal to the distance between objects associated with the events. This conceptual difficulty seems to be responsible for the indiscriminate application of the formula for length contraction. We have also found that some students' identification of spatial separation with object length is so strong that they may reject the Lorentz (or Galilean) transformation in cases in which it contradicts the result they expect.

In responding to the Eruptions question, some students recognized that the relation $\delta x' = \delta x / g$ is in general inconsistent with the Lorentz transformations and struggled to reconcile the inconsistency. In the following excerpt from an interview with a graduate student, the student has assigned the primed frame to be the spacecraft frame and the unprimed frame to be the ground frame and has written the relation $?z' = ?z/g$

"Okay. This is a part that confuses me. I know length contraction. And so I set

up this formula [$?z' = ?z/g$]…If I were to write down the full Lorentz

transformation it's $?z' = g(?z - v?t/c^2)$ [sic]…if I pick a frame where two things

happened at the same time, where my original frame had two things happening at

the same time but different places, then I see that the length in the moving frame is also longer than the length in the rest frame [writes $?z´ = \boldsymbol{g}?z$], to which I say no, length *contraction,* so I use this [$?z´ = ?z/\boldsymbol{g}$]." (graduate student)

The student did not reconcile the inconsistency during the interview.

When the Eruptions question was administered as an interview task, the interviewer sometimes asked students to produce an event diagram for the situation. Most students were eventually able to produce a correct event diagram, including a representation of the fact that the eruptions are not simultaneous in the spacecraft frame. However, not all students were able to accept the idea that the distance between the eruptions could be different than the distance between the mountains, even though they themselves produced a diagram illustrating that fact. The following student drew a correct event diagram (see Figure 4-1), correctly calculated the spatial separation between the eruptions in the spacecraft frame by means of the Lorentz transformations, and yet rejected the correct answer based on her conviction that length contraction was the relevant relationship.

"That [application of the Lorentz transformation] says that [the spacecraft pilot] sees a longer distance between the two mountains – a greater spacing between the mountains than Alan and Bob [on the ground] do. Which doesn't quite seem right, but I don't see what's wrong with this procedure. It doesn't seem consistent with the pictures [event diagrams] on the one hand and it doesn't seem consistent in a different way with time-dilation-length-contraction…Hm. So if this were a test I would go with my length contraction and set it up in that manner." (graduate student)

Such responses illustrate the strength of the belief that the distance between the eruptions is equal to the distance between the mountains, regardless of the time order of the eruptions. Other students were unable to relate the event diagram to the formalism they had learned:

"I don't see much connection with this picture [event diagram] and this label [$?z´$] and these equations. But I think they're both right." (graduate student)

Examples from section *a* above illustrate that students incorrectly equate spatial separation with length in nonrelativistic as well as relativistic contexts. As illustrated below, students reject the Galilean transformation in contexts in which it contradicts their belief that spatial separation is equal to length.

In all of the classes in which the nonrelativistic version of the Eruptions question was administered, about two-thirds of the students answered the question correctly (or nearly correctly, mistaking only the sign of the velocity). In each class, the most common incorrect answer was that the distance between the events in the airplane frame is equal to that in the runway frame, consistent with the idea that the distance between the events is identically equal to the length of the runway. This answer was given by 25-30% of each class. Other incorrect answers included incorrect identification of the spatial separation between the events with the displacement of the runway in the airplane frame. This response was given by about 10% of students. The results of the nonrelativistic version of the Eruptions question are summarized in Table 4-7.

Table 4-7:   Results of the nonrelativistic version of the Eruptions question.

	Written question		
	Introductory students		Advanced undergrad. students
	Before instruction Sp98 (N=11)	After instruction Sp99[12], Au99[13], Au99 (N=108)	After instruction Au00 (N=27)
Correct (Galilean transformation: $\delta x' = \delta x - v\delta t$)	55% (6)	35% (40)	35% (9)
Nearly correct ($\delta x' = \delta x + v\delta t$)	0	15% (14)	20% (5)
Consistent with identifying spatial separation with length ($\delta x' = \delta x$)	25% (3)	30% (31)	30% (8)
Consistent with identifying spatial sep. with displacement ($\delta x' = v\delta t$)	10% (1)	15% (15)	15% (4)
Other	10% (1)	5% (8)	5% (1)

One student discussed the Galilean transformation for spatial separations but asserted that it does not affect the distance between two events:

"At nonrelativistic velocity, we simply use the Galilean transformation $x - vt$, but this does not affect the *distance between two observed events* with respect to their measured separation in a non-moving frame." (introductory student; emphasis in original)

Other students' responses indicated difficulties with the nature of events. Some, for example, claimed that "the eruptions are at rest in the runway frame." Such difficulties are discussed in section 2.b below.

2. Difficulties with reference frames and the determination of the position of an event

We have seen that students' indiscriminate use of length contraction appears to be based on an incorrect belief that the spatial separation between two events is identically equal to the length of the object associated with those events. Our observations led us to consider whether other difficulties underlie that inappropriate identification. In order to gain additional insight, we carefully probed student understanding of basic ideas of spatial measurements in a given reference frame: *e.g.,* measurement procedures for the length of an object and the determination of the position of an event. Our observations suggest the presence of student difficulties with reference frames in the context of spatial measurements.

a. Failure to apply spontaneously the formalism of a reference frame in measuring spatial quantities

In the Measurement question, detailed in section C.3 (page 107), we ask students to identify or describe an appropriate procedure for measuring the length and speed of an object. The results indicate serious and persistent difficulties with these measurements that suggest difficulties with the concept of a reference frame. In particular, students fail to recognize that a reference frame by its nature has spatial extent.[14]

The correct responses we expected for the Measurement question all rely on using events that occur at different locations to determine the length or displacement of the described object. Hence, these correct answers rely on the concept of a reference frame that has spatial (as well as temporal) extent. The spatial extent is among the defining features of a reference frame.

In responding to the Measurement interview task, only one of the nine graduate students initially suggested using events at different locations to measure the speed or length of the ship. Not all of the methods students suggested were incorrect; two students, for example, initially stated that if you knew the speed of the spacecraft, you could measure its length by timing how long it took to pass you. One of these students, however, seemed to feel that this was the *only* way to measure the length of the spacecraft:

I: How could he [the observer on the landing strip] measure the length of the ship?

S: If he didn't know how fast it was moving? It seems to me he would need to know either the length – if he knew the length of Beth's ship, then he could figure out how fast it was moving. And if he knew how fast it was moving, he could measure the length of the ship.

I: So is there a way that he could measure the speed of the ship without knowing its length or measuring its length first?

S: [long pause] I can't think of anything off the top of my head…I guess I can't think of any easy way to do it. (graduate student)

Another student was explicit about his inability to use clocks at different locations. In his effort to limit his measurement procedure to a single location, he explored and discarded several possible approaches.

I: How would Alan [on the landing strip] go about measuring Beth's speed [in the spacecraft]?

S: Trying to think...I have something in the back of my head that tells me I want to be making two time measurements; if I'm able to make two time measurements and I know the distance between I can get a velocity. However, I also have something in the back of my head that says in order to get a measurement of the time interval relativistically you need to be using one clock to measure both the initial and final time. In which case, the way I can see that would be to have to revise things to have Alan not actually making any of the measurements but to have Beth...there can be a clock in here in the spacecraft and Beth could then record the measurement t_1 and t_2 when she crossed these lines [on the landing strip]. The measurement she would be making would be on the same clock. However her clock will be running at a different rate than the clock on the landing strip. So I could use this clock [on the landing strip]…It would appear to me that I somehow want to use light. In looking at different clocks in any reference frame, wouldn't constitute a measurement in that frame. Light, however, will travel at the speed of light independent of frame, so…

I: So you are saying that Alan would have to use this clock over here? Alan would
     have to use a single clock? Could Alan use this clock and that clock? [Indicates
     clock at rest with respect to landing strip at a point further down the landing strip
     from Alan.]

S: Um, I want to say only if there's something correlated, and I don't know what I
     mean by that. I guess I don't know why I'm holding to the idea that he needs to
     use only one clock aside from the fact that if there's no problem to using two
     separate clocks it would appear that the measurement would be the same as the
     measurement is classically, which I relatively strongly feel is not the case.
     (graduate student)

This student seemed to feel that the considerations of special relativity limited him to the use of a single clock. Although he had a recollection of something that may have been clock synchronization, he remained convinced that a relativistic measurement would be inherently different than a nonrelativistic one in that it would necessarily involve only one clock. It may be the case that this student had the belief described in Chapter Two that each clock on the landing strip constitutes its own reference frame. In any case, his ideas about clocks at different locations prevented him from recording the locations of two simultaneous events at different locations.

In stating the Measurement question, we did not explicitly ask students to consider a lattice of rulers and clocks by which to determine the position and time of events. We expected students to generate such a lattice spontaneously in order to make the measurements we requested. Very few students applied this formalism in determining the length or speed of an object.

b.   *Tendency to associate the location of an event with the location of an object*

In the discussion of the determination of the position of an event in the introduction to this dissertation, we spoke in terms of a coordinate system and associated each position with a number. In responding to the Measurement question, students did not typically describe locations in terms of coordinates. Instead, they tended to describe locations in terms of *objects*. In particular, they mostly limited themselves to events that occurred at the location of one of the very few objects distinguished in the problem statement: the observer on the landing strip.

Such an identification of event locations with objects is not incorrect. An object that is (1) co-located with the event and (2) is at rest in the reference frame of interest specifies the position of the event as fully as does a coordinate label. In the reference frame of the ground, for example, it is equally legitimate to say that a certain event occurred in Seattle "at 6th and Pine" or "in front of the Convention Center." We will refer to an object such as the Convention Center as a "marker object."[15] In this section, we examine the extent to which students associate event locations with objects.

i.   Belief that the location of an event can change with time

In cases in which the Eruptions question was administered as an interview task, interviewers often requested that students draw an event diagram showing the eruptions in the spacecraft frame. Interviewers introduced students to the features of event diagrams during the course of the interview. Students typically began by drawing the relevant objects (the spacecraft and the volcanoes) without indicating event locations on their diagram. In these cases, the interviewer prompted students to indicate the event locations, typically with a numerical label (e.g., "Event 1" for the eruption of Mt. Rainier). A complete event diagram for the spacecraft frame of the Eruptions question appears in Figure 4-1 (page 100).

Several students indicated *multiple locations* for one or the other eruption, which could be interpreted as a location that was changing with time. The following excerpt is from an interview with an advanced undergraduate; the italics indicate vocal emphasis.

I: Where does event 1 [Mt. Rainier's eruption] happen?

S: Here. [Indicates Mt Rainier erupting; writes "event 1."] And it happen*ed* here.
     [Indicates Mt Rainier at a later instant; writes "event 1 happen_ed_" (see Figure 4-10).]

I: And how about event 2 [Mt. Hood's eruption]?

S: Okay, 'event 2' [indicates Mt. Hood erupting], and 'event 2 *will happen*'
     [indicates Mt. Hood at an earlier instant]. (advanced undergraduate)

The student's incorrect event diagram is shown in Figure 4-10.
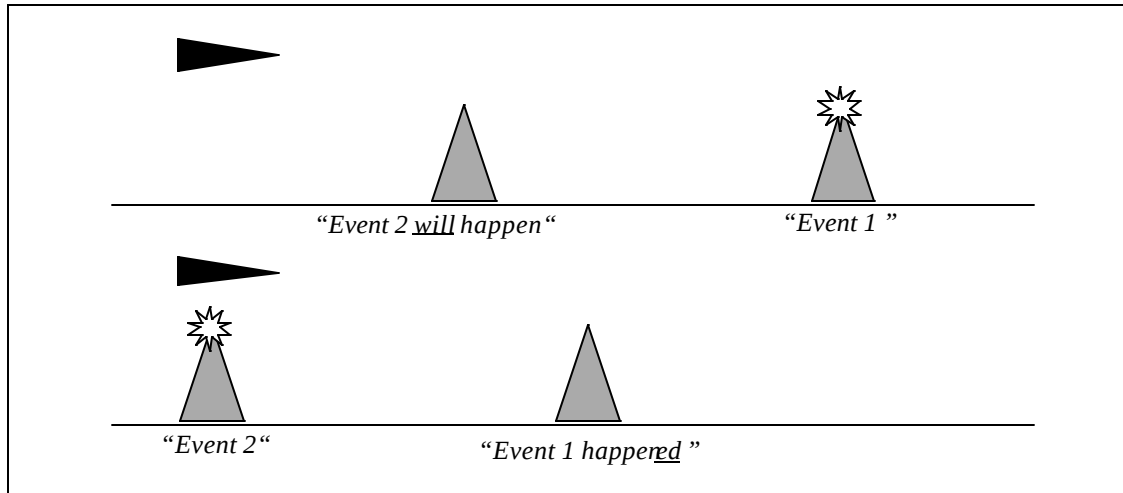
Figure 4-10:   Incorrect event diagram drawn by a student for the Eruptions question.

A graduate student, in producing an event diagram similar to the one shown above, had the following interaction with the interviewer:

I: What's event 1?

S: [Draws event 1 twice, at each appearance of Mt. Rainier in the diagram.]

I: So event 1 is moving to the left?

S: Because the spacecraft is moving to the right in the rest frame. (graduate
      student)

Students who responded like the two above seemed to strongly associate the position of the *eruption* with the position of the *volcano*. They apparently believed the event was moving in the spacecraft frame, just as the volcano was moving in that frame. Other students described the eruptions as being "at rest" in a certain reference frame:

"?t would be the time difference between the two eruptions as seen by somebody
in the rest frame of the eruptions. Of course that would be complicated if the two

weren't in the same frame to begin with – if the mountains were moving relative to each other." (graduate student)

"?z and ?t are the time interval and the space interval in the rest frame of the two explosions." (graduate student)

"Same reference frame as the event means you're not moving relative to them." (graduate student)

Students who answered as though the locations of the eruptions were "attached" to the volcanoes had a strong (incorrect) basis that they used to claim that the spatial separation between the eruptions was equal to the distance between the mountains. An introductory student, responding to the Eruptions question, stated explicitly that δx and L were "equal" because

"…Event 2 occurs at Hood regardless of *when* and event 1 occurs at Rainier regardless of *when*. Since Hood and Rainier are both "moving" relative to Beth [on the spacecraft] the same, relativity does not affect this for her." (introductory student)

The belief that the location of an event can change with time was apparent in nonrelativistic contexts as well as relativistic ones. In section 1.b we illustrated that in the nonrelativistic version of the Eruptions question many students claimed that the spatial separation between the eruptions was equal to the distance between the mountains in the airplane frame. Some students stated explicitly that the events moved with the mountains:

"An observer in the airplane's frame would know that the locations of the explosions were moving relative to the airplane (rather than attached to the airplane's frame) and would be able to transform the two points using the ?t and speed." (introductory student)

"As the west explosion location becomes further away the location of the right (east) explosion gets closer at the same rate." (introductory student)

"The two explosions are at set places, the mountains." (introductory student)

The apparent belief that the spatial separation between events is identically equal to the distance between objects associated with the events is consistent with the tendency to inappropriately associate the location of an event with the location of a moving object.

ii.   Failure to recognize the motion of an object associated with an event

We have seen that students tend to associate event locations with marker objects rather than coordinate systems. We have seen further that in some cases students inappropriately associate event locations with objects that they recognize as moving in the reference frame of interest. In other cases, students associate the location of an event with a moving object without apparently recognizing that the object is moving. For example, in interviews with the Eruptions question, students were asked to produce an event diagram for the spacecraft frame. About one-quarter of the students failed to represent the spacecraft at rest in the spacecraft frame. Instead, they sketched an event diagram for the spacecraft frame in which the ground was at rest and the spacecraft was moving, as shown in Figure 4-11.[16]
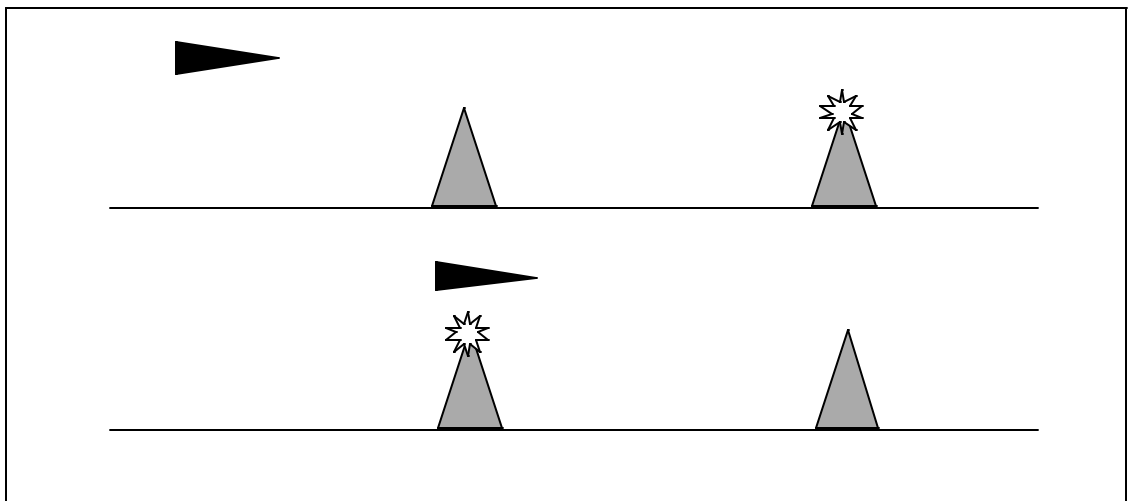
Figure 4-11:   Incorrect event diagram for the spacecraft frame of the Eruptions question.

Table 4-8: Results of the Eruptions question administered as an interview task, in which students drew an event diagram for the spacecraft frame.

	Interview task
	Graduate students Sp99 (N=10)
Correct: spacecraft at rest, mountains moving	30%*
Incorrect: mountains at rest, spacecraft moving	70%*
Other incorrect	0

* These results represent students' initial responses. All students corrected their event diagrams during the course of the interview, with assistance from the interviewer.

All of the graduate students who drew incorrect event diagrams eventually corrected their responses with prompting from the interviewer. However, it is interesting that seven out of ten graduate students do not spontaneously treat an object as not changing position in its own rest frame. Such difficulties with relative motion are consistent with an apparent lack of functional understanding of the concept of a reference frame.

Additional evidence that many students do not treat an object as stationary in its own frame arose in responses to the Measurement question. An appropriate measurement procedure for the length of an object includes consideration of the motion of the object in the frame of interest. In particular, if the frame of interest is one in which the object is moving, the ends of the object must be marked simultaneously in order for the spatial separation between the events to equal the length of the object. We have observed that students proposing measurement procedures for the length of a moving object tend to neglect the motion of the object in their discussion.

None of the students interviewed initially proposed a method for measuring the length of the spacecraft involving simultaneous measurement of the ends of the spacecraft. Three students initially proposed other correct methods that depended on prior knowledge of the speed of the spacecraft. Asked if there was a means by which one might measure the length of the spacecraft without prior knowledge of its speed, one student (who was attempting the equivalent measurement of the length of the landing strip in the spacecraft frame) replied,

"Hold on, this might be easier than it looks. Let's [assume] that somebody has put some reflectors onto the runway. There are reflectors on the end of the runway, and here's Beth coming in [on the spacecraft]. Beth sends out a short pulse of light, the pulse of light propagates down, hits the first reflector, comes back, some of it heads on down hits the second reflector, and what Beth gets back is two pulses, separated in time. She knows that the time between these pulses divided by c should give her the – better watch that – divided by 2c. Call L the length of the runway. T is this time delay, between these two pulses. That's what Beth thinks the length of the runway is in her frame." (graduate student)

In addition to erring by a factor of two, the student fails to account for the fact that in the frame of the spacecraft (Beth's frame), the landing strip will move forward in the time between the two reflections. The time between the spacecraft observer's reception of the two pulses will be less than the time it would take light to travel the length of the landing strip. Another student suggested a similar method.

Two other students proposed to measure the length of the spacecraft in the runway frame by means of a photographic image. This method, while sufficient for spacecraft moving with nonrelativistic speed relative to the observer, is inaccurate for large relative speeds. Light that reaches the camera while the shutter is open would leave the different locations on the object at different times, during which the object would move relative to the observer.

F. SUMMARY

We have seen in the course of our investigation that students do not spontaneously apply the formalism of a reference frame when measuring kinematical quantities. Asked to measure the length or speed of an object, students do not automatically generate the concept of a coordinate system by which to mark the position of events. Students not fluent with the means by which one measures the location and time of events distant from one another lack a functional understanding of reference frame. It seems likely that such students would have difficulty interpreting quantities such as the spatial separation between events. Furthermore, students who do not spontaneously

generate appropriate coordinate systems may have limited means by which to judge whether an object is in motion or at rest in a certain frame of reference.

Lacking the concept of a reference frame, students resort to marker objects to specify event locations. This potentially useful approach is problematic for students who are unable to apply the idea that a marker object must be at rest in the frame of interest in order to specify an event location properly. Some students appear not to recognize whether an object is moving in the frame of interest; other students inappropriately associate an event with a moving object, to the extent of claiming that events, like objects, may be in motion or at rest. These errors may prevent students from correctly identifying event locations or interpreting quantities such as the spatial separation between events.

The strong association of event locations with marker objects may be the basis for the apparent belief that the spatial separation between two events is identically equal to the distance between two objects. This inappropriate equation appears to be at the root of the indiscriminate application of length contraction and the failure to apply the Lorentz transformations.

NOTES TO CHAPTER FOUR

[1] B. Stanwyck, as quoted in *A Dictionary of Scientific Quotations,* edited by Alan L. Mackay (Institute of Physics Publishing, Bristol and Philadelphia, 1991), p. 229.

[2] E. Saltiel and J.L. Malgrange, "'Spontaneous' ways of reasoning in elementary kinematics," Eur. J. Phys. **1**, 73 (1980).

[3] A. Villani and J.L.A. Pacca, "Students' spontaneous ideas about the speed of light," Int. J. Sci. Educ. **9**, 55 (1987); "Spontaneous reasoning of graduate students," Int. J. Sci. Educ. **12**, 589 (1990).

[4] T.E. O'Brien-Pride, "An investigation of student difficulties with two dimensions, two-body systems, and relativity in introductory mechanics," Ph.D. dissertation, Department of Physics, University of Washington, 1997.

[5] R. Resnick, D. Halliday, and K.S. Krane, *Physics,* 4th edition (John Wiley & Sons, New York, NY, 1992), Chapter 21.

[6] D.J. Griffiths, *Introduction to Electrodynamics* (Prentice-Hall, Englewood Cliffs, NJ, 1989), Chapter 10.

[7] See, for example, p. 476 of ref. 5.

[8] One text that does discuss measurement procedures for spatial quantities is E.F. Taylor and J.A. Wheeler, *Spacetime Physics* (W.H. Freeman, New York, NY, 1992). See, for example, p. 63.

[9] *Ibid.* p. 64.

[10] Data from Oregon State University.

[11] In other interviews, students have made similar claims about the time durations between events, *i.e.,* that $\delta t' = k\delta t$.

[12] Data from Oregon State University.

[13] Data from Massachusetts Institute of Technology.

[14] See also S. Panse, J. Ramadas, and A. Kumar, "Alternative conceptions in Galilean relativity: frames of reference," Int. J. Sci. Educ. **16**, 63 (1994); J. Ramadas and A. Kumar, "Alternative conceptions in Galilean relativity: inertial and non-inertial observers," Int. J. Sci. Educ. **18**, 615 (1996). The authors document (in Galilean contexts) a widespread belief that the extent of an observer's reference frame is the extent of the physical object on which that observer is located (*e.g.,* the deck of a boat).

[15] It is possible to classify a coordinate label as a special case of a "marker object." In this chapter, however, we will reserve the term "marker object" to refer to a physical object.

[16] These results are consistent with results in refs. 2 and 3 that suggest student belief in a preferred frame of reference. However, the results of the Ratios question (section D), in which students perform poorly on questions about both frames, suggest that belief in a preferred frame is not solely responsible for the difficulties we observe.

CHAPTER FIVE:

ADDRESSING STUDENT DIFFICULTIES WITH SPATIAL MEASUREMENTS

Our imagination is stretched to the utmost, not, as in fiction, to imagine things which are not really there, but just to comprehend those things which are *there.*

– Richard Feynman[1]

A. INTRODUCTION

The investigation of student understanding of spatial measurements discussed in Chapter Four suggested that many introductory and advanced students who have studied special relativity do not have a functional understanding of basic spatial quantities such as length and displacement. The observed tendency to apply length contraction indiscriminately appears to be related to a failure to apply the formalism of a reference frame in determining the position of an event. Specific difficulties include treating the location of an event with a moving with an object associated with the event. Such errors appear to prevent students from correctly applying the Lorentz transformations.

This chapter describes the development and assessment of a series of tutorials that we have designed to address some of the specific difficulties with spatial measurements that were identified in our study. We begin by describing a tutorial sequence in which students articulate appropriate procedures for spatial measurements in nonrelativistic contexts. We then describe a tutorial sequence that guides students to interpret spatial quantities in more challenging relativistic contexts. In the process, students begin to recognize the conditions under which spatial separations are related by the formula for length contraction and when a more general relationship (*e.g.,* the Lorentz transformations) is appropriate. Assessment of each part of the series of tutorials is given immediately following the description of the curriculum.

We have used the series of tutorials described below in courses for introductory students and advanced undergraduate students. The materials are designed for use as a supplement to lecture instruction; the tutorials are not a stand-alone curriculum, but assume that students are introduced to certain ideas (*e.g.,* length contraction) in other parts of the course. The series of tutorials takes about two hours of class time to complete. Associated homework, completed outside of class, follows each tutorial.

The series of tutorials on spatial measurements is in fact integrated with the tutorial series described earlier (*Events and reference frames* and *Relativistic kinematics,* Chapter Three). Many of the contexts that we will describe in this chapter are similar to those presented in Chapter Three. We discuss here only those aspects of the instructional strategies that pertain to helping students develop an understanding of spatial measurements. The complete sequence of integrated exercises is in Appendix C.

B. ADDRESSING STUDENT DIFFICULTIES WITH SPATIAL MEASUREMENTS IN NONRELATIVISTIC CONTEXTS

Evidence presented in Chapter Four indicates that many student difficulties with spatial measurements may have their basis in difficulties applying the formalism of a reference frame and, in particular, determining the position of an event. We have designed a tutorial sequence, titled *Spatial measurement,* to help students develop a functional understanding of event location in the context of measurement procedures for spatial quantities such as length and displacement.

The tutorial sequence is specifically designed to address some of the underlying difficulties with events and their spatial coordinates that were discussed in Chapter Four. Among these difficulties is the tendency to associate the location of an event with the location of an object that is associated with that event, even if the object is moving in the reference frame of interest. In some cases, students appear not to recognize the motion of the associated object. In other cases, students claim that the event itself is moving. The *Spatial measurement* tutorial sequence contains exercises specifically designed to challenge these beliefs.

1. **Tutorial sequence:** *Spatial measurement*

a. *Eliciting the belief that the spatial separation between two events is identically equal to the length of an object*

In order to elicit students' incorrect ideas about spatial measurements, we ask a pretest question identical to the nonrelativistic version of the Eruptions question described in Chapter Four. In this question, shown in Figure 5-1, students are given the spatial separation and time duration between two events in the ground frame and are asked to calculate the spatial separation between the events in the frame of an airplane that moves with nonrelativistic speed relative to the ground.

Two volcanoes, Mt. Rainier and Mt. Hood, suddenly erupt on the same day. The volcanoes are 300 km apart in their rest frame. In the frame of a seismologist at rest on the ground, Mt. Hood erupts first; Mt. Rainier erupts exactly 100 minutes (6000 seconds) later.

An airplane flies with a constant speed of 50 m/s past Rainier towards Hood. Determine the spatial separation between the two eruptions in the reference frame of the airplane. Show your work.

Figure 5-1: *Spatial measurement* pretest.

Since the speed of the airplane relative to the ground is nonrelativistic, a correct answer to the *Spatial measurement* pretest may be obtained by application of the Galilean transformation for spatial separations. Choosing the unprimed frame to be the ground frame, the primed frame to be the frame of the airplane, and the positive direction to be the direction of the airplane's velocity in the ground frame:

$$\delta x' = \delta x - v\delta t$$
$$= 300 \text{ km} - (50 \text{ m/s}) \, (-6000 \text{ s}))$$
$$= 600 \text{ m}$$

As described in Chapter Four, student performance on this pretest is typically poor, with only about half of the students at introductory or advanced levels responding correctly or nearly correctly. The most common incorrect answer is that the distance between the locations of the explosions in the reference frame of the airplane is equal to the distance in the ground frame.

b. Guiding students to identify events

Analysis of the Measurement interview task described in Chapter Four indicated that even students who are able to use correct measurement procedures for spatial quantities such as length and displacement can have difficulty relating those procedures to events and their coordinates. The *Spatial measurement* tutorial sequence thus begins with an exercise in which students identify specific events, distinguish between related events, and recognize the differences between events, objects, locations, instants, *etc*. The exercise is designed to help students articulate and apply a definition for events.

The context for the exercise is one described in Chapter Three, in which a student, Alan, is some distance from a beeper that beeps once. Students are informed that in special relativity we commonly refer to material objects, locations in space, instants in time, and events. An event is described as being associated with a single location in space and a single instant in time. Students are presented with the list of items shown in Figure 5-2 and are asked to identify whether each item is an object, a location, an event, or none of these.

In the study of relativity we refer to material *objects, locations* in space, *instants* in time, and *events.* An *event* is associated with a single location in space and a single instant in time.

State whether each of the items below is *an object, a location, an instant, an event,* or *none of these.*

- The beeper of Part I
- The beeper emits a beep
- Alan hears the beep

- The exact time at which the beeper beeps
- A sound wave travels from the beeper to Alan
- Two beepers beep at the same time

Figure 5-2:   Tutorial exercise to identify and distinguish events.

Students should recognize that a beeper is an object; that "the beeper emits a beep" and "Alan hears a beep" are events (and in fact are distinct events, since they occur at different times and places); that "the exact time at which a beeper beeps" is an instant; and that "a sound wave travels from the beeper to Alan" and "two beepers beep at the same time" are combinations of events (in one case a series of events, and in the other case a pair). Students exhibit little difficulty with this task.

c.   Guiding students to construct and analyze event diagrams

We have found *event diagrams* to be a powerful tool for helping students understand events. In the process of constructing event diagrams, students must decide when and where to place a specific event in its physical context. The *Spatial measurement* tutorial sequence makes heavy use of event diagrams in addition to other representations (*e.g.,* algebraic).

Students are formally introduced to event diagrams in an exercise in which a 12-meter-long train moves with constant nonrelativistic speed along a long, straight stretch of train track. Alan stands at rest relative to the track. His assistant, Andy, is also at rest relative to the track and stands 12 meters from Alan as shown in Figure 5-3. Beth stands at rest on the train. Events 1, 2, and 3 are defined to be "the front of the train is next to Alan," "the front of the train is next to Andy," and "the rear of the train is next to Alan." Students are asked to sketch a picture showing Alan, Andy, Beth, and the train at the instant of each event in Alan's frame, to correctly represent the motion of each object in that frame, and to indicate the location of each event on the appropriate picture. A correct event diagram is shown in Figure 5-4.

A train moves with constant nonrelativistic speed along a straight track. The train is 12 meters long. Alan and Andy stand 12 meters apart at rest on the track (see figure). Beth and Becky stand at rest at the front and rear of the train, respectively.

Define events 1, 2, and 3 as follows:
   Event 1: The front of the train is next to Alan
   Event 2: The front of the train is next to Andy
   Event 3: The rear of the train is next to Alan

Sketch an event diagram showing Alan, Andy, Beth, and Becky at the instants of events 1, 2, and 3 in Alan's frame. (That is, sketch a separate picture for each different instant; sketch pictures for successive instants one below the other; and indicate the location of each event on the appropriate picture.)
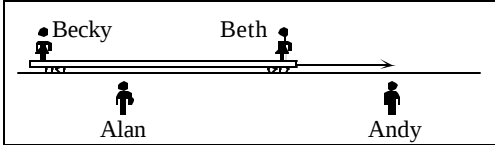
Figure 5-3:   Tutorial exercise to introduce students to event diagrams.
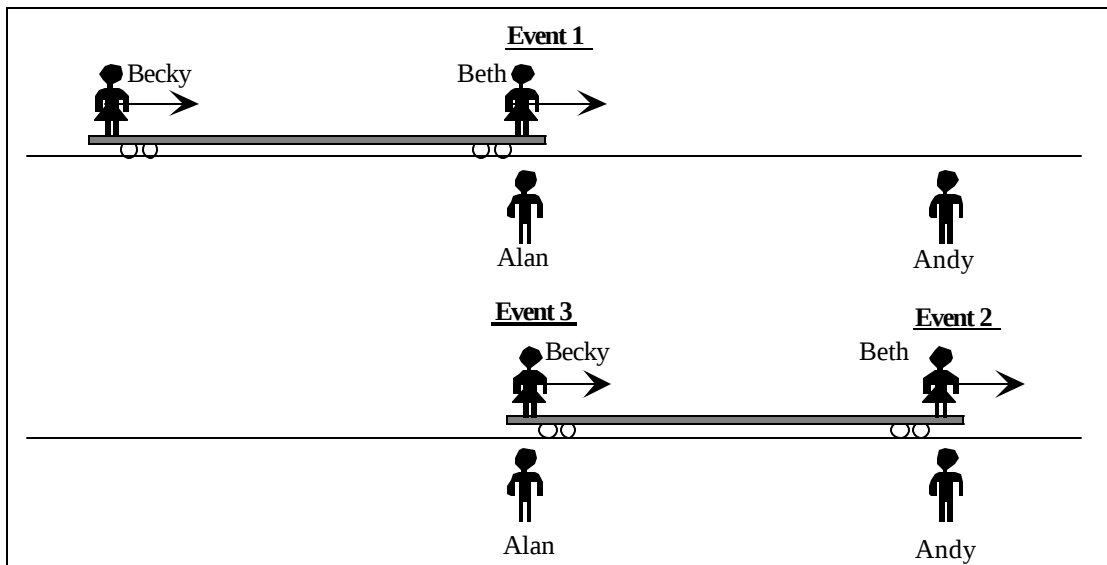
Figure 5-4:   Correct event diagram for the tutorial exercise of Figure 5-3.

We ask students several questions intended to help them interpret event diagrams and to extract information from this representation.   The questions also provide practice with distinguishing events from objects and instants. First, we ask under what circumstances more than one event would appear in a single picture in an event diagram. Students usually recognize that events that are simultaneous occur in the same picture (*i.e.,* at the same instant), typically at different locations. A correct answer to this question can help students answer the more difficult question of whether the first picture in their event diagram represents an object, a location, an instant, an event, or none of these. (Many students initially identify the picture as an event, but eventually recognize that it must represent an instant since it includes all locations and can contain more than one event.) Finally, we ask students whether it is possible for a single event to occur in more than one picture in an event diagram. To answer this question correctly (that it is not possible, since an event only exists at a single instant) students must distinguish an event from an object associated with the event.

d. Addressing the failure to recognize the motion of an object associated with an event

Event diagrams also present a context in which to address difficulties that some students seem to have in recognizing the motion of objects in a certain reference frame. Once students

have produced and analyzed an event diagram for the ground frame, we ask them what features of their diagram indicate that it is a diagram for Alan's frame. Most students quickly recognize that the fact that Alan is at the same position at every instant indicates that the diagram is for Alan's reference frame. Next, we ask students to produce an event diagram for the same context but in Beth's reference frame. Students are reminded explicitly to represent correctly the motion of the train in this frame. This exercise is shown in Figure 5-5.

- What feature(s) of your event diagram indicate that it is a diagram for Alan's reference frame?

- Sketch an event diagram showing events 1, 2, and 3 in Beth's frame. Be sure your diagram correctly represents the motion of the train in this frame.

Figure 5-5: Tutorial exercise regarding relative motion in an event diagram.
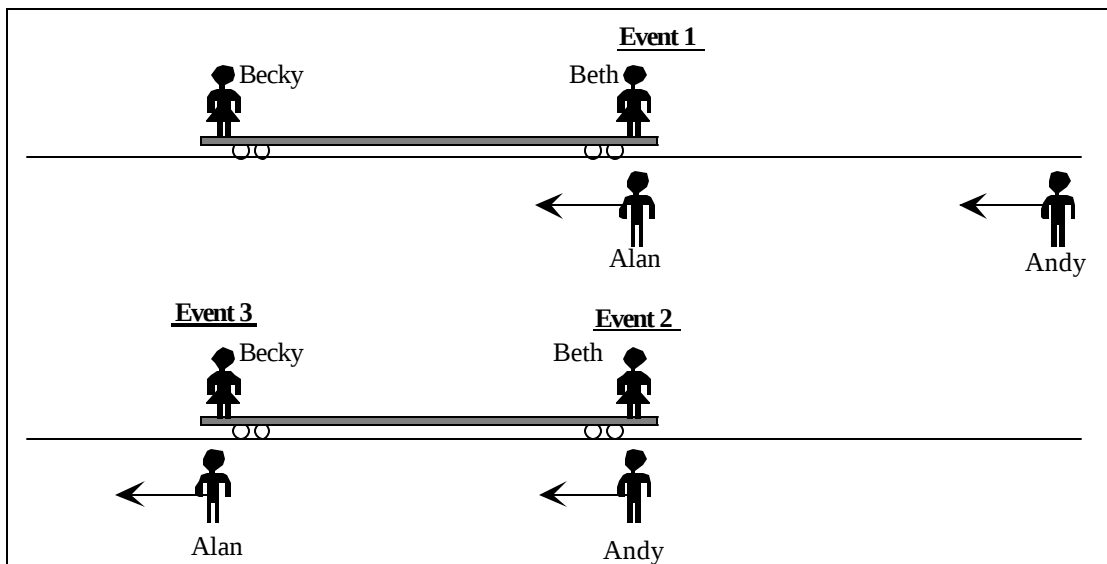
Figure 5-6: Correct event diagram for the exercise shown in Figure 5-5.

It has been our experience that students often neglect to indicate the train as being at rest in the train frame. However, they appear to be familiar with the idea when reminded of it, and are usually quick to correct their diagrams with prompting from an instructor. It is worth noting that there is no explicit instruction on relative motion in the *Spatial measurement* tutorial sequence due to time constraints.

e. Addressing the belief that the spatial separation between nonsimultaneous events is equal to the length of an object involving those events

As we have seen in Chapter Four, the belief that the spatial separation between two events is identically equal to the length of an object is widespread and persistent. The *Spatial measurement* tutorial sequence attempts to address this belief in Galilean contexts, before the complications of length contraction, relativity of simultaneity, and so on arise.

i.   Guiding students to identify event locations in an event diagram

Event diagrams present advantages over other representations of events (such as spacetime diagrams) in that events are situated in the physical context in which they occur. In order to help students identify the locations of events, we ask students to calculate spatial separations between pairs of events in the event diagrams shown in Figure 5-4 and Figure 5-6. The exercise is shown in Figure 5-7. In the classroom, we find that many students are surprised to find that the spatial separation between events 1 and 3 is zero in Alan's frame, as is the spatial separation between events 1 and 2 in Beth's frame.

How far apart are the locations of the following pairs of events in Alan's reference frame?
- Events 1 and 2
- Events 2 and 3
- Events 1 and 3

How far apart are the locations of the following pairs of events in Beth's reference frame?
- Events 1 and 2
- Events 2 and 3
- Events 1 and 3

Figure 5-7:   Tutorial exercise to calculate spatial separations between events.

ii.   Guiding students to appropriate interpretations of spatial separation

Spatial separations between events can, in general, have different interpretations depending on the events to which they refer; *e.g.,* they may be equal to the length of an object, the displacement of an object, or neither. The next exercise in the tutorial sequence introduces notation for spatial separations: for instance, $\delta x_{12}^{(A)} = x_2^{(A)} - x_1^{(A)}$ denotes the spatial separation between events 1 and 2 in Alan's frame. Students are then asked to provide *interpretations* for

each spatial separation relevant to the physical context described above. An interpretation is described as telling the meaning of the number in the physical context. One interpretation is provided as an example: the quantity $\delta x_{12}^{(A)}$ is the displacement of the train in Alan's frame, since events 1 and 2 occur at the front of the train. Students are told that some quantities may have more than one valid interpretation. The exercise is shown in Figure 5-8.

The symbol $\delta x_{12}^{(Alan)}$ indicates the spatial separation between events 1 and 2 as measured in Alan's reference frame: $\delta x_{12}^{(Alan)} = x_2^{(Alan)} - x_1^{(Alan)}$, where $x_1^{(Alan)}$ and $x_2^{(Alan)}$ are the positions of events 1 and 2 in Alan's reference frame. Note that the spatial separation between events is a signed quantity (it may be positive or negative).

Give *interpretations* for the magnitude of each of the following quantities; that is, tell the meaning of the number in this physical context. One has been provided as an example. Some quantities may have more than one interpretation.

- $\delta x_{12}^{(Alan)}$ The displacement of the train
- $\delta x_{13}^{(Alan)}$
- $\delta x_{23}^{(Alan)}$

- $\delta x_{12}^{(Beth)}$
- $\delta x_{13}^{(Beth)}$
- $\delta x_{23}^{(Beth)}$

Figure 5-8:   Tutorial exercise to interpret spatial separations between events.

Students should be able to recognize that $\delta x_{12}^{(B)}$ is the displacement of the train in Beth's frame; $\delta x_{13}^{(A)}$ and $\delta x_{13}^{(B)}$ are Alan's displacement in Alan's and Beth's frames respectively; $\delta x_{23}^{(A)}$ is the distance from Alan to Andy, which is equal to the length of the train, $\delta x_{23}^{(B)}$. Other correct interpretations are possible.

Most students find this exercise very challenging. They have not been asked to provide interpretations for spatial separations previously, and tend to answer by calculating the value of the spatial separation (often incorrectly) instead of by stating its relationship to physical quantities. Students eventually arrive at appropriate responses with assistance from instructors and each other.

f.   Guiding students to recognize correct measurement procedures for object length

Once students have been introduced to spatial separations and their possible interpretations in a physical context, the tutorial guides them to construct an appropriate measurement procedure

for object length. Next, students generalize by identifying the circumstances under which the spatial separation between two events is equal to the length of an object.

i.   Guiding students to apply the formalism of a reference frame in constructing measurement procedures for object length

The tutorial describes a train of unknown length that moves with constant nonrelativistic speed on the same track as that described above. Alan and his assistants are described as standing shoulder to shoulder along the track. (This explicit establishment of a reference frame serves as a hint for students in this exercise.) The tutorial asks students to describe a method by which Alan can determine the length of the train in his frame (i) if he knows the speed of the train in his frame and (ii) without knowing or measuring its speed first. In each case students are asked to specify the two events relevant to the measurement procedure by describing them in words. The exercise is shown in Figure 5-9.

A train of unknown length moves with constant nonrelativistic speed on the same track. Alan's assistants stand shoulder to shoulder along the track.

i. Describe a method by which Alan can determine the length of the train in his frame if he knows the speed of the train in his frame. Specify two events associated with this measurement proce dure.

ii. Describe a method by which Alan can determine the length of the train in his frame without knowing or measuring its speed first. Specify two events associated with this measurement pro cedure.

Figure 5-9:   Tutorial exercise guiding students to construct an appropriate measurement procedure for length.

A correct answer to part (i) should include a description of events that occur at the same location, *e.g.,* "the front of the train is next to Alan" and "the rear of the train is next to Alan." The length of the train may be calculated by multiplying the time between these two events by the speed of the train. The fact that students are usually quick to recognize this method is consistent with the results of the Measurement interview task reported in Chapter Four.

A correct answer to part (ii) should refer to events that occur at front and rear of the train at the same time, *e.g.,* "the front of the train is next to Alan at exactly noon" and "the rear of the

train is next to another assistant at exactly noon." The length of the train in the ground frame is identically equal to the spatial separation between these two events in the ground frame. Although this second method is more difficult for students to develop, the previous exercises lay useful groundwork for their discussions in class.

ii.   Guiding students to recognize the circumstances under which the spatial separation between two events is equal to the length of an object

In the next exercise, students are asked to articulate the conditions under which the spatial separation between two events is equal to the length of an object. Event X occurs at the front of a spaceship, and event Y occurs at the rear of the spaceship. Students are asked to describe the circumstances in which δx_{XY} is equal to the length of the spaceship (i) in the frame in which the spaceship is at rest and (ii) in a frame in which the spaceship is moving. Students are requested to sketch event diagrams supporting their answers. The exercise is shown in Figure 5-10.

Suppose event 4 occurs at the front of a long ship, and event 5 occurs at the rear of the same ship. Describe the circumstances in which δx_{45} is equal to the length of the ship:

i. in the frame in which the ship is at rest

ii. in frame $F,$ in which the ship is moving

Draw event diagrams to support your answers.

Figure 5-10:   Tutorial exercise guiding students to articulate the conditions under which the spatial separation between two events is equal to the length of an object.

Students should recognize that in the frame in which the spaceship is at rest, the spatial separation between events occurring at the front and rear of the spaceship is always equal to the length of the spaceship. In a frame in which the spaceship is moving, the spatial separation between events occurring at the front and rear of the spaceship is equal to the length of the spaceship if and only if the events are simultaneous. Illustrative event diagrams are shown in Figure 5-11.
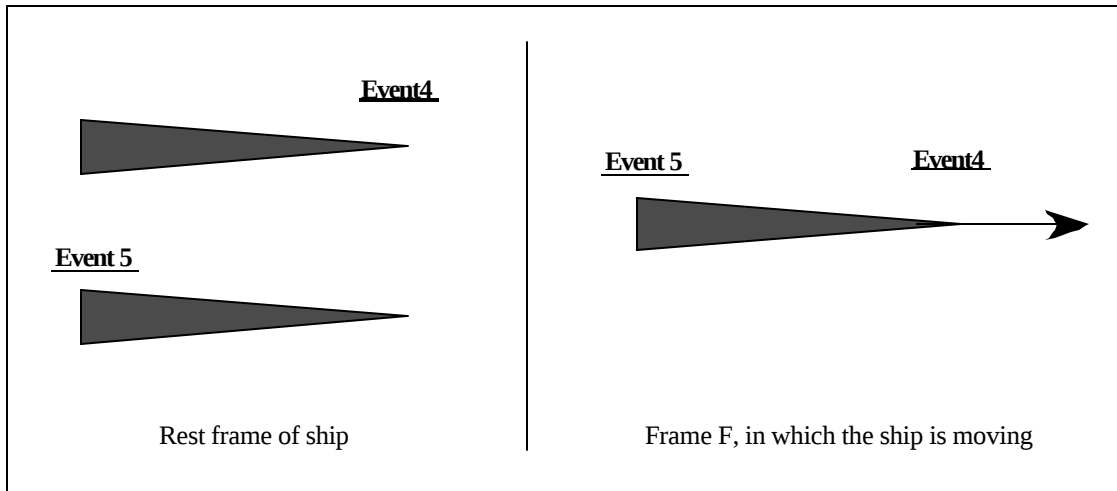
Figure 5-11:   Event diagrams illustrating a correct response to the tutorial exercise shown in Figure 5-10. (In the rest frame of the ship, the events may have any time ordering or be simultaneous; the spatial separation between them will still equal the length of the ship.)

g. *Reinforcing student understanding of the spatial separation between events*

Because the concept of spatial separation appears to be so difficult for students, the *Spatial measurement* tutorial sequence includes a number of exercises that review and reinforce the key ideas.   These exercises, discussed below, are assigned as homework following the tutorial sequence described above.

i. Guiding students to recognize an incorrect measurement procedure for length

Students are asked to criticize a statement that gives a suggested method for measuring the speed of a moving spacecraft. The exercise is shown in Figure 5-12. The method described is incorrect in that the locations of the ends of the spacecraft are recorded at different times; because the spacecraft moves in the time between the events, the spatial separation between them is not equal to the length of the spacecraft.

A physics student describes an incorrect method for measuring the length of a spaceship that is moving directly toward him with a speed v .

"Suppose there is a half-silvered mirror on each end of the spaceship. If I send a short pulse of light toward the spaceship, some of the light will be reflected back to me from the near end of the spaceship, and the rest will travel to the far end of the spaceship and be partially reflected back to me from that end. So I will receive two light pulses, separated in time. One-half of the time between these pulses times the speed of light is equal to the length of the spaceship."

Explain why the method described is incorrect.

Figure 5-12:   Tutorial exercise in which students criticize a measurement procedure for object length.

The student statement included in the exercise is adapted from an incorrect student response to the Measurement interview task (see Chapter Four). We have found that it is often useful for students to explicitly consider common incorrect student statements as a means to refine their own understanding.

ii.　　　Guiding students to identify an object whose location indicates the location of an event

We have seen that students often associate the location of an event with the location of an object that is moving in the reference frame of interest. To help address this difficulty, we have designed an exercise to help students identify appropriate "marker objects" for events (objects whose locations are always the same as the locations of the events; see Chapter Four). In this exercise, firecrackers explode harmlessly at either end of a flatcar that moves on a train track. The firecrackers explode at different times. Each explosion leaves ash on the end of the flatcar and on the track at the instant that it occurs. Students are asked to sketch event diagrams for the scenario in the track and the train frames. They are then asked to identify the object(s) whose position in that frame is always the same as the position of each explosion. Finally, students are asked to generalize their criteria for choosing marker objects by stating the circumstances under which the position of an object in a certain reference frame indicates the position of an event in that reference frame. Students are also asked the corresponding questions about spatial separation. The exercise is shown in Figure 5-13.

Firecrackers explode harmlessly at either end of a flatcar that moves along a long, straight stretch of train track. The firecracker at the front of the flatcar explodes several seconds before the one at the rear of the flatcar. Ash from each firecracker sticks to the flatcar *and* falls onto the ground at the instant each firecracker explodes.

A. Sketch an event diagram for the frame of the track. Include all four piles of ash in your pictures.

1. Identify the object(s) whose position, in this reference frame, is *always* the same as the position of:
      a. the first explosion
      b. the second explosion

2. In this reference frame, the magnitude of the spatial separation between the explosions is equal to the distance between two particular objects. Identify these objects.

B. Sketch an event diagram for the frame of the flatcar. Include all four piles of ash in your pictures.

In this reference frame, the magnitude of the spatial separation between the explosions is equal to the distance between two particular objects. Identify these objects.

C. Under what circumstances does the position of an object in a certain reference frame indicate the position of an event in that reference frame?

D. Under what circumstances is the distance between two objects equal to the magnitude of the spatial separation between two events?

Figure 5-13:   Tutorial exercise in which students identify appropriate marker objects.

Correct event diagrams for the above exercise are shown in Figure 5-14. Students should be able to recognize that in each frame, the ashpiles that are at rest in that frame mark the locations of the explosions in that frame; for example, the ashpiles that stick to the train mark the locations of the events in the train frame. In general, the position of an object in a certain reference frame indicates the position of an event in that reference frame if and only if the object (1) is at the location of the event at the time the event occurs and (2) is at rest in that reference frame.
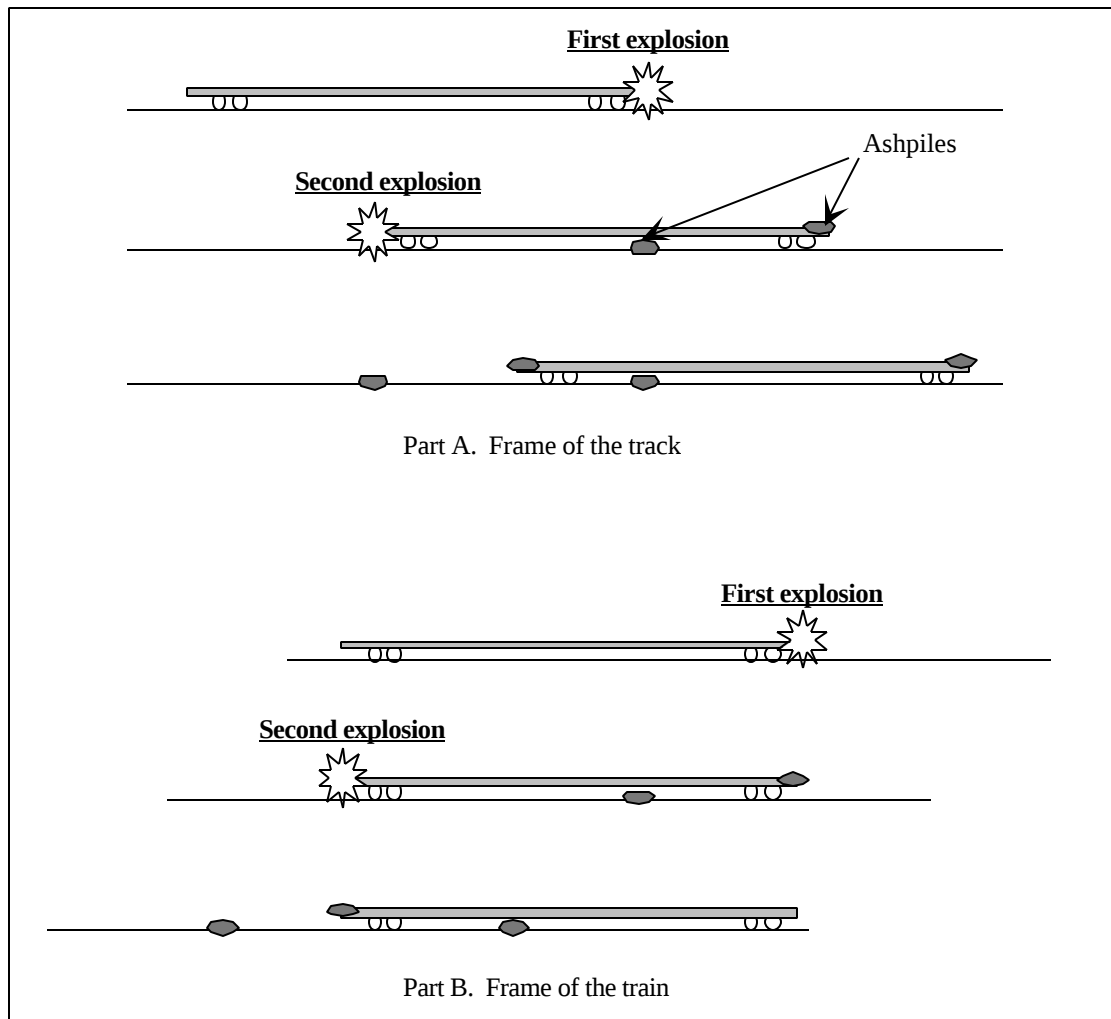
Figure 5-14:   Event diagrams for the tutorial exercise of Figure 5-13.

iii.   Addressing the belief that the location of an event can change with time

As we have mentioned, we have found event diagrams to be an especially useful representation for events and their spatial coordinates. In the following homework exercise, we present students with flawed event diagrams and student statements, and ask them to identify and correct the errors they find. The setup for the exercise is shown in Figure 5-15.

A city bus moves from left to right relative to the road.

Events 1, 2, and 3 are as follows: *Event 1:* The bus driver drops his hat onto the road
Event 2: A passenger on the bus sticks his gum
to the wall
Event 3: A drop of oil drips from the rear of the bus
onto the road

Events 1, 2, and 3 occur one after the other in numerical order. After events 1, 2, and 3
have occurred, the hat is to the right of the drop of oil on the road.

Figure 5-15:   Setup for tutorial exercise in which students criticize flawed event
diagrams.

In the first part of the exercise (Figure 5-16), the events are incorrectly represented on the same picture even though they are not simultaneous. In the second part (Figure 5-17), the events are incorrectly represented as moving along with certain objects. In each case, students are asked to identify the flaw in the diagram and produce correct event diagrams (shown in Figure 5-18).

A student sketches the event diagram shown below for the road frame. The diagram is flawed.
Identify the error(s) in the event diagram. Draw a correct event diagram for the road frame.

Event 3   Event 2   Event 1

Oil
(just dropped to ground)

Gum
(on the wall)

hat
(just dropped)

Figure 5-16:   Tutorial exercise in which students criticize a flawed event diagram.
The events are incorrectly represented on the same picture even though
they are not simultaneous.

Another student sketches the event diagram shown at right for the road frame. The diagram is flawed. Identify the error(s) in the event diagram. Correct the diagram appropriately.

Event 1

Event 2

Event 1

Event 2
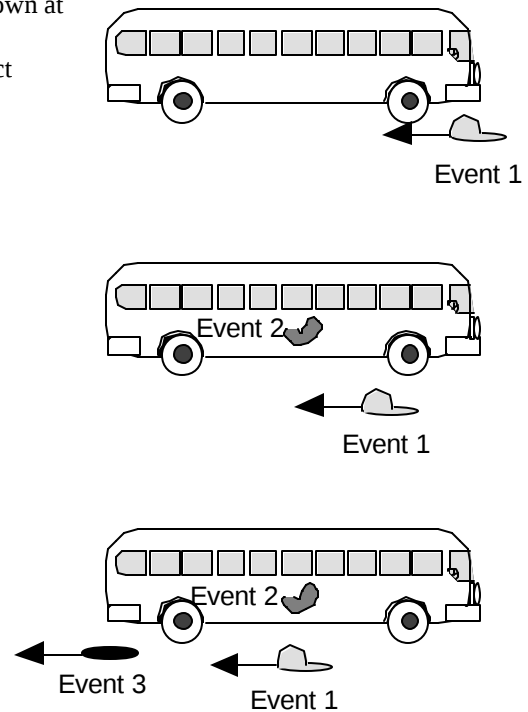
Event 3

Event 1

Figure 5-17: Tutorial exercise in which students criticize a flawed event diagram. The events are incorrectly represented as moving along with certain objects.
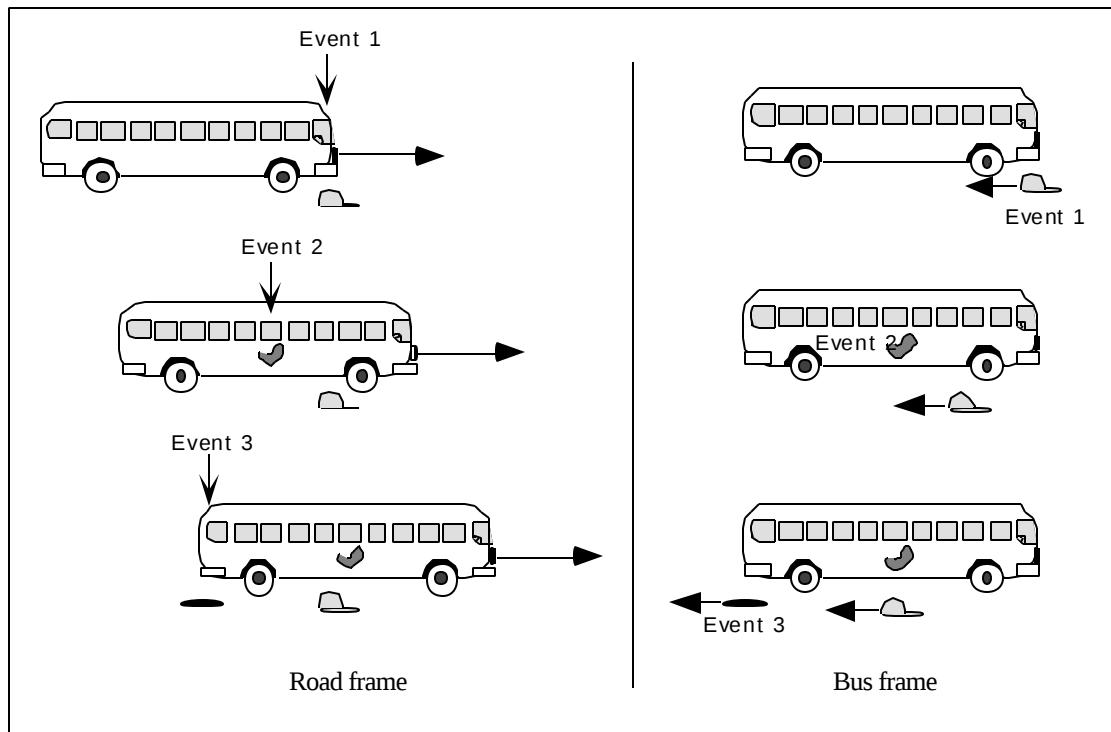
Figure 5-18:   Corrected event diagrams for the situations of Figure 5-16 and Figure
5-17.

In the third part (Figure 5-19), fictional student statements incorrectly equate the length of
an object with the spatial separation between events at the front and rear of the object. In
responding to question 1, students should recognize that although the hat is dropped from the
front of the bus and the oil drips from the back, the spatial separation between events 1 and 3 in
the road frame is *not* equal to the length of the bus, because the bus moves in the time between
the events. In responding to question 2, students should recognize that although the hat moves in
the time between events 1 and 3, the location of event 1 does not change; the distance between
the locations of events 1 and 3 is, therefore, the length of the bus.

Criticize the following statements.

1. "The hat is dropped from the front of the bus and the oil drips from the back. So the spatial separation between event 1 (the hat drops) and event 3 (the oil drips) in the road frame is approximately equal to the length of the bus."

2. "The distance between the location of event 1 and the location of event 3 in the bus frame is definitely less than the length of the bus, because the hat moves toward the back of the bus in the time between the events."

Figure 5-19:   Tutorial exercise in which students criticize fictional student statements regarding the spatial separation between events.

This exercise has proved to be quite a difficult one for students, even with the preparation provided by the earlier tutorial exercises. On the homework, only about half of the students answer either question 1 or question 2 correctly; about a third answer both parts correctly. In question 1, many students redraw the events as occurring at separate instants (in separate pictures) but fail to label them as occurring at a specific location (Figure 5-20). In question 2, some students redrew the event labels to indicate the events (*e.g.,* event 1, "the bus driver drops his hat onto the road") as being at rest in that frame (Figure 5-21(a)). Other students redrew the *hat* (and its label, "Event 1") to indicate its being at rest in the bus frame (Figure 5-21(b)). These students are failing to distinguish events from the objects associated with them.
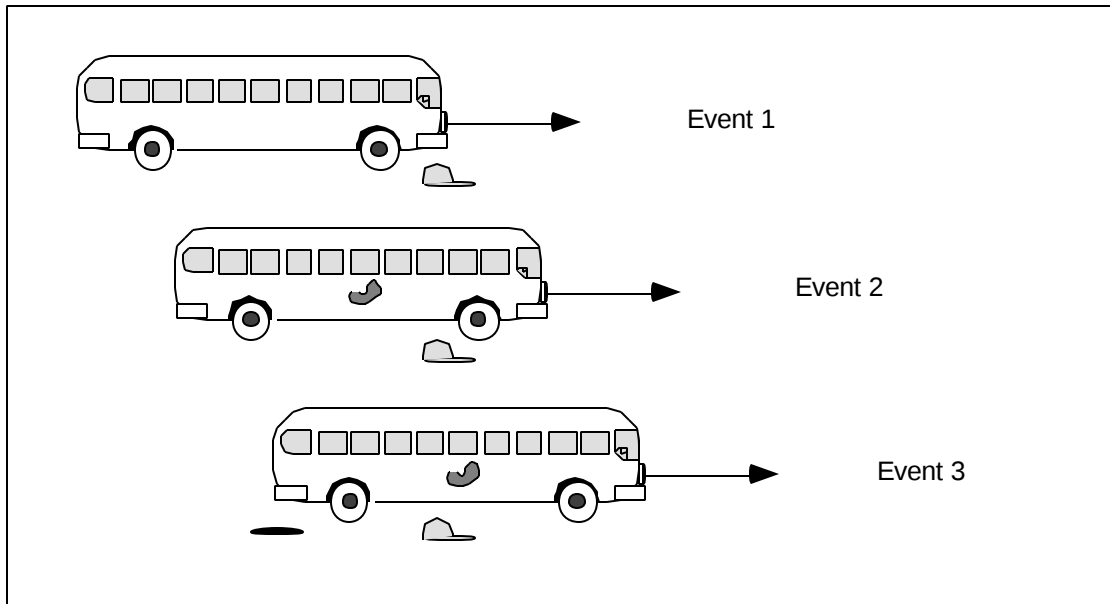
Figure 5-20: Incorrect student response to question 1 of the exercise shown in Figure 5-19. The student has failed to label the events as occurring at specific locations.
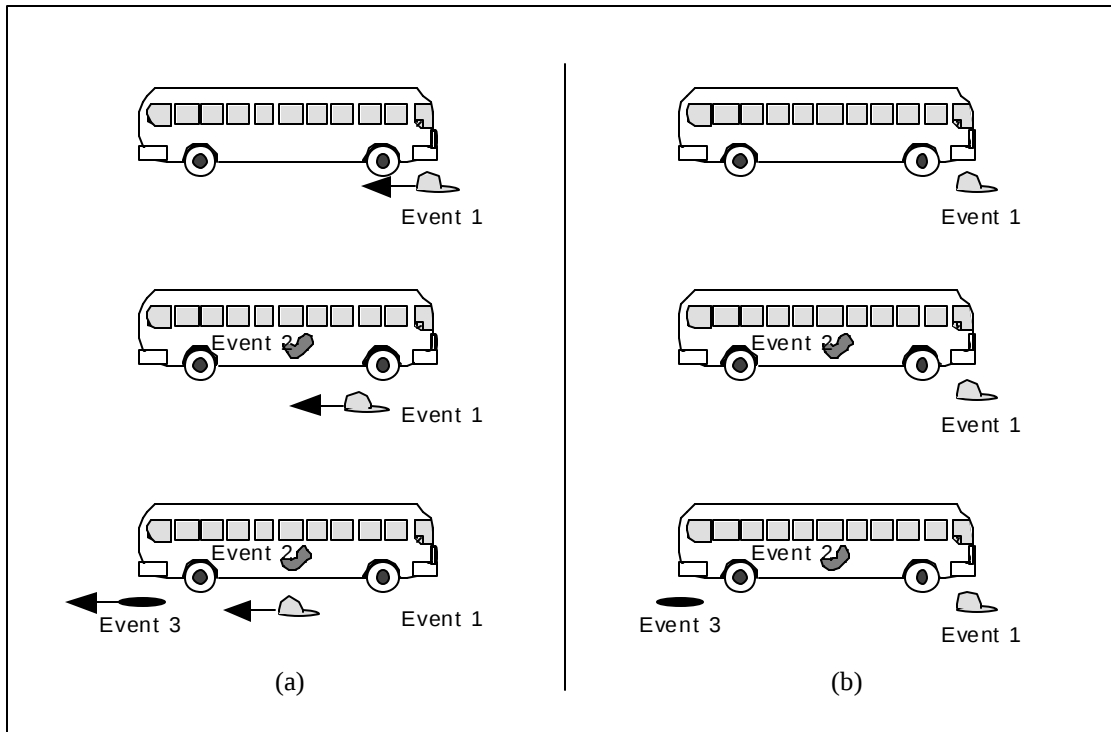
Figure 5-21:   Incorrect student responses to question 2 of the exercise shown in
Figure 5-19. (a) The student has indicated the event as being "at rest"
in the bus frame, as though an event could exist for an extended period
of time. (b) The student has indicated the hat as being at rest in the bus
frame, and has labeled the hat "Event 1" at each instant.

Interestingly, several students explained their responses with statements that are not incorrect:

 "Event 1 in reference to the bus occurs at the front of the bus and does not move

to the left." (introductory student)

"The hat may be moving backward relative to the bus, but in the bus frame the

location of the event stays constant at the front." (introductory student)

The students' superficially acceptable explanations conceal the errors in thinking that their incorrect event diagrams make clear. We often find that students' thinking is best illustrated by use of a variety of representations.

2. Assessing student understanding after the *Spatial measurement* tutorial sequence

We have assessed student understanding after the *Spatial measurement* tutorial sequence with a variety of questions. Three are discussed below. The first two are in a nonrelativistic context; the third is in a relativistic context. One question, the "calculation post-test," assesses student ability to calculate the spatial separation between two nonsimultaneous events. Another question, the "marker-object post-test," assesses student recognition of the motion of objects in a given reference frame. The third question, the "relativistic post-test," assesses student ability to extend their understanding of spatial separation to relativistic scenarios. In that question, students are asked to compare the spatial separation between events to the length of an object moving with relativistic speed in the frame of interest.

a. Question requiring calculation of spatial separation

i.   Description of question

In the *calculation post-test* for the *Spatial measurement* tutorial, students are given the spatial separation and time duration between two events in the ground frame and are asked to calculate the spatial separation between the events in the frame of an airplane that moves with nonrelativistic speed *v* relative to the ground. The post-test is identical to the pretest described on page 135 (Figure 5-1). However, no student saw the same question twice.

ii.   Correct response

As detailed on page 135, a correct answer to the calculation post-test may be obtained by application of the Galilean transformation for spatial separations.

$$\delta x' = \delta x - v\delta t$$
$$= 300 \text{ km} - (50 \text{ m/s}) (-6000 \text{ s}))$$
$$= 600 \text{ m}$$

As discussed in Chapter Four, students tend to fail to calculate the spatial separation correctly because of their intuitive idea that the spatial separation between the events is equal to the distance between the objects associated with the events.

iii.   Administration of question

The post-test described above has been given in courses for introductory and advanced undergraduate students. In some cases, the question was given after traditional instruction in Galilean relativity and served as a pretest of the *Spatial measurement* tutorial. In other cases, the question was given after students had completed the *Spatial measurement* tutorial and served as a post-test of that tutorial. Student performance after only traditional instruction was reported in detail in Chapter Four. We repeat these post-traditional-instruction results below for comparison with post-tutorial-instruction results.

When used as a post-test, the question described above was either part of the homework following the *Spatial measurement* tutorial or part of the pretest for the next tutorial (*Length contraction,* described below). The results were very similar. Therefore, they have been combined.

iv.   Student performance

Student performance on the calculation post-test is substantially improved after completion of the *Spatial measurement* tutorial sequence. The proportion of students answering correctly is approximately doubled in both introductory and advanced courses. The fraction of students claiming that the spatial separation between the events is invariant and equal to the distance between the mountains is reduced by about half.

Table 5-1:   Results of the calculation post-test of the *Spatial measurement* tutorial
sequence.

	Written question			
	Traditional instruction		SPM tutorial instruction	
	Introductory students Sp99[2], Au99[3], Au99 (N=108)	Advanced undergrad. Au00 (N=27)	Introductory students Au98 (N=37)	Advanced undergrad. Au99, Au00 (N=48)
Correct $(\delta x' = \delta x - v\delta t)$	35% (40)	35% (9)	60% (22)	65% (31)
Nearly correct $(\delta x' = \delta x + v\delta t)$	15% (14)	20% (5)	15% (5)	5% (2)
$\delta x' = \delta x$	30% (31)	30% (8)	15% (6)	15% (7)
$\delta x' = v\delta t$	5% (8)	15% (4)	5% (2)	10% (5)
Other	15% (15)	5% (1)	5% (2)	5% (3)

The results of the calculation post-test are especially notable in that the focus of the tutorial sequence is on qualitative comparisons, not quantitative applications. The fact that completion of the *Spatial measurement* tutorial strengthens student performance on the calculation post-test provides additional evidence that student difficulties with this question are in part conceptual difficulties with measurement procedures for object length.

b. Question about the motion of "marker objects"

i.   Description of question

In the marker-object post-test of the *Spatial measurement* tutorial sequence, two volcanoes, Mt. Rainier and Mt. Hood, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes. A spacecraft flies directly from Mt. Rainier to Mt. Hood with constant relativistic speed relative to the ground. Students are

asked, among other tasks, to sketch an event diagram illustrating the scenario in the spacecraft frame. We observe particularly whether students correctly represent the motion of objects in the spacecraft frame. The marker-object post-test, shown in Figure 5-22, is similar to the explicit version of the Eruptions question described in Chapter Four. However, no student saw the same question twice.

Two volcanoes, Mt. Rainier and Mt. Hood, erupt simultaneously in the reference frame of a seismologist at rest midway between them. The volcanoes are 300 km apart in their rest frame. A spacecraft flying with constant speed v =0.8c past Rainier towards Hood is directly over Rainier when it erupts.

 Sketch an event diagram for the spacecraft frame.

Figure 5-22:   Marker-object post-test for *Spatial measurement* tutorial sequence.

ii.   Correct response

A correct response is shown in Figure 5-23. A correct event diagram has several key features. For the purposes of this question, we observed only whether students indicated that in the spacecraft frame, the positions of the volcanoes change with time while the spacecraft remains at rest.
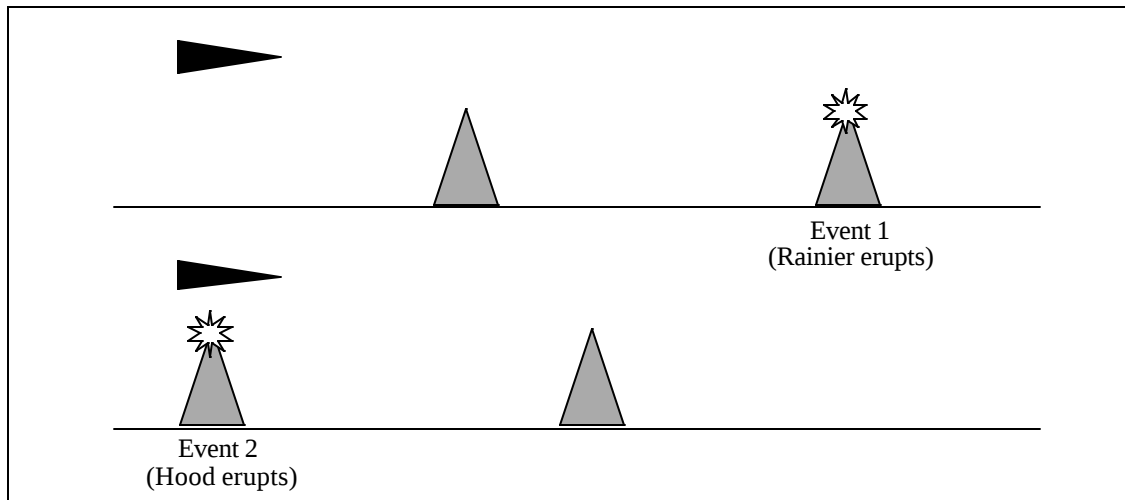
Figure 5-23:   Correct event diagram for the marker-object post-test.

iii.   Administration of question

We have administered the marker-object post-test as a written question to about 200 students in six introductory and advanced undergraduate physics courses. The question was administered after the *Spatial measurement* tutorial, as part of the pretest for the following tutorial (*Length contraction,* described below).

iv.   Student performance

Three-quarters of students answered the marker-object post-test correctly. Such performance represents substantial improvement over the results of the graduate student interviews discussed in Chapter Four, suggesting that the *Spatial measurement* tutorial provides effective instruction regarding marker object motion.

Table 5-2:   Results of marker-object post-test of the *Spatial measurement* tutorial sequence.

	Interview task	Written question	
	Traditional instruction	SPM tutorial instruction	
	Graduate students Sp99 (N=10)	Introductory students Au98, Au99, Sp99[4] (N=142)	Advanced undergraduates Wi98, Wi99, Au99, Au00 (N=93)
Correct: spaceship at rest, mountains moving	30% (3)[*]	70% (100)	75% (72)
Incorrect: ground at rest, spaceship moving	70% (7)[*]	15% (23)	15% (16)
Other incorrect	0	15% (19)	5% (5)

*   These results represent students' initial responses. All students corrected their event diagrams during the course of the interview, with assistance from the interviewer.

c. Question comparing spatial separation to length in a relativistic context

We have observed repeatedly that students tend to equate the spatial separation between events with the distance between objects associated with those events. As part of our assessment of the *Spatial measurement* tutorial sequence, we asked a question designed to probe this difficulty directly. In order to observe whether students were able to extend the ideas of the *Spatial measurement* tutorial sequence to situations they had not previously studied, we set the question in a relativistic context. (The students had already studied relativistic kinematics in other parts of the course.)

i.　　Description of question

The *relativistic post-test* of the *Spatial measurement* tutorial sequence takes place in the same context as the calculation post-test, but the speed of the vehicle passing over the mountains is relativistic, and the eruptions occur simultaneously in the ground frame. Students are asked (i) to determine the order of the eruptions in the spacecraft frame and (ii) to compare the spatial separation between the eruptions in the spacecraft frame to the distance between the mountains in that frame (and explain their reasoning). The relativistic post-test is shown in Figure 5-24.

Two volcanoes, Mt. Rainier and Mt. Hood, erupt simultaneously in the reference frame of a seismologist at rest midway between them. The volcanoes are 300 km apart in their rest frame. A spacecraft flying with constant speed $v = 0.8c$ past Rainier towards Hood is directly over Rainier when it erupts.

1. In the reference frame of the spacecraft, does Rainier erupt *before, after,* or *at the same time as* Hood? Explain your reasoning.

2. Determine the spatial separation between the two eruptions in the reference frame of the space craft. Show your work.

Figure 5-24:　Relativistic post-test of the *Spatial measurement* tutorial sequence.

Part 1 of this question is identical to the location-specific Spacecraft question described in Chapter Two and used to assess the effectiveness of the *Relativistic kinematics* tutorial sequence described in Chapter Three. Here, we discuss the results of this part only to the extent that they relate to student performance on part 2 of the question, which relates to the material in the *Spatial measurement* tutorial.

ii.   Correct response

In the spacecraft frame, Mt. Hood erupts first.   (For a detailed discussion of the determination of the order of events in the spacecraft frame, see Chapter Two.)   Because mountains move away from the location of Hood's eruption in the time between the eruptions, the spatial separation between the eruptions is greater than the distance between the mountains. An event diagram illustrating the situation in the spacecraft frame appears in Figure 5-23 (page 157).

iii.   Administration of question

We have administered the relativistic post-test of the *Spatial measurement* tutorial as a written question to about 200 students in five introductory and advanced undergraduate physics courses. The question was administered after the *Spatial measurement* tutorial, as part of the pretest for the following tutorial (*Length contraction,* described in section C.1 below). For reasons that will be discussed in section iv below, it is important to note that the question came after students had completed the first half of the *Relativistic kinematics* tutorial sequence described in Chapter Three.

iv.   Student performance

About one-third of students answered part 1 correctly.[5] (For a detailed discussion of the results of part 1, see the discussion of the (identical) location-specific spacecraft question in Chapter Two and the *Relativistic kinematics* post-test in Chapter Three.)

Student performance on part 2 of the relativistic post-test (comparing $\delta x,$ the spatial separation between the eruptions in the spacecraft frame, to $d,$ the distance between the mountains in that frame) is poor. Although we have no pretest data with which to compare the results, it is difficult to imagine that much improvement could have taken place as a result of *Spatial measurement* tutorial instruction. Many students claimed that the spatial separation between the events was equal to the distance in the spacecraft frame. They failed to recognize that this would only be true if the events were simultaneous. A smaller group of students claimed that the two quantities were related by the formula for length contraction, even though the two quantities are measured in the same frame of reference. Student approaches are summarized in Table 5-3.

Table 5-3:   Results of part 2 of the relativistic post-test of the *Spatial measurement* tutorial sequence.

	Written question	
	SPM tutorial instruction	
	Introductory students Au98, Sp99, Au99 (N=130)	Advanced undergraduate students Wi98, Au99, Au00 (N=69)
Correct ($\delta x > d$)	5% (8)	10% (7)
Reasoning based on incorrect measurement of length ($\delta x = d$)	50% (63)	60% (41)
Reasoning based on length contraction ($\delta x < d$)	25% (31)	15% (10)
Other or no reasoning	20% (28)	15% (11)

It might be questioned whether the students recognized that the two events were not simultaneous in the spacecraft frame. For this reason, we also examined student responses to part 1 of the comparison post-test in this context. Part 1 requests the order of eruptions in the spacecraft frame. As stated above, about one-third of students answered correctly.[6] Those students' responses to part 2 of the comparison post-test are summarized in Table 5-4. This sub-group did, in fact, do better on the question; the fraction of students claiming that $\delta x = d$ is lower in this sub-group of students. However, overall performance is still not impressive. Only 15-20% answered correctly. This result is consistent with our observations in Chapter Four that student difficulties with spatial measurements are apparently not limited to difficulties with the relativity of simultaneity.

Table 5-4:   Results of part 2 of the relativistic post-test of the *Spatial measurement* tutorial sequence. Includes only students who answered correctly regarding the relativity of simultaneity.

	Written question	
	SPM tutorial instruction	
	Introductory students Au98, Sp99, Au99 (N´=44)	Advanced undergraduate students Au99, Au00 (N´=35)
Correct ($\delta x > d$)	20% (8)	15% (5)
Reasoning based on incorrect measurement of length ($\delta x = d$)	25% (10)	45% (16)
Reasoning based on length contraction ($\delta x < d$)	30% (13)	25% (9)
Other or no reasoning	30% (13)	15% (5)

Although an understanding of length contraction is not necessary for (or relevant to) a correct answer to this post-test, the idea appears to present a major distraction for students. The *Spatial measurement* tutorial sequence, which deals only with nonrelativistic contexts, does not present opportunities to confront incorrect applications of length contraction. The above results suggest that explicit consideration of this idea may be essential to improving student understanding of measuring length.

v.   Commentary

The *Spatial measurement* tutorial sequence appears to be fairly effective in addressing some common student difficulties with spatial measurements in nonrelativistic contexts. For example, after completion of the tutorial sequence, 60-65% of students apply the Galilean transformations correctly to calculate a spatial separation between events, as compared with about 35% correct after traditional instruction. However, about one-third of students still do not calculate spatial separations correctly after the tutorial. We intend to add quantitative exercises to the tutorial sequence to the as a next step in improving the curriculum.

It is interesting to note that completion of the *Spatial measurement* tutorial sequence is not sufficient for students to successfully respond to questions comparing spatial separation to object length in relativistic contexts. Other ideas seem to arise in those contexts that prevent students from applying the ideas of spatial measurement introduced by the tutorial. As discussed below, the combination of the *Spatial measurement* and *Length contraction* tutorials seems to be necessary to help students apply measurement procedures for length in relativistic contexts.

C. ADDRESSING STUDENT DIFFICULTIES WITH SPATIAL MEASUREMENTS IN RELATIVISTIC CONTEXTS

1. Tutorial sequence: *Length contraction*
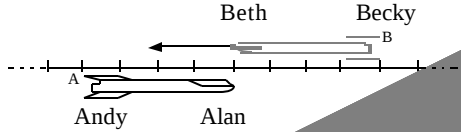
We have seen that students have difficulty with quantitative relationships among spatial quantities, especially in relativistic contexts. In particular, students tend to apply the formula for length contraction indiscriminately. In the *Length contraction* tutorial sequence, we try to address this difficulty by providing experience with situations in which the length contraction formula is not relevant and having students reflect on the conditions that must be met in order that it can be applied.

a. *Eliciting indiscriminate applications of length contraction*

The *Length contraction* pretest serves the purpose of eliciting students' tendency to apply length contraction indiscriminately. The pretest, shown in Figure 5-25, is identical to the relativistic version of the Ratios question described in Chapter Four. Two spaceships (A and B) pass one another with relativistic relative speed. Students are asked to determine numerical values for the ratios $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$ and $\delta x_{13}^{(A)}/\delta x_{13}^{(B)}$. As detailed in Chapter Four, since the relevant events for the numerator of each ratio occur at the same location, both ratios are equal to zero. Event diagrams for the *Length contraction* pretest are shown in Figure 5-26.

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Two spaceships, A and B, pass very close to each other. Alan is at rest in the front of spaceship A and Beth is at rest in the font of spaceship B. Andy and Becky are at rest in the backs of spaceships A and B respectively.

Define events 1, 2, and 3 as follows:
Event 1: Alan and Beth are adjacent
Event 2: Andy and Beth are adjacent
Event 3: Alan and Becky are adjacent

In Alan's frame, the speed of spaceship B is $0.6c$ ($g = 1.25$) and spaceships A and B each have length 120 c-ns. (One light-nanosecond, abbreviated c-ns, is the distance light travels in one nanosecond.)

Determine numerical values for the following ratios, in which, for example,
$\delta x_{23}(B) = x_3(B) - x_2(B)$ = the spatial separation between events 2 and 3 in Beth's frame.

1. $\delta x_{12}(B)/\delta x_{12}(A)$ 2. $\delta x_{13}(A)/ \delta x_{13}(B)$

Figure 5-25:   The *Length contraction* pretest.
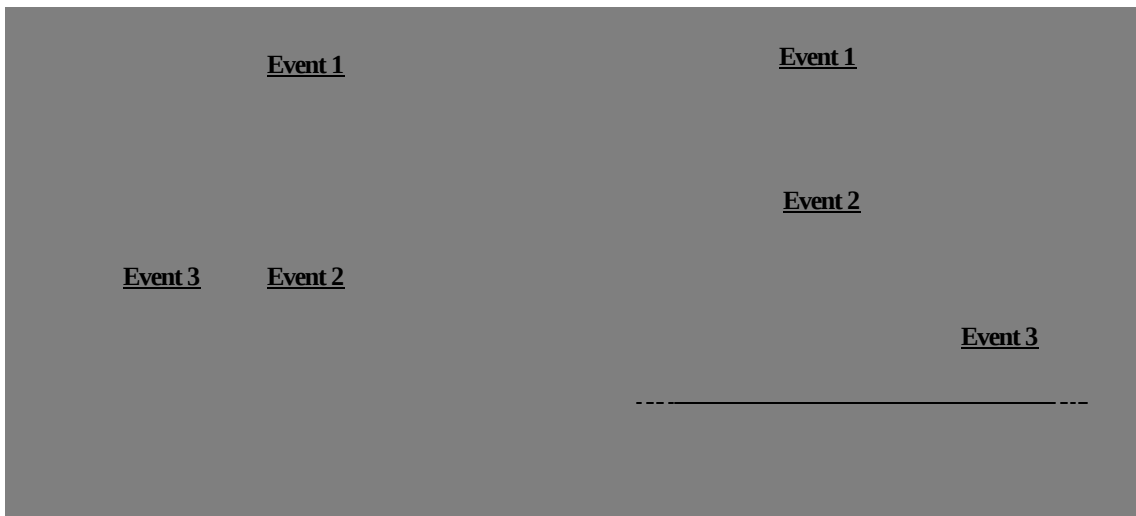
Figure 5-26:   Event diagrams for the *Length contraction* pretest.

The majority of students at all levels are unable to answer the *Length contraction* pretest correctly. Most state that the ratio is equal to the factor *g* or its reciprocal.

b. *Addressing the belief that a ratio of spatial separations is a ratio of lengths*

The *Length contraction* tutorial sequence begins with an exercise in which students produce event diagrams for both frames in the same situation used in the *Length contraction* pretest.[7] Students are prompted to include the fact that lengths are contracted in their diagrams

with subsidiary questions such as "Determine the proper length of spaceship B," and "In Beth's frame, is the length of spaceship A greater than, less than, or equal to 120 c-ns?" In the classroom, students show no difficulty representing contracted lengths in their event diagrams. The correct event diagrams appear in Figure 5-26.

i.   Exercise in which ratio of spatial separations is zero

After having produced event diagrams, students are asked to determine a numerical value for the ratio $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$ and to check that their answer is consistent with their event diagrams. In the classroom, almost no students answer this question correctly, despite having produced correct event diagrams on the same page of the tutorial worksheet. For this reason, we have included a "checkpoint" in the tutorial sequence at this exercise: students are instructed to check their answers with a tutorial instructor before proceeding to the rest of the worksheet. With such assistance, students are able to recognize that the value of the ratio requested is zero. The exercise is shown in Figure 5-27.

Determine the numerical value of the ratio $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$.

4   Check your answer with a tutorial instructor.
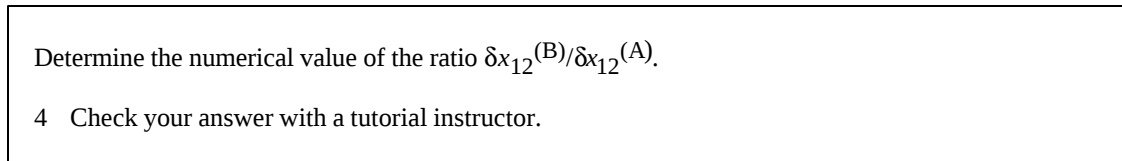
Figure 5-27:   Tutorial exercise in which students identify a ratio of spatial
                separations as being equal to zero.

ii.   Exercise in which ratio of spatial separations is the reciprocal of the expected ratio

The second exercise in the *Length contraction* tutorial sequence refers to the event diagrams just described, but is in the form of a student dialogue regarding the ratio $\delta x_{23}^{(B)}/\delta x_{23}^{(A)}$. Student 1 claims (correctly) that the length of spaceship A in Beth's frame is less than the length of spaceship A in Alan's frame, since in Beth's frame, spaceship A is contracted from its proper length. Student 2 goes on to deduce (incorrectly) that the ratio $\delta x_{23}^{(B)}/\delta x_{23}^{(A)}$ is therefore less than one. Students in the classroom are told that student 2's conclusion is incorrect and are asked to identify the error in Student 2's reasoning. The exercise is shown in Figure 5-28.

Consider the following student dialogue
.
Student 1: "The length of spaceship A in Beth's frame is less than the length of spaceship A in Alan's frame, since in Beth's frame, the length of spaceship A is contracted from its proper length."

Student 2: "Exactly. So $\delta x_{23}^{(B)} / \delta x_{23}^{(A)}$ is less than one."

Student 2's conclusion is *incorrect*. Identify the error in the student's reasoning.

Figure 5-28: Tutorial exercise in which a ratio of spatial separations is the
reciprocal of the expected ratio.

Students can use their event diagrams (Figure 5-26) to determine that the ratio $\delta x_{23}^{(B)}/\delta x_{23}^{(A)}$ is actually greater than one, since it is the ratio of the proper length of ship B to the contracted length of ship B. They should recognize that Student 2's error is in equating δx_{23} with the length of ship A; in fact, it is equal to the length of ship A only in the frame in which events 2 and 3 are simultaneous (Alan's frame).

In the classroom, students have little difficulty identifying the particular error in Student 2's reasoning. However, they seem to classify Student 2's error as careless, not conceptual; he or she made a mistake as to *which ship length* the spatial separations referred to. The exercise provides an example of a ratio of spatial separations that is the reciprocal of the expected ratio, but it is still a ratio of lengths. It is perhaps for this reason that the students find the exercise easy to complete.

c.   Reinforcing student understanding of spatial separation in the context of quantitative relationships

We have found that in special relativity it is important to provide students with practice in interpreting quantitative relationships. The practice provides a context in which to reinforce and extend student understanding of spatial separations between events. Below, we describe exercises emphasizing applications of the invariant spacetime interval. These exercises are assigned as homework after completion of the tutorial sequence described above.

i.   Applying the invariance of the spacetime interval to derive the Lorentz transformations

In one exercise, students work through a derivation of the Lorentz transformations. (In some cases, they have already seen a similar derivation in another part of the course.) They use the invariance of the spacetime interval and basic kinematic relationships (*e.g.,* the definition of velocity). The exercise describes a situation that includes three events and two reference frames. The events and frames are chosen so that the spatial and temporal separations between them have particular interpretations in the physical context described. Students complete a series of questions in which they relate these spatial and temporal separations to one another using the invariance of the spacetime interval and identify certain physically meaningful quantities (*e.g.,* lengths, velocities) in the process. By rewriting spatial and temporal separations in terms of these physical quantities and assuming linear transformations, students are able to determine the coefficients for both spatial and temporal Lorentz transformations. Because of the length of the exercise, it is not included here; the full exercise appears in Appendix C.

ii.   Interpreting spatial separations in the context of timelike, spacelike, and lightlike spacetime intervals

The last exercise that we describe has students consider timelike, spacelike, and lightlike intervals (*e.g.,* intervals for which the quantity $c^2 \delta t_{12}^2 = - c^2 \delta t_{12}^{(F)\,2} + \delta x_{12}^{(F)\,2}$ is positive, negative, or zero, respectively).[8] Several short exercises require students to interpret the terms in the spacetime interval by relating them to a generic physical context. For example, two events that are measured to occur 10 c-ns apart in frame A are observed to have a lightlike interval between them. Students are asked to determine the time duration between the events. For events that occur 10 c-ns apart in space and 15 ns apart in time in frame A, students are asked if there is a frame in which the events occur at the same place. The complete exercise is shown in Figure 5-29.

Pairs of events for which the quantity $c^2\delta t_{12}^2 = -c^2\delta t_{12}^{(F)\,2} + \delta x_{12}^{(F)\,2}$ (the "square of the spacetime interval between events 1 and 2") is positive are referred to as having a *spacelike* separation; pairs of events for which that quantity is negative are referred to as having a *timelike* separation. If the spacetime interval between two events is zero, their separation is said to be *lightlike*.

Two events, 1 and 2, are measured to occur 10 c-ns apart in frame A.

A. Suppose events 1 and 2 are separated by a *lightlike* interval. Determine the time duration (in ns) between the events in frame A.

B. Suppose instead that in frame A, event 1 occurs 15 ns before event 2.

1. Is there a reference frame in which the events occur at the same place? If so, describe the motion of that frame relative to frame A. If not, explain why not.

2. Is there a reference frame in which the events occur at the same time? If so, describe the motion of that frame relative to frame A. If not, explain why not.

3. Is there a reference frame in which event 1 occurs after event 2? If so, describe the motion of such a frame relative to frame A. If not, explain why not.

C. Suppose instead that in frame A, event 1 occurs 6 ns of time before event 2. Determine the *minimum* spatial separation (in c-ns) between the events in any frame.

Figure 5-29:   Tutorial exercise regarding interpretation of spatial separations in the context of timelike, spacelike, and lightlike intervals.

2. Assessing student understanding after *Length contraction*

We have assessed student understanding after the *Length contraction* tutorial with both qualitative and quantitative questions that require students to distinguish between spatial separation and object length. One, the "comparison post-test," asks students to compare the spatial separation between nonsimultaneous events to the length of a moving object. The "calculation post-test" and "ratios post-test" assess student ability to determine the spatial separation between events in situations in which the spatial separation does not correspond to the length of an object.

a. *Question comparing spatial separation to length*

i.   Description of question

The *comparison post-test* for the *Length contraction* tutorial is identical to the relativistic post-test for the *Spatial measurement* tutorial sequence, shown in Figure 5-24 on page 159. In

that question, a spacecraft flies with relativistic speed relative to volcanoes that erupt simultaneously in the ground frame. Students are asked (i) to determine the order of the eruptions in the spacecraft frame and (ii) to compare the spatial separation between the eruptions in the spacecraft frame to the distance between the mountains in that frame (and explain their reasoning).

ii.   Correct response

A correct response to the comparison post-test is described on page 160. Mt. Hood erupts first in the spacecraft frame. Since, in that frame, the mountains move away from the location of the first eruption in the time between the two eruptions, the spatial separation between the events is greater than the distance between the mountains.

iii.   Administration of question

We have administered the comparison post-test of the *Length contraction* tutorial sequence as an examination question to about a hundred students in three advanced undergraduate physics courses. The question was given on an examination after the *Length contraction* tutorial sequence. (Students had also completed the *Relativistic kinematics* tutorial sequence described in Chapter Three at that time.)

iv.   Student performance

Student performance on part 2 of the comparison post-test is substantially improved with completion of the *Length contraction* tutorial sequence. Table 5-5 and Table 5-6 report the performance of introductory and advanced students, respectively. In each table, *N* represents all students taking the post-test; *N´* represents the number of students taking the post-test who responded correctly to part 1 regarding the order of events in the spacecraft frame. (See page 161 for a discussion of the relevance of event order to this comparison.)

Table 5-5: Results of part 2 of the comparison post-test of the *Length contraction* (LC) tutorial sequence, administered to introductory students. Results after the *Spatial measurement* (SPM) tutorial are shown for comparison. *N* includes all students responding to the post-test; *N´* includes only those students who answered correctly regarding the relativity of simultaneity in part 1.

	Written question			
	Introductory students			
	SPM tutorial instruction Au98, Sp99, Au99		SPM and LC tutorial instruction Au99	
	(N=130)	(N´=44)	(N=70)	(N´=44)
Correct ($\delta x > d$)	5% (8)	20% (8)	35% (24)	45% (19)
Reasoning based on incorrect measurement of length ($\delta x = d$)	50% (63)	25% (10)	20% (15)	20% (8)
Reasoning based on length contraction ($\delta x < d$)	25% (31)	30% (13)	35% (23)	25% (12)
Other or no reasoning	20% (28)	30% (13)	10% (8)	10% (5)

Table 5-6: Results of part 2 of the comparison post-test of the *Length contraction* (LC) tutorial sequence, administered to advanced undergraduate students. Results after the *Spatial measurement* (SPM) tutorial are shown for comparison. *N* includes all students responding to the post-test; *N´* includes only those students who answered correctly regarding the relativity of simultaneity in part 1.

	Written question			
	Advanced undergraduate students			
	SPM tutorial instruction Wi98, Au99, Au00		SPM and LC tutorial instruction Wi98, Au98, Au99	
	(N=69)	(N´=35)	(N=67)	(N´=49)
Correct ($\delta x > d$)	10% (7)	15% (5)	40% (28)	55% (27)
Reasoning based on incorrect measurement of length ($\delta x = d$)	60% (41)	45% (16)	15% (10)	10% (5)
Reasoning based on length contraction ($\delta x < d$)	15% (10)	25% (9)	25% (16)	15% (9)
Other or no reasoning	15% (11)	15% (5)	20% (13)	20% (9)

Student performance on this question is improved after working through the *Length contraction* tutorial sequence. Among both introductory and advanced students, the percentage reasoning based on an incorrect measurement of length ($\delta x = d$) is reduced.

The percentage of students reasoning based on length contraction is not dramatically reduced and in some cases may increase as a result of tutorial instruction. This error on students' part is not well understood; students appear to be applying length contraction to compare two quantities that are measured in the same frame. Further research is needed to better identify and respond to this particular student difficulty.

b. Question requiring calculation of spatial separation

i. Description of question

The calculation post-test for the *Length contraction* tutorial sequence is identical to the implicit version of the Eruptions question described in Chapter Four. In that question, a spacecraft flies with relativistic speed relative to volcanoes that erupt simultaneously in the ground frame. Students are first asked whether there is a frame in which the events are simultaneous and, if so, to determine the velocity of that frame relative to the ground. Second, students are asked to determine the spatial separation between the eruptions in the spacecraft frame. The calculation post-test appears in Figure 5-30.

Two volcanoes, Mt. Rainier and Mt. Hood, suddenly erupt on the same day. The volcanoes are 300 km apart in their rest frame. In the frame of a seismologist at rest on the ground Mt. Hood erupts first; Mt. Rainier erupts at a time $c^2 t = 120$ km later.

1. Is there a frame in which the eruptions occur simultaneously? If so, determine the magnitude and direction of the velocity of this frame relative to the ground. If not, explain why not.

2. A spacecraft flies with constant speed past Rainier towards Hood at v 0.8c. Determine the spatial separation between the two eruptions in the reference frame of the spacecraft. Show your work.

Figure 5-30: Calculation post-test for the *Length contraction* tutorial sequence.

Student responses to part 1 of this question reflect their understanding of the relativity of simultaneity and are discussed in Chapter Two (section E.3). Here, we discuss the results of part 1 only to the extent that they relate to student performance on part 2 of the question, which relates to the material in the *Length contraction* tutorial. The discussion below parallels that in Chapter Four (section D).

ii. Correct response

Correct responses to parts 1 and 2 may be obtained by application of the Lorentz transformations. The positive direction is from Rainier to Hood. In the solution to part 1 below, the unprimed frame is the ground frame, the primed frame is the frame in which the events are simultaneous, and *v* is the velocity of that frame relative to the ground. The eruptions are simultaneous in a frame moving with speed 0.4c from Hood to Rainier.

Part 1: $\delta t' = 0 = \boldsymbol{g}(\delta t - v\delta x/c^2)$

$= 5/3\ (-400\ \text{ns} - v\ (1000\ c\text{-ns})/c^2)$

$v = -0.4c$

In the solution to part 2 below, the unprimed frame is the ground frame, the double-primed frame is the frame of the spacecraft, and V is the velocity of the spacecraft relative to the ground. The eruptions are 2200 c-ns apart in the spacecraft frame.

Part 2: $\delta x'' = \boldsymbol{g}(\delta x - V\delta t)$

$= 2200\ \text{c-ns}$

iii.   Administration of question

We have administered the calculation post-test of the *Length contraction* tutorial sequence as an examination question to students in three introductory and advanced undergraduate physics courses.

iv.   Student performance

Table 5-8 summarizes student performance on part 2 of the post-test. Student performance is improved over pretest performance. Two-thirds of introductory students use a correct approach after tutorial instruction, compared to about one-fourth after traditional instruction; among advanced undergraduate students, performance improves from about one-third to about one-half using a correct approach. Although a significant number of students still use length contraction inappropriately in response to this question, the fraction of students doing so is reduced. Overall, student performance is substantially better than graduate student performance without tutorial instruction.

As was the case after traditional instruction, most students answer part 1 of the calculation post-test for the *Length contraction* tutorial sequence in a manner consistent with the idea that the eruptions are not simultaneous in the spacecraft frame (see Chapter Four). Student performance among those who recognized the relativity of simultaneity (denoted by N' in the table) is slightly better than the performance of the entire group (denoted by N).

Table 5-8:   Results of part 2 of the calculation post-test of the *Length contraction*
               tutorial sequence given to introductory students. The second row is a
               subset of the first. Graduate student results are included for comparison. *N*
               includes all students responding to the post-test; *N´* includes only those
               students who answered correctly regarding the relativity of simultaneity in
               part 1.

	Traditional instruction			SPM and LC tutorial instruction	
	Written question		Interview task	Written question	
	Introductory students Au98, Sp99, Au99		Graduate students Sp98, Sp99	Introductory students Au98, Au99	
	(N=127)	(N´=114)	(N=N´=16)	(N=116)	(N´=107)
Correct approach (Lorentz trans.)	25% (30)	25% (29)	20% (3)	65% (74)	65% (72)
Correct answer	*15% (21)*	*15% (20)*	*20% (3)*	*45% (55)*	*50% (53)*
Approach based on length contraction	60% (78)	55% (66)	80% (13)	30% (34)	25% (27)
Other incorrect approach	15% (19)	15% (19)	0	5% (8)	5% (8)

Table 5-9: Results of part 2 of the calculation post-test of the *Length contraction* tutorial sequence given to advanced undergraduate students. The second row is a subset of the first. Graduate student results are included for comparison. *N* includes all students responding to the post-test; *N´* includes only those students who answered correctly regarding the relativity of simultaneity in part 1.

	Traditional instruction			SPM and LC tutorial instruction
	Written task		Interview task	Written task
	Advanced undergraduate students Sp98		Graduate students Sp98, Sp99	Advanced undergraduate students Wi98
	(N=34)	(N´=25)	(N=N´=16)	(N=N´ =22)
Correct approach (Lorentz trans.)	30% (10)	35% (9)	20% (3)	50% (11)
Correct answer	*25% (8)*	*30% (7)*	*20% (3)*	*35% (8)*
Approach based on length contraction	60% (20)	50% (12)	80% (13)	45% (10)
Other incorrect approach	10% (4)	15% (4)	0	5% (1)

c. *Question requiring calculation of a ratio of spatial separations*

i. Description of question

The *ratios post-test* for the *Length contraction* tutorial sequence is identical to part (i) of the pretest described on page 163 (Figure 5-25). The post-test is also extremely similar to the tutorial exercise described on page 164. In the question, Alan and Andy are in the front and rear of spaceship A, and Beth and Becky are in the front and rear of spaceship B. The two spaceships

pass one another with given relativistic speed. Students are asked to determine the numerical value of the ratio $\delta x_{12}^{(B)}/\delta x_{12}^{(A)}$, where events 1 and 2 both occur at Beth.

ii.   Correct response

Because the events in question occur at the same location in the frame of ship B, the value of the ratio requested is zero.

iii.   Administration of question

We have administered the ratios post-test for the *Length contraction* tutorial sequence as an examination question to students in one introductory and one advanced undergraduate physics course.

iv.   Student performance

The ratios question appears to be singularly difficult for students. Performance on the post-test is poor, showing little or no improvement (perhaps even a reduction) from pretest performance. This is despite the fact that students had seen a nearly identical question previously in tutorial or on the pretest. As we have seen in other contexts (*e.g.,* in Chapter Three), repeated exposure to the same question does not reliably result in improved performance.

Table 5-10:   Results of the ratios post-test of the *Length contraction* tutorial sequence
given to introductory students.

	Written question			
	Introductory students		Advanced undergraduate students	
	Traditional instruction Sp98, Au98, Sp99, Au99 (N=65)	SPM and LC tutorial instruction Au99 (N=56)	Traditional instruction Au98, Sp99 (N=26)	SPM and LC tutorial instruction Au99 (N=19)
Correct (zero)	10% (5)	15% (7)	40% (10)	30% (6)
Incorrect: g or $1/g$	75% (49)	75% (43)	60% (16)	70% (13)
Other incorrect	15% (11)	10% (6)	0	0

Further research is needed to identify instructional strategies that will enable students to identify spatial separations that are equal to zero.

D.   SUMMARY

Based on the research described in Chapter Four, we developed a tutorial sequence to address students' difficulties with event position, spatial measurements, and applications of quantitative relationships such as length contraction. The two-part sequence focuses first on applications of the concept of a reference frame to measurement procedures for the spatial separation between events in general and object length in particular. The second part requires students to identify the conditions under which length contraction is and is not the appropriate relationship between pairs of spatial separations. We have found that students who have completed this sequence are more likely to be able to correctly compare spatial separation to object length, and are better able to perform calculations of spatial separation by means of the Galilean or Lorentz transformations.

Some difficulties with spatial measurements are persistent, and thus far we have only partially succeeded in addressing them within a few hours of tutorial instruction. Student

performance on applications of the Lorentz transformations leaves room for improvement, and students have particular difficulty recognizing spatial separations that are equal to zero. Further research is needed to identify effective instructional strategies to address persistent student difficulties with spatial measurements.

NOTES TO CHAPTER FIVE

[1] R.P. Feynman, *The Character of Physical Law* (MIT Press, Cambridge, MA, 1967), p.127.

[2] Data from Oregon State University.

[3] Data from Massachusetts Institute of Technology.

[4] Data from Oregon State University.

[5] Students had completed the first half of the *Relativistic kinematics* tutorial described in Chapter Three (parts a-d). After completion of the entire *Relativistic kinematics* tutorial sequence, student performance is about 50% correct on similar questions.

[6] Note that part (i) of the comparison post-test is identical to the extremely challenging location-specific spacecraft question discussed in Chapter Two. By using this version of the question, we hoped to distinguish students with a genuine understanding of the relativity of simultaneity from those with a superficial recollection of the idea.

[7] The same exercise serves as a reinforcement of the relativity of simultaneity (see Chapter Three, section C.1.e).

[8] Several different definitions of the spacetime interval, timelike intervals, *etc.* are common in undergraduate physics courses. Each time we assigned this exercise, we consulted with the instructor to make sure that the definitions we used were consistent with his or her definitions and with the text for the course. The resulting modifications to the exercises were minor.

CHAPTER SIX:

CONCLUSION

In this dissertation we have presented results from a systematic investigation of student understanding of special relativity. This work is part of the ongoing effort of the Physics Education Group at the University of Washington to establish a research base on the teaching and learning of physics that can serve as a resource for instruction and curriculum development. During this investigation, which was conducted among students enrolled in physics courses ranging from the introductory to the graduate level, we identified persistent difficulties with the definitions of the position and time of an event and with the concept of a reference frame. We have applied the results from this research to guide the design of instructional materials to address some of the specific difficulties that we identified.

Our investigation has shown that many students do not think of a reference frame as a system of observers that determine the same position and time for any event. Instead, they appear to equate a reference frame with a "point of view." They interpret statements of the frame-dependence of the time of an event to mean that observers at different locations receive signals from events at different times. Popular culture does not distinguish well between relativity and *relativism,* and the idea that "everything is relative" may find expression in modern physics classrooms. However, the difficulties that we identified do not appear to arise simply from inappropriately colloquial interpretations of technical terms. Students appear to believe strongly that it is in the nature of relativity for observers to disagree about the reality that surrounds them. Such a belief unravels the very fabric of relativity theory, which depends on invariance as much as it allows for variability. However, the incorrect belief has the tremendous advantage of allowing students to avoid the true relativity of simultaneity, which is easily among the most disturbing results of modern physics.

Student difficulties with spatial measurements also appear to have their roots in difficulties with the concept of a reference frame. In the context of spatial measurements, however, instead of being burdened with an incorrect concept of reference frame, students appear to lack such a concept. For example, they do not spontaneously apply the formalism of a reference frame when asked to measure the length or speed of an object. They use objects to specify event locations, a practice that suggests that they imagine space as being "studded" with items of interest, rather than uniformly blanketed with a coordinate grid. Lacking a reliable sense of what is meant by the position of an event, students tend to associate events with moving objects, an association that may be the basis for the apparent belief that the spatial separation between two events is identically equal to the distance between two objects. This inappropriate equality seems to be at the root of the indiscriminate application of length contraction and failure to apply the Lorentz transformations.

Traditional instruction in relativity appears to have little effect on these ideas, which are present even among graduate students in physics. In fact, instruction in relativity may inadvertently reinforce such beliefs. New vocabulary, equations, and slogans such as "moving clocks run slow" may be assimilated without understanding and may be used incorrectly to justify a worldview that is Newtonian at best, and may perhaps be better described as medieval.[1]

The results of this investigation strongly suggest that a meaningful understanding of relativity requires a sound basis in nonrelativistic kinematics. This generalization has guided us in the development of tutorials to supplement instruction in special relativity. For example, the instructional sequence that attempts to address student difficulties with applications of length contraction begins with applications of the concept of a reference frame to measurement procedures for the spatial separation between events in nonrelativistic situations. Similarly, the sequence that addresses student difficulties with the relativity of simultaneity focuses first on the development of the concept of a reference frame and measurement procedures for the time of an event. Later parts of each sequence work to help students apply these basic ideas in a rigorous manner to challenging relativistic scenarios.

After students have completed instruction using the materials that we developed, we have posed examination problems designed to test student conceptual understanding. By comparing

student performance on these questions to that after standard instruction, we have been able to assess the effectiveness of the curriculum. In several cases, we have found that student performance on these problems is significantly better in courses in which our materials have been used than in courses in which students have completed standard instruction. Students who have completed the series of tutorials on special relativity are more likely to answer correctly basic questions about simultaneity, both for observers at rest relative to one another and for those in motion relative to one another. They are also more likely to be able to compare spatial separation to object length and to be able to perform calculations of spatial separation by means of the Galilean or Lorentz transformations. We have also identified other difficulties that appear to be very persistent. Thus far we have only partially succeeded in addressing these through tutorial instruction. Further research is needed to identify effective instructional strategies.

The work reported in this dissertation provides an example of the three-part process by which we attempt to improve the teaching and learning of physics. By investigating student difficulties, developing curriculum to address these difficulties, and assessing student performance after instruction is completed, we hope to gain an increasingly detailed and coherent understanding of how students learn physics. Our experience indicates that the type of research and research-based curriculum development described greatly increases the likelihood that there will be cumulative improvement in the effectiveness of instruction at all levels of physics.

NOTE TO CHAPTER SIX

[1] See A. Crosby, *The Measure of Reality: Quantification and Western society, 1250-1600* (Cambridge University Press, Cambridge, UK, 1997) for an interesting discussion of the medieval conception of space.

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APPENDIX A:

EVENT DIAGRAMS

In this dissertation (specifically, in Chapters Four and Five), we introduced a new representation called an *event diagram.* Event diagrams are comparable to Minkowski (spacetime) diagrams in that they are a graphical representation of spacetime, providing a visual means for organizing events, objects, locations, and so on. An event diagram consists of a series of pictures, each of which represents events and objects at a single instant. Each picture in an event diagram is therefore a "snapshot" of the objects, with their relative sizes and distances from one another indicated.[1] Events that occur at the instant of each picture are indicated in the appropriate picture at the location of the event. Successive instants appear one below the other. Event diagrams for different reference frames are drawn separately.

An example of a pair of event diagrams for a scenario described in Chapter Four appears in Figure A-1. Two volcanoes, Mt. Rainier and Mt. Hood, erupt simultaneously in the ground frame (shown in Figure A-1(a)). Event 1 is "Mt. Rainier erupts," and event 2 is "Mt. Hood erupts." The two events are indicated at the locations at which they occur in the ground frame. The event diagram for the ground frame contains only a single picture, since there is only a single instant of interest in this case. In the spacecraft frame, shown in Figure A-1(b), event 2 occurs before event 1.[2] Because the spacecraft is at rest in this frame, it appears at the same location at each instant; the mountains move to the left.
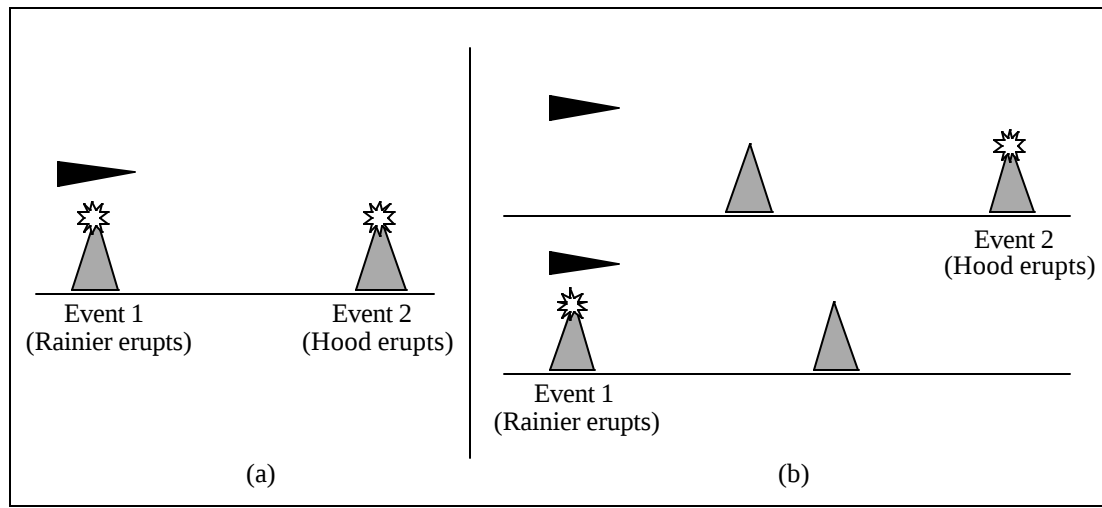
Figure A-1:   Event diagrams for (a) the ground frame and (b) the spacecraft frame
for the scenario described above and in Chapter Two.

Event diagrams have much in common with Minkowski (spacetime) diagrams.   The
spacetime diagrams corresponding to the event diagrams shown in Figure A-1 appear in Figure
A-2. Events appear as points in either type of diagram. A vertical line through either diagram
indicates a fixed position over a range of times; a horizontal line indicates a single time at a range
of positions. A line tracing an object's location at successive times in an event diagram (*e.g.,* a
line connecting Mt. Hood at successive times in Figure A-1(b)), is comparable to the worldline
for that object in a spacetime diagram.
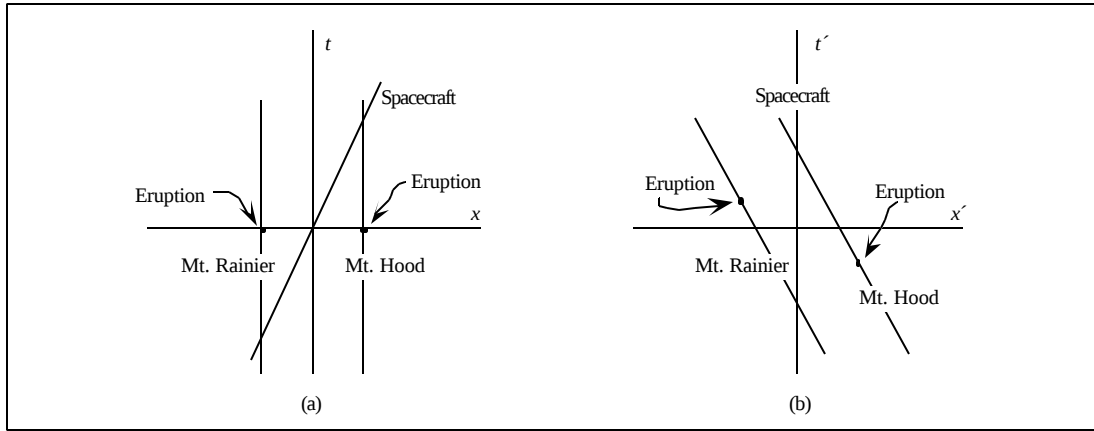
Figure A-2:   Spacetime diagrams for (a) the ground frame and (b) the spacecraft
frame for the scenario described above and in Chapter Two.

The principal difference between event and spacetime diagrams is that spacetime diagrams present a simplified, abstract, global representation of spacetime; event diagrams represent events in a more concrete physical context. Objects such as mountains appear as pictures of mountains instead of as idealized points in a coordinate grid. For this reason event diagrams are more cumbersome than spacetime diagrams. To the same extent, however, event diagrams situate events more directly in the physical context in which they occur. Event 1, for example, occurs at the spaceship (and at Mt. Rainier) at the instant that the spaceship is over Mt. Rainer, instead of at the coordinate location on a grid where the spaceship worldline crosses the Mt. Rainier worldline.

Other differences between event diagrams and spacetime diagrams include the representation of time as discrete in event diagrams; individual instants are pictured separately instead of on a continuum. Additionally, in event diagrams, time progresses vertically downward instead of upward as is conventional for spacetime diagrams.

Event diagrams are useful in that they elicit student difficulties with events more effectively than spacetime diagrams do; for examples, see Chapter Four. In addition, they provide a tool that students can use as they work to relate events to physical objects, particular observers, measurement procedures, and so on. Chapter Five discusses ways in which we have used event diagrams in addressing student difficulties with spatial measurements.

Related graphical representations have been promoted by Boas[3] and by Evett[4] as useful aids to teaching relativity. These authors' representations are similar to event diagrams in that they picture instants separately instead of continuously. They each represent objects as idealized rods along coordinate axes, and Evett's diagrams do not show event locations explicitly. We believe there are advantages to using a representation that closely approximates a physical situation. Both authors suggest picturing clocks at a range of locations, some at rest and some in motion, and showing the clocks visually as being synchronized or not as appropriate. This addition seems appropriate for students advanced enough to have deduced the relativity of clock synchronization.

NOTES TO APPENDIX A

[1] The term "snapshot" is used in its sense of "representation at a single instant;" an actual photograph of the objects would differ from the representation shown in the diagram because of angular effects, light travel time from object to camera, and so on.

[2] See Chapters Two and Three for detailed discussions of the relativity of simultaneity.

[3] M. Boas, "Events as the key to a graphic understanding of special relativity," Am. J. Phys. **47**, 938 (1979).

[4] A. Evett, "An aid for clarifying space-time concepts in special relativity," Am. J. Phys. **39**, 44 (1971).

APPENDIX B:

RESEARCH TASKS

This appendix contains versions of the research tasks discussed in Chapters Two and Four and some of the post-test questions that appear in Chapters Three and Five. (In the cases that post-test questions are identical to pretest questions described in Chapters Three and Five, the questions appear in Appendix C along with the tutorials and tutorial homework.) For many of the tasks, several versions have been administered that differ in wording, context, *etc.;* in the cases that the different versions appear to be equivalent as indicated by student performance, only a representative version is included here.

The tasks presented here are exact reproductions of tasks administered in classes and in interviews. Their formatting is preserved from the original. The tasks may differ from those discussed in the body of the dissertation in the following respects: they may differ in context or wording in ways that do not appear to affect student performance, and they may appear in combination instead of singly as they do in the body of the dissertation. For example, the location-specific Spacecraft question, the Seismologists question, and the relativistic version of the Ratios question all appear within a single problem given on an examination to an advanced undergraduate physics class.

Because questions often appear in combination within a single problem, the problems are numbered with roman numerals to distinguish them from the individual questions described in the body of the dissertation. The questions contained within each problem are identified in the list below. Some tasks appear more than once. Other tasks appear within the problems presented here that are not discussed in this dissertation.

Problem I: *Written question, advanced undergraduate students, Autumn 1999*
Location-specific version of the Spacecraft question
Seismologists question
Comparison post-test of *Length contraction* tutorial sequence
Relativistic version of the Ratios question

Problem II: *Interview protocol, graduate students, Spring 1997*
Measurement question
Undirected version of the Spacecraft question

Problem III: *Written question, introductory students, Autumn 1998*
Directed version of the Spacecraft question
Explicit version of the Eruptions question

Problem IV: *Interview protocol, graduate students, Autumn 2000*
Explicit version of the Spacecraft question
Seismologists question

Problem V: *Written question, advanced undergraduate students, Spring1998*
Implicit version of the Eruptions question

Problem VI: *Written question, advanced undergraduate students, Autumn 1998*
Numerator version of the Ratios question

Problem VII: *Written question, introductory students, Autumn 1998*
Relativistic version of the Ratios question

Problem VIII: *Written question, advanced undergraduate students, Autumn 2000*
Nonrelativistic version of the Eruptions question
Nonrelativistic version of the Ratios question

Appendix B: Research tasks   *Problem I*

---

1. Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the frame of a seismologist at rest in a laboratory midway between the volcanoes.
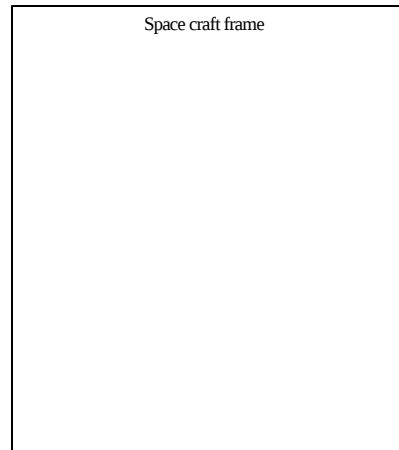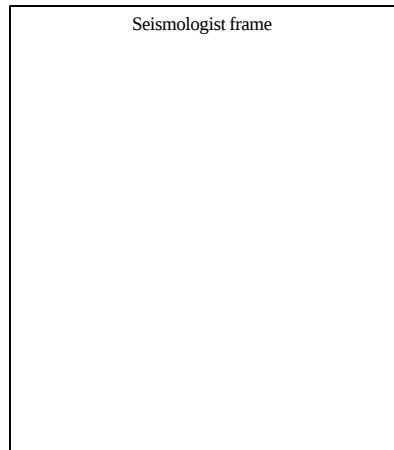
    The seismologist's assistant is at rest in a lab near Rainier when it erupts. A fast spacecraft, flying directly from Rainier toward Hood at $v=0.8c$ relative to the ground, is above Rainier when it erupts.

    A. Does Rainier erupt *before, after,* or *at the same time as* Hood:

        i.   in the frame of the seismologist's assistant? Explain.

        ii. in the frame of the space craft? Explain.

    B. Sketch event diagrams showing Mt. Rainier, Mt. Hood, and the spacecraft at the instant(s) of the eruptions. Sketch one diagram for the seismologist's frame and one for the space craft's frame.
        Sketch a different picture for each different instant. Indicate the *location* of each eruption on the appropriate picture, and indicate the *motion* of each object in each frame.

Seismologist frame	Space craft frame

    C. In the spacecraft frame, is the distance between the eruptions *greater than, less than,* or *equal to* the distance from Rainier to Hood? Explain.

    D. Let event 1 be "the space craft passes Mt. Rainier" and event 2 be "the space craft passes Mt. Hood." Determine the magnitude of the ratio $\delta x_{12}^{\text{(space craft)}}/\delta x_{12}^{\text{(seismologist)}}$. ($\delta x_{12}^{\text{(space cmft)}}$ is the spatial separation between events 1 and 2 in the space craft's frame.) Explain.

Physics 311   Autumn 1999

Interview Protocol

(Need 6 copies of blank Spacetime diagrams per interview   )

- Name, physics and math courses taken at UW or elsewhere.

Part I

Space landing strip/spaceship moving with respect to each other.
- Ask for operational definition of speed of spaceship wrt person on landing strip
- Ask for operational definition of speed of landing strip wrt person on ship
- How, if at all, would operational definition of speed change if speeds involved are relativistic?

- If op. defs are in terms of displacement/distance then ask for operational definition of displacement/distance
- Operational definition of length of spaceship as measured by person on board spacecraft.
- Operational definition of length of spaceship as measured by person on landing strip.
- If length involves simultaneous measurements, ask how one could make such measurements.

- If $v_{\text{craft, strip}} = c/2$, what is $v_{\text{strip,craft}}$? Explain.

Part II

The space landing strip and the star Hoth are at rest with respect to each other. Alan, at rest on the landing strip has measured the distance to Hoth to be 12 light years. Captain Outbound, on spaceship moving with constant speed $v = 0.6$ c ($\boldsymbol{g}= 1.25$; Don't give to interviewee yet), past the landing strip toward Hoth, passes Alan, on the strip, at the instant that both of their clocks read 0.

The (spacetime) diagram at right is one that **Alan** uses to locate when and where something happens. Note that a scale for the vertical and horizontal axes has been provided (1 tick = 2 years or 2 light years). On this (spacetime) diagram:

- Choose and label the location of Alan when Captain Outbound passes him.
- Label event 1, "Outbound arrives at Hoth."
- Mark the line segment that represents the time lapse on Alan's watch while Outbound moves from Earth to Hoth.
- Label event 2, "the reading on Alan's watch is equal to 'whatever the interviewee said' ."

The (spacetime) diagram below is the one **Outbound** uses to locate when and where something happens. Note that this diagram uses the *same scale* for the vertical and horizontal axes as the diagram for part A. On this (spacetime) diagram:

- Mark and label the location of Outbound when Alan passes him.
- Label event 1. Explain your reasoning and show your work.
- Label event 2. Explain your reasoning and show your work.
- On **Outbound's** (spacetime) diagram, mark the line segment that represents the time lapse on Outbound's watch while Alan and Outbound move apart. Explain how you knew to mark this line segment as you did.

1. [25points total]

   Mt. Rainier and Mt. Hood, which are 300 km apart in their rest frame (the Earth), suddenly erupt at the same time as determined by observers on Earth.

   A. [8 pts] What is the time interval between the two eruptions as determined by observers in a fastspace craft (V=0.8c) flying directly from Rainier toward Hood? Show your work.

   B. [4 pts] Which eruption occurs first according to the observers in the space craft? Explain.

   C. [5 pts] Answer part B if the space craft were flying from Hood toward Rainier. Explain.

   D. [8 pts] How far apart in space are the eruptions, according to observers in the space craft of part A? Show your work.

   Physics 121BH   Autumn 1998

Appendix B: Research tasks   *Problem IV*

Interview protocol   Autumn 2000

In this problem, all events and motions occur along a single line in space. Non-inertial effects on the surface of the Earth may be neglected.

All observers are *intelligent* observers, *i.e.,* they correct for signal travel time to determine the time of events in their reference frame. Each observer has synchronized clocks with all other observers in his or her reference frame.

Two volcanoes, Mt. Rainier and Mt. Hood, are 300 km apart in their rest frame. Each erupts suddenly in a burst of light. Alan, a seismologist at rest in a laboratory midway between the volcanoes, receives the light signals from the volcanoes at the same time. Bob, the seismologist assistant, is at rest in a lab at the base of Mt. Rainier.

Define event 1 to be "Mt. Rainier erupts," and event 2 to be "Mt. Hood erupts."

I. Caroline is the pilot of a fast space craft flies past Mt. Rainier toward Mt. Hood with constant velocity v=0.8c relative to the ground (g = 5/3). At the instant Mt. Rainier erupts, the space craft is directly above it and so Caroline receives the light from Mt. Rainier instantaneously.

    i. For each intelligent observer below, does event 1 occur *before, after,* or *at the same time as* event 2? Explain.

- Alan

- Bob

- Caroline

    ii. For each intelligent observer below, determine the time that elapses between events 1 and 2. Explain.

- Alan

- Bob

- Caroline

II. David is in a space craft that flies from Mt. Rainier to Mt. Hood with the same velocity as Caroline's ship (v=0.8c relative to the ground). David's ship is directly above Alan at the instant Alan receives the light signals from the volcanoes.

For David, determine the time that elapses between events 1 and 2.

III. Mt. Baker, which is 360 km from Mt. Rainier in their rest frame, erupts 400 microseconds after Mt. Rainier as recorded by a geologist at rest in a lab midway between Mt. Rainier and Mt. Baker.

Is there an observer for whom Mt. Rainier and Mt. Baker erupt simultaneously? If so, determine the velocity of this observer relative to the ground (magnitude and direction). If not, explain why not.

#5 (35) For your information, 1 nanosecond, which is abbreviated 1 ns, is equal to 1×10^{-9} s. A light-nanosecond, the distance that light travels in one nanosecond, $(3 \times 10^8 \ m/s)(1 \times 10^{-9} \ s) = 0.3 \ m$, will be abbreviated 1 c-ns.

Consider the following two events in a frame on the ground : At $x = 0$ and $t = 0$, a firecracker goes off at one end of an alley. Forty nanoseconds later another firecracker goes off at the other end of the alley, a distance 100 c-ns down the positive x axis. An observer in the air travels down the positive x axis at a speed 0.8c relative to the ground.

(a)(19) According to *this* observer, what is the spatial separation, in light–nanoseconds, between these two events; *i.e.* what is the distance between the two places at which the two events occurred?

(b)(8) Is there a frame in which the two events occur at the same place? If so, what is the speed of this frame relative to the ground?

(c)(8) Is there a frame in which the two events occur at the same time? If so, what is the speed of this frame with respect to the ground?
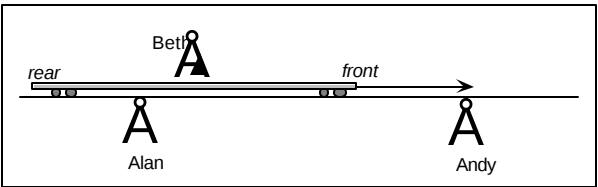
Physics 311　　　　　　　　　　　　　　**Pretest**　　　　　　　　Name _____

A 12-meter long train moves with constant nonrelativistic speed relative to a long, straight stretch of track.

Alan stands at rest relative to the track. His assistant, Andy, is also at rest relative to the track and stands 12 meters from Alan (see figure). Beth stands at rest on the train.

Define events 1, 2, and 3 as follows:　　*Event 1:* The front of the train is next to Alan
　　　　　　　　　　　　　　　　　　　Event 2: The front of the train is next to Andy
　　　　　　　　　　　　　　　　　　　Event 3: The rear of the train is next to Alan

The *spatial separation* between events 1 and 2 in Alan's frame is designated $\Delta x_{12}^{(A)}$ and is equal to $x_2^{(A)} - x_1^{(A)}$. Determine numerical values for the following quantities. Briefly explain your reasoning in each case.

- $\Delta x_{12}^{(B)}$

- $\Delta x_{13}^{(B)}$

- $\Delta x_{12}^{(A)}$

- $\Delta x_{13}^{(A)}$
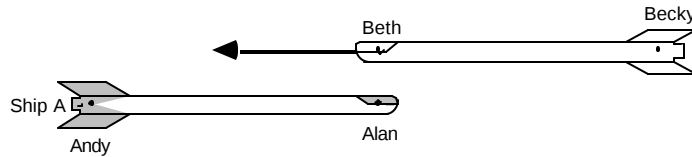
PRETEST: SPECIAL RELATIVITY I   Name _____

Physics 121B(H)

Two spaceships, A and B, pass very close to each other. Alan is at rest in the front of spaceship A and Beth is at rest in the front of spaceship B. Andy and Becky are at rest in the backs of spaceships A and B respectively.

In Alan's frame, the speed of spaceship B is 0.6 c ($g = 1.25$) and spaceships A and B each have length 120 c-ns. (One light-nanosecond, abbreviated c-ns, is the distance light travels in 10^{-9} seconds.)

1. Define events 1, 2, and 3 as follows: *Event 1:* Alan and Beth pass each other
                                                Event 2 : Andy and Beth pass each other
                                                Event 3: Alan and Becky pass each other

The diagram above represents the spaceships at the instant of event 1 in Alan's frame.

Determine numerical values for the following ratios, in which the subscripts refer to the events defined above. Use the notation $^2 x_{23}^{(B)} = x_3^{(B)} - x_2^{(B)}$ = the spatial separation between events 2 and 3 in Beth's frame.

Explain your reasoning and show your work.

- $^2 x_{12}^{(B)} / {}^2 x_{12}^{(A)}$

- $^2 t_{12}^{(B)} / {}^2 t_{12}^{(A)}$

- $^2 x_{13}^{(A)} / {}^2 x_{13}^{(B)}$

PRETEST: MEASUREMENT   Name _____

1. A straight runway is 100 m long. A small explosion occurs at the east end of the runway; 10 seconds later, a small explosion occurs at the west end of the runway. An airplane moves from west to east with speed 25 m/s relative to the runway.

How far apart in space are the locations of the explosions:

• in the frame of the run way? Explain.

• in the frame of the airplane? Explain.

2. Two spaceships, A and B, pass very close to each other. Alan is in the front of ship A and Beth is in the front of ship B. Andy and Becky are at rest at the backs of ships A and B respectively. In Alan's frame, ship B moves with speed $v = 3$ m/s and ships A and B each have length 12 m.

Define events 1, 2, and 3 as follows:

Event 1: Alan and Beth are adjacent
Event 2: Andy and Beth are adjacent
Event 3: Alan and Becky are adjacent

The diagram at right represents the ships at the instant of event 1 in Alan's frame.

Beth   Ship B   Becky

Andy   Ship A   Alan

Determine numerical values for the following ratios, in which the subscripts refer to the events defined above. Use the notation $\delta x_{12}^{(A)} = x_2^{(A)} - x_1^{(A)}$ = the (signed) distance between the locations of events 1 and 2 in Alan's frame. Explain your reasoning.

• $\delta x_{12}^{(B)} / \delta x_{12}^{(A)}$

• $\delta x_{13}^{(A)} / \delta x_{13}^{(B)}$

APPENDIX C:

PRETESTS, TUTORIALS AND TUTORIAL HOMEWORK

This appendix presents an integrated sequence of pretests, tutorials, and tutorial homework that includes the *Events and reference frames, Spatial measurement, Relativistic kinematics,* and *Length contraction* tutorial sequences described in Chapters Three and Five. The post-tests for each tutorial sequence appear in Appendix B *(Research tasks)* except in the cases that they are identical to the pretests included here.

Each time the tutorials are used with a particular group of students, they are modified slightly to suit the particular circumstances. Notation, wording, and other details may be changed to match the text or the lecturer's presentation. The sequence presented here is a representative version of the tutorials used in the classroom. The exercises are presented in the order in which students typically complete them.

The titles of the tutorials reproduced here are those used in the classroom and do not match those in the body of the dissertation. Exercises described in Chapter Three and Five as the *Events and reference frames* and *Spatial measurement* tutorials appear here in the tutorials titled *Events and reference frames* and *Measurement.* Exercises described in Chapters Three and Five under the titles *Relativistic kinematics* and *Length contraction* appear here within the *Simultaneity* and *Synchronization and causality* tutorials. The four pretests shown here are identical to those described in the body of the dissertation for the *Events and reference frames, Spatial measurement, Relativistic kinematics,* and *Length contraction* tutorials.

- *Events and reference frames* pretest

- *Events and reference frames* tutorial

- *Measurement* pretest

- *Measurement* tutorial

- *Measurement* tutorial homework

- *Simultaneity* pretest

- *Simultaneity* tutorial

- *Synchronization and causality* pretest

- *Synchronization and causality* tutorial

- *Synchronization and causality* tutorial homework
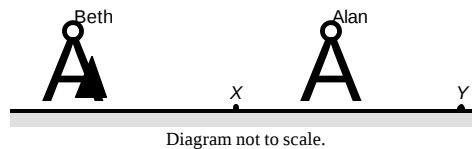
PRETEST:
EVENTS AND REFERENCE FRAMES   Name _____

Two physics students, Alan and Beth, are shown in the diagram at right. Alan and Beth have measured their exact relative distances from points *X* and *Y*.

Sparks jump at the points marked   *X* and *Y*. When each spark jumps, it emits a flash of light that expands outward in a spherically symmetric pattern. Alan, who is equidistant from points *X* and *Y,* receives the wavefront from each spark at the same instant.

Diagram not to scale.

Answer each of the following questions for the observers listed.

(i)   Does he or she receive the wavefront from the spark that jumped at point X   *before, after,* or *at exactly the same time as* the wavefront from the spark that jumped at point Y?

(ii) In his or her reference frame, does the spark that jumped at point X jump   *before, after,* or *at exactly the same time as* the spark that jumped at point Y?

Explain your reasoning in each case.

• Alan
  (i)

  (ii)

• Beth
  (i)

  (ii)

EVENTS AND REFERENCE FRAMES   Name _____

I. Reference frames

A physics student named Alan and   a beeper are arranged as shown at right. The beeper is about to emit a beep, and Alan wants to determine the exact time at which it does so. However, he is a long way from the beeper and unable to travel to it.

A. Alan is equipped with accurate meter sticks and clocks, and there are a number of other physics students available to assist him if necessary.

  1. Describe a set of measurements by which Alan can determine the time at which the beep is emitted using his knowledge of the speed of sound in air.

  2. Describe a method by which Alan can determine the time at which the beep is emitted   *without* knowing or measuring the speed of sound first. (*Hint:* Alan's assistants are free to stand at any location.)

B. A fugitive from justice is at large in Seattle. His identity and exact whereabouts are unknown. A reporter has reason to believe that the fugitive will soon confess to his crime, and wishes to record as exactly as possible the time and place of the confession. Her funding for this project is excellent.

  1. Describe an arrangement of observers and equipment with which the reporter may record the position and time of the confession.

An observer's *reference frame* is an arrangement of assistants and equipment with which the observer may record the position and time of anything that occurs.

  2. Justify the claim that the reporter's arrangement of observers and equipment is the reporter's reference frame.

C. A horn is located between Alan and the beeper. The beeper beeps once and the horn honks once. Alan hears the two sounds at the same instant in time.

  1. Describe a method by which Alan can measure the time separation between the emission of the beep and the emission of the honk   *in his reference frame* *without* knowing or measuring the speed of sound first.

  2. In Alan's reference frame, is the beep emitted   *before, after,* or *at the same instant as* the honk is emitted? Explain.

Events and reference frames

An *intelligent observer* is equipped with measuring devices (such as meter sticks, clocks, and assistants) and is able to use them to make correct and accurate determinations of where and when something occurs. All observers in the study of relativity are intelligent observers.

II. Events

A. In the study of relativity we refer to material *objects, locations* in space, *instants* in time, and *events*. An *event* is associated with a single location in space and a single instant in time.

State whether each of the items below is *an object, a location, an instant, an event,* or *none of these.*
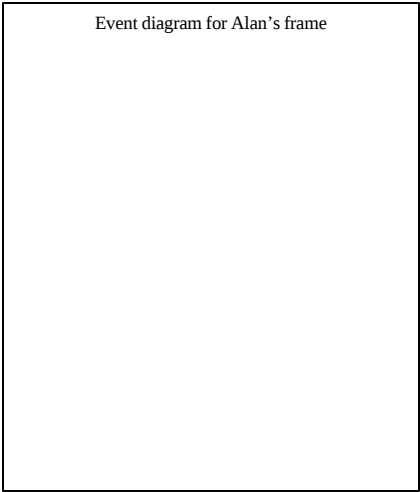
- The beeper of Part I
- The beeper emits a beep
- Alan hears the beep

- The exact time at which the beeper beeps
- A sound wave travels from the beeper to Alan
- Two beepers beep at the same time

B. Sketch a picture showing Alan, the beeper, the horn, and any other objects of interest at the instant the beeper beeps. Indicate the location of the event "the beeper beeps" on the picture.

Below that picture, sketch another picture showing the objects of interest at the instant the horn honks. Indicate the position of the event "the horn honks" on this picture.

Below that picture, sketch picture(s) showing the objects of interest at the instant(s) of the remaining events of interest. Sketch a separate picture for each different instant. Indicate the location of each event on the appropriate picture.

> Event diagram for Alan's frame

C. A diagram such as the one you drew above is called an   *event diagram.*   An event diagram has the following characteristics:

- The objects of interest are shown at the instants of the events of interest.
- Successive instants are shown one below the other.
- The location of each event of interest is indicated in the appropriate picture.

1. Does the first picture in your event diagram represent   *an object, a location, an instant, an event,* or *none of these?*

2. Is it possible for a single event to appear in more than one picture in an event diagram? Explain why or why not.

3. Describe the circumstances under which more than one event should appear in a single picture in an event diagram.

III. Synchronization of clocks

Alan and Beth are exactly 10 meters apart relative to the floor. Each of them wears a watch. Both watches are extremely accurate, run at the same rate, and measure time in *meters.* (One meter of time is the   amount of time it takes light to travel one meter.) However, the reading on Beth's watch is not the same as the reading on Alan's watch.

A. Determine the amount of time, in meters, that it will it take a light signal to travel from Alan to Beth.

Beth and Alan decide in advance that at the instant Alan's watch reads 50 meters, Alan's laser pointer will emit a pulse of light in Beth's direction.

B. What time will Alan's watch read at the instant Beth first   receives the light from the laser pointer?

C. Describe a method by which Beth could synchronize her watch with Alan's (*i.e.,* make her watch have the same reading as Alan's at every instant).

D. Another physics student, Caroline, is at rest with respect to Alan and Beth but is very far away from them. Caroline looks at the reading on Alan's watch with a powerful telescope, and finds that at every instant, the reading she sees on Alan's watch through the telescope is identical to the reading on her watch.

Is Caroline's watch synchronized with Alan's? Explain why or why not.

PRETEST: MEASUREMENT   Name _____

1. A straight runway is 100 m long. A small explosion occurs at the east end of the runway; 10 seconds
    later, a small explosion occurs at the west end of the runway. An airplane moves from west to east with
    speed 25 m/s relative to the runway.

   How far apart in space are the locations of the explosions:

    • in the frame of the run way? Explain.

    • in the frame of the airplane? Explain.

2. Two spaceships, A and B, pass very close to each other. Alan is in the front of ship A and Beth is in the
    front of ship B. Andy and Becky are at rest at the backs of ships A and B respectively. In Alan's frame,
    ship B moves with speed $v = 3$ m/s and ships A and B each have length 12 m.

    Define events 1, 2, and 3 as follows:

    Event 1: Alan and Beth are adjacent
    Event 2: Andy and Beth are adjacent
    Event 3: Alan and Becky are adjacent

    The diagram at right represents the ships at
    the instant of event 1 in Alan's frame.

    Determine numerical values for the following ratios, in which the subscripts refer to the events defined
    above. Use the notation $\delta x_{12}^{(A)} = x_2^{(A)} - x_1^{(A)} =$ the (signed) distance between the locations of events 1 and 2
    in Alan's frame. Explain your reasoning.

    • $\delta x_{12}^{(B)} / \delta x_{12}^{(A)}$

   • $\delta x_{13}^{(A)} / \delta x_{13}^{(B)}$

MEASUREMENT

Name _____

I. Spatial measurements

A. A train moves with constant nonrelativistic speed along a straight track. The train is 12 meters long.

Alan and Andy stand 12 meters apart at rest on the track (see figure). Beth and Becky stand at rest at the front and rear of the train, respectively.

Define events 1, 2, and 3 as follows:

Event 1: Alan and Beth pass each other.
Event 2: Andy and Beth pass each other.
Event 3: Alan and Becky pass each other.

1. On a large sheet of paper, sketch   an event diagram showing Alan, Andy, Beth, and Becky at the instants of events 1, 2, and 3 in Alan's frame. (That is, sketch a separate picture for each different instant; sketch pictures for successive instants one below the other; and indicate the location of each event on the appropriate picture   .)

    a. What feature(s) of your event diagram can be used to indicate that it is a diagram for Alan's frame?

    b. How would an event diagram for Andy's reference frame compare to the one you drew above? Explain.

    c. What procedure could Alan (or Andy) follow to measure the distance between the locations of events 1 and 2   ?

    d. How far apart in space are the locations of the following pairs of events in Alan's frame?

        •   Events 1 and 2
        •   Events 2 and 3
        •   Events 1 and 3

> Copy event diagram for Alan's frame here after discussion.

> Copy event diagram for Beth's frame here after discussion.

2. Sketch an event diagram showing events 1, 2, and 3 in Beth's frame. Be sure your diagram correctly represents the motion of the train in this frame.

    How far apart in space are the locations of the following pairs of events in Beth's reference frame?

        •   Events 1 and 2
        •   Events 2 and 3
        •   Events 1 and 3

Measurement

---

3. How does Beth's procedure for measuring the distance between the positions of two   events compare to Alan's procedure?

4. On the basis of your answers above, develop a general rule that uses an event diagram to determine how far apart the locations of two events are in a given reference frame.

In tutorial, we will use the symbol $\delta x_{12}^{(Alan)}$ to indicate the spatial separation between events 1 and 2 as measured in Alan's reference frame. $\delta x_{12}^{(Alan)} = x_2^{(Alan)} - x_1^{(Alan)}$, where $x_1^{(Alan)}$ and $x_2^{(Alan)}$ are the positions of events 1 and 2 in Alan's reference frame. Note that the spatial separation between events is a signed quantity (it may be positive or negative).

B. Give *interpretations* for the magnitude of each of the following quantities; that is, tell the meaning of the number in this physical context. One has been provided as an example. Some quantities may have more than one interpretation.

• $\delta x_{12}^{(Alan)}$ The displacement of Beth (or the displacement of the train) as measured by Alan (or Andy)	• $\delta x_{12}^{(Beth)}$
• $\delta x_{13}^{(Alan)}$	• $\delta x_{13}^{(Beth)}$
• $\delta x_{23}^{(Alan)}$	• $\delta x_{23}^{(Beth)}$

Measurement

C. A train of unknown length moves with constant nonrelativistic speed on the same track. Alan and his assistants stand shoulder-to-shoulder on the track.

1. Describe a method by which Alan can determine the length of the train in his frame if he knows the speed of the train in his frame. Specify two events associated with this measurement procedure.

Event a:

Event b:

2. Describe a method by which Alan can determine the length of the train in his frame without knowing or measuring its speed first. Specify two events associated with this measurement procedure.

Event c:

Event d:

D. Suppose event 4 occurs at the front of a long ship, and event 5 occurs at the rear of the same ship. Describe the circumstances in which the absolute value of δx_{45} is equal to the length of the ship:

Event diagram for ship frame

• in the frame in which the ship is at rest

• in frame *F,* in which the ship is moving

Event diagram for frame F

In the spaces provided at right, draw event diagrams to support your answers.

Homework: Measurement

Name _____

I. Ashpiles

Firecrackers explode harmlessly at either end of a flatcar that moves along a long, straight stretch of train track. The firecracker at the front of the flatcar explodes several seconds before the one at the rear of the flatcar. Ash from each firecracker sticks to the flatcar *and* falls onto the ground at the instant each firecracker explodes.

A. Sketch an event diagram for the frame of the track. Include all four piles of ash in your pictures.

Track frame

    1. Identify the object(s) whose position, in this reference frame, is *always* the same as the position of:

       a. the first explosion

       b. the second explosion

    2. In this reference frame, the magnitude of the spatial separation between the explosions is equal to the distance between two particular objects. Identify these objects.

B. Sketch an event diagram for the frame of the flatcar. Include all four piles of ash in your pictures.

Flatcar frame

   In this reference frame, the magnitude of the spatial separation between the explosions is equal to the distance between two particular objects. Identify these objects.

C. Under what circumstances does the position of an object in a certain reference frame indicate the position of an event in that reference frame?

Under what circumstances is the distance between two objects equal to the magnitude of the spatial separation between two events?

Homework: Measurement

II. Measuring the length of a spaceship

A physics student describes an incorrect method for measuring the length of a spaceship that is moving directly away from him with a speed v. (One light-second is the distance light travels in one second.)

> "Suppose there is a half-silvered mirror on each end of the spaceship. If I send a short pulse of light toward the spaceship, some of the light will be reflected back to me from the back of the spaceship, and the rest will travel to the front of the spaceship and be partially reflected back to me from that end. So I will receive two light pulses, separated in time. The length of the spaceship in light-seconds is one-half of the time between these pulses times the speed of light."

Explain why the method described is incorrect.

III. City bus

A city bus moves from left to right relative to the road.
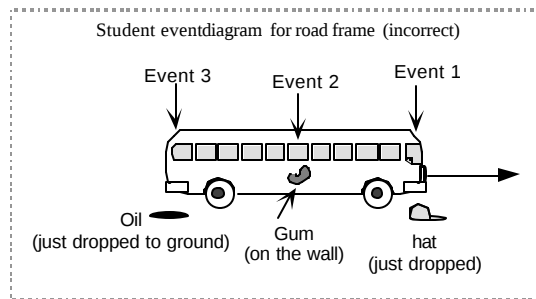
Events 1, 2, and 3 are as follows: *Event 1:*   The bus driver drops his hat onto the road
                                       Event 2:   A passenger on the bus sticks his gum to the wall
                                         Event 3:   A drop of oil drips from the rear of the bus onto the road

Events 1, 2, and 3 occur one after the other in numerical order. After events 1, 2, and 3 have occurred, the hat is to the right of the drop of oil on the road.

A. A student sketches the event diagram shown below for the road frame. The diagram is flawed.

    Identify the error(s) in the event diagram.

    Draw a correct event diagram for the road frame.

Event diagram for road frame

Student eventdiagram for road frame (incorrect)

Event 3   Event 2   Event 1

Oil
(just dropped to ground)

Gum
(on the wall)

hat
(just dropped)

216
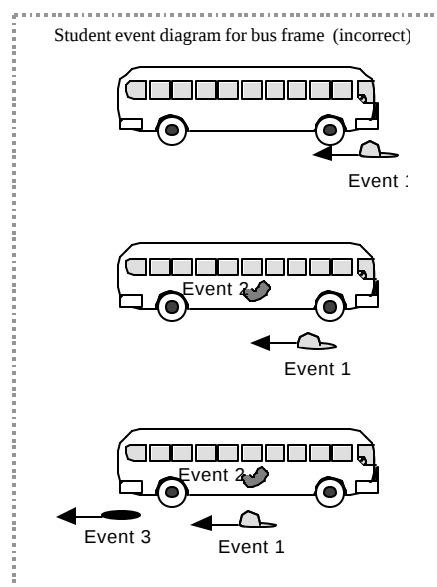
Homework: Measurement

B. Another student sketches the event diagram shown below for the bus frame. The diagram is flawed.

Identify the error(s) in the event diagram. Correct the diagram appropriately.

Student event diagram for bus frame (incorrect)

Event 1

Event 2

Event 1

Event 2

Event 3   Event 1

C. Criticize the following statements.

1. "The hat is dropped from the front of the bus and the oil drips from the back. So the spatial separation between event 1 (the hat drops) and event 3 (the oil drips) in the road frame is approximately equal to the length of the bus."

2. "The distance between the location of event 1 and the location of event 3 in the bus frame is definitely less than the length of the bus, because the hat moves toward the back of the bus in the time between the events."

PRETEST: SIMULTANEITY   Name _____

Two spaceships, A and B, pass very close to each other with relativistic relative speed. Alan is at rest in the front of spaceship A and Beth is at rest in the front of spaceship B. Andy and Becky are at rest in the backs of spaceships A and B respectively.

The diagram below shows the two spaceships in Alan's frame. At the instant shown, two sparks jump between the spaceships and make char marks on both ships. One spark marks an ´, and the other marks an **0**. When each spark jumps, it emits a flash of light that expands outward in a spherically symmetric pattern. The sparks jump at the same instant in the reference frame of ship A.

Answer each of the following questions for the observers listed.

(i) Does he or she receive the wavefront from the spark that marks the ´ *before, after,* or *at exactly the same time as* the wavefront from the spark that marks the **0**?

(ii) In his or her frame, does the spark that marks the ´ jump *before, after,* or *at exactly the same time as* the spark that marks the **0**?

Briefly explain your reasoning for each case.

• Alan

 (i)

 (ii)

• Andy

 (i)

 (ii)

• Beth

 (i)

 (ii)

• Becky

 (i)

 (ii)

SIMULTANEITY

Name _____

I. Spherical Wavefronts

Two physics students, Alan and Beth, move past each other. At the instant that they pass, a spark jumps between them. The spark emits a flash of light that travels outward in a spherically symmetric pattern.

The first diagram at right represents the wavefront of the flash of light a short time after the spark jumps in Alan's frame.
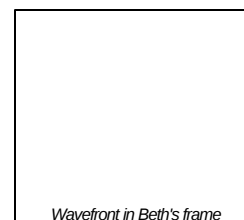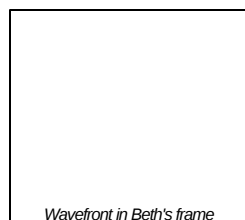
A. Explain how this picture is consistent with the fact that Alan observes the speed of light to be the same in all directions.

Wavefront in Alan's frame   *Wavefront in Alan's frame*

1. In the second diagram above, sketch the wavefront at a later time in Alan's frame. Include Alan's and Beth's positions in your sketch.

2. Is there a time at which the distance from Alan to Beth is greater than the distance from Alan to the wavefront? Explain.

B. In the spaces at right, sketch the wavefront a short time after the spark jumps in Beth's frame and at a later time in Beth's frame. Include   Beth's and Alan's positions in your sketches.

Is your sketch consistent with the fact that Beth observes the speed of light to be the same in all directions? If not, modify your diagram so that it is consistent with this observation.

Wavefront in Beth's frame   *Wavefront in Beth's frame*
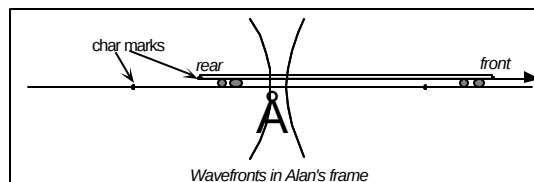
Simultaneity

II. Relativity of Simultaneity

A spark jumps between the front end of a train and the track (spark F), and another spark jumps between the rear end of the train and the track (spark R). When each spark jumps, it emits a flash of light (wavefronts F and R). Each spark also leaves a char mark on both the train and the track.

A. Alan is equidistant from the char marks on the track. Wavefronts F and R hit him at the same instant. The diagram at right represents this instant in Alan's frame.

Wavefronts in Alan's frame
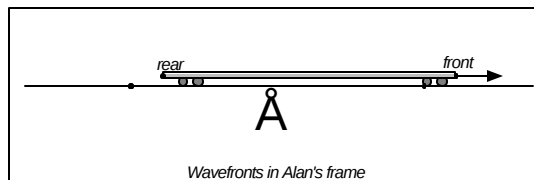
1. In Alan's frame, does spark F jump *before, after,* or *at the same instant as* spark R? Explain your reasoning.

2. Alan's assistant Andy stands at rest relative to the track, near the front char mark on the track.

    In Andy's frame, does spark F jump   *before, after,* or *at the same instant as* spark R? Explain your reasoning.

3. The diagram at right represents an instant a short time after the spark jumps between the front of the train and the track in Alan's frame (before he receives either wavefront). Complete the diagram by drawing the wavefronts at this instant. Explain your reasoning.

Wavefronts in Alan's frame

Beth stands at rest relative to the train, exactly halfway between the front and rear of the train. Let event 1 be "spark F jumps," event 2 be "Alan and Beth pass each other," and event 3 be "spark R jumps."

4. In Alan's frame, does event 1 happen   *before, after,* or *at the same instant as* event 2? Explain your reasoning.

5. In Alan's frame, does wavefront F hit Beth   *before, after,* or *at the same instant as* wavefront R? Explain your reasoning.

Simultaneity

B. A cassette tape player sits at Beth's feet.   In Alan's frame, when wavefront F hits the tape player, it plays the opening chords of Beethoven's Fifth Symphony at top volume; when wavefront R hits the tape player, it is silenced. If both wavefronts hit the tape player at the same instant in Alan's frame, it remains silent.

Does the tape player play the opening chords of the symphony:

- in Alan's frame?

- in Beth's frame?

Check that your responses are consistent with your answers to the following questions.

1. Later in the day, Beth ejects the cassette from the tape player. She descends from the train, and she and Alan examine the cassette together. Will the cassette have wound at all from its starting position?

2. Will Beth hear the opening chords of the symphony?

C. In Beth's frame, does wavefront F hit Beth   *before, after,* or *at the same instant as*   wavefront R? Explain.

1. In Beth's frame, is the speed of wavefront F   *greater than, less than,* or *equal to* the speed of wavefront R? Explain.

In Beth's frame, event 1 occurs at $t = t_1^{(B)}$ and event 3 occurs at $t = t_3^{(B)}$.

2. Is $t_3^{(B)}$ *greater than, less than, or equal to* $t_1^{(B)}$? Explain.

3. The diagram at right represents an instant a short time after both sparks have jumped in Beth's frame (but before the wavefront from either spark hits her). Add to the diagram by drawing the wavefronts at this instant. Explain.

Wavefronts in Beth's frame
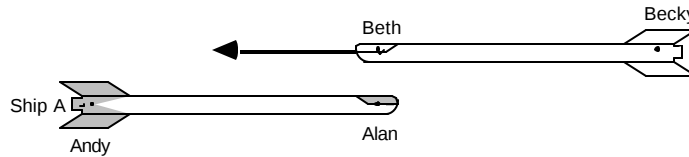
4. Beth's buddy Becky stands at the rear of the train, at rest relative to the train. In Becky's frame, event 1 occurs at $t = t_1^{(Becky)}$.

Is $t_1^{(Becky)}$ *greater than, less than, or equal to* $t_1^{(B)}$? Explain your reasoning.

PRETEST: SYNCHRONIZATION AND CAUSALITY   Name _____

Two spaceships, A and B, pass very close to each other. Alan is at rest in the front of spaceship A and Beth is at rest in the front of spaceship B. Andy and Becky are at rest in the backs of spaceships A and B respectively.

In Alan's frame, the speed of spaceship B is 0.6 c ($g = 1.25$) and spaceships A and B each have length 120 c-ns. (One light-nanosecond, abbreviated c-ns, is the distance light travels in 10^{-9} seconds.)

1. Define events 1, 2, and 3 as follows: *Event 1:* Alan and Beth pass each other
                                              Event 2: Andy and Beth pass each other
                                              Event 3: Alan and Becky pass each other

The diagram above represents the spaceships at the instant of event 1 in Alan's frame.

Determine numerical values for the following ratios, in which the subscripts refer to the events defined above. Use the notation $\delta x_{23}^{(B)} = x_3^{(B)} - x_2^{(B)}$ = the spatial separation between events 2 and 3 in Beth's frame.

Explain your reasoning and show your work.

- $\delta x_{12}^{(B)} / \delta x_{12}^{(A)}$

- $\delta t_{12}^{(B)} / \delta t_{12}^{(A)}$

- $\delta x_{13}^{(A)} / \delta x_{13}^{(B)}$

SYNCHRONIZATION AND CAUSALITY   Name _____

I. Length contraction

Two spaceships, A and B, pass very close to each other. Alan is at rest in the front of spaceship A and Beth is at rest in the front of spaceship B. Andy and Becky are at rest in the backs of ships A and B respectively.

In Alan's frame, the speed of ship B is 0.6 (g = 5/4) and ships A and B each have length 120 ns. (One nanosecond of distance is the distance light travels in one nanosecond.)
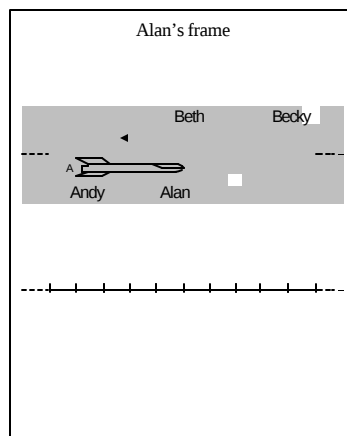
Define events 1, 2, and 3 as follows: *Event 1:* Alan and Beth pass each other
Event 2 : Andy and Beth pass each other
Event 3: Alan and Becky pass each other
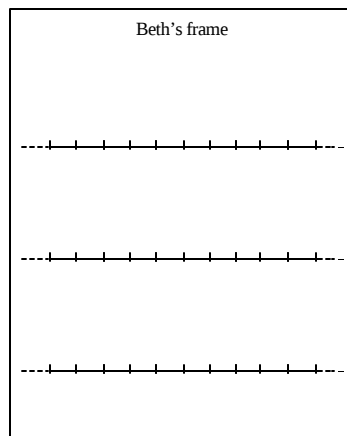
A. The instant of event 1 is shown at right in Alan's frame.

    1. Does event 2 occur   *before, after,* or *at the same time as* event 3 in Alan's frame? In Andy's frame? Explain.

    2. Sketch an event diagram showing events 1, 2, and 3 in Alan's frame in the space provided.

    3. Determine a numerical value for $\delta x_{23}{}^{(A)}$, the spatial separation between events 2 and 3 in Alan's frame.

B. Give an explanation of the phrase "the proper length of spaceship B."

    1. Determine the proper length of spaceship B in c-ns.

    2. In Beth's frame, is the length of spaceship A   *greater than, less than,* or *equal to* 120 *c*-ns? Explain.

    3. Sketch an event diagram showing events 1, 2, and 3 in Beth's frame. Make your diagram consistent with your answers to the above questions.

Alan's frame

Beth's frame

Synchronization and causality

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4. Determine a numerical value for the ratio $\delta x_{23}^{(B)} / \delta x_{23}^{(A)}$. Explain your reasoning.

4 *Check your answer with a staff member before proceeding.*

5. Consider the following student dialogue .

   Student 1: "The length of spaceship A in Beth's frame is less than the length of spaceship A in Alan's frame, since in Beth's frame, the length of spaceship A is contracted from its proper length."

   Student 2: "Exactly. And since event 2 occurs at the back of ship A and event 3 at the front, $\delta x_{23}^{(B)}$ is the length of ship A in Beth's frame, and $\delta x_{23}^{(B)} / \delta x_{23}^{(A)}$ is less than one."

   Student 2's conclusion is *incorrect*. Identify the error in the student's reasoning.

II. Relative Synchronization of Clocks

A. Alan has synchronized clocks at the front and back of spaceship A, and Beth has synchronized clocks at the front and back of spaceship B. When Andy and Beth pass each other, both of their clocks read 12 noon.

   1. Determine the reading on the clock at the front of ship B at the instant of event 2.

   2. When event 3 occurs, is reading on the clock at the back of ship B *before* 12 noon, *after* 12 noon, or *exactly* 12 noon? Explain your reasoning.

   Recall that, in Alan's frame, events 2 and 3 occur at the *same instant* .

   3. In Alan's frame, do the clocks on Beth's spaceship all have the *same reading* at a given instant? If not, which clock on ship B is "ahead" of the other? Explain.

III. Causality

A. Later, Alan (in the front of ship A) and his assistant Andy (in the back of ship A) are asleep. Alan's bedside alarm clock goes off and instantly wakes Alan up. Andy wakes up at exactly the same instant.

   Is it possible that Andy woke up because he heard Alan's alarm? Explain why or why not.

B. An empty spaceship of proper length L_o, moving with relativistic speed V_o relative to a large asteroid, collides head-on with the asteroid. At an instant shortly after the collision $(t = t_o)$, the front of the spaceship is brought to rest relative to the surface of the asteroid.

   1. Is it possible that the collision brings the back of the spaceship to rest relative to the asteroid at $t = t_o$? Explain why or why not.

   A space probe behind the spaceship moves with constant speed V_o toward the asteroid at all times.

   2. Describe the motion of the back of the spaceship during a time interval equal to L_o/c on the space probe clock after the collision. Explain.

   3. Identify the relevant events in the scenario described. Sketch event diagrams for the asteroid frame and the space probe frame.

Asteroid frame	Space probe frame

HW: Synchronization and causality   Name _____

I.   Train in a tunnel paradox

(In the following problem, consider one dimension only.) A train moves with constant velocity down a straight track that passes through a tunnel. When the train is at rest with respect to the tunnel, the train is exactly the same length as the tunnel. However, the train in this case is moving relative to the tunnel at nearly the speed of light.

The engineer on the train says: "The tunnel is Lorentz-contracted and is shorter than the train; therefore at no time can the train be wholly within the tunnel."

The keeper of the tunnel says: "The train is Lorentz-contracted and is shorter than the tunnel; therefore, there will be a time at which the train is wholly within the tunnel."

They are both infuriated by their failure to reach an agreement.

A.   The engineer decides to settle the issue by placing rockets on the front and rear of the train, equipped with timing devices such that the rockets will be launched simultaneously, in a vertical direction, when the midpoint of the train passes the midpoint of the tunnel. (The engineer synchronizes these timing devices while the train is approaching the tunnel.)

   1.   Sketch event diagrams for the train frame and the tunnel frame, showing the train and the tunnel at the instant(s) associated with the following events. Show qualitatively correct relative lengths of objects and relative times of events.

   - The midpoint of the train is at the midpoint of the tunnel
   - The front rocket fires
   - The rear rocket fires

Train frame	Tunnel frame

   2.   Do the rockets fire inside or outside the tunnel   in the train frame?   in the tunnel frame?

Homework: Synchronization and causality

B. The tunnel keeper then tries to settle the issue (permanently) by erecting giant indestructible iron gates at the front and rear of the tunnel, equipped with timing devices such that the gates will close simultaneously when the midpoint of the train passes the midpoint of the tunnel. (The tunnel keeper knows that this will destroy the train and perhaps the tunnel when the front end of the train plows into a gate, but is so infuriated as to not care.)

1. Sketch event diagrams for the train frame and the tunnel frame, showing the train and the tunnel at the instant(s) associated with the following events.

   - The midpoint of the train is at the midpoint of the tunnel
   - The front gate closes
   - The rear gate closes

Train frame	Tunnel frame

C. Consider the instant at which the front of the train is brought to rest relative to the front gate.

1. Is it possible that the collision brings the back of the train to rest relative to the front gate at exactly the same instant? Explain why or why not.

A flatcar far, far behind the train moves with constant speed V_o toward the tunnel.

2. Describe the motion of the back of the train during a time interval equal to L_d/c on the flatcar clock after the collision. Explain.

3. Does the rear gate close on the back of the train, or does it close on the train track?

Homework: Synchronization and causality

II. Timelike, spacelike, and lightlike intervals

Pairs of events with a possible causal connection are referred to as having a *timelike* separation; the square of the spacetime interval between such events is written $\delta t_{12}^2 = \delta t_{12}^{(F)\,2} - \delta x_{12}^{(F)\,2}/c^2$. Pairs of events without a possible causal connection are referred to as having a *spacelike* separation; the square of the spacetime interval between such events is written $\delta s_{12}^2 = \delta x_{12}^{(F)\,2} - c^2 \delta t_{12}^{(F)\,2}$. If the spacetime interval between two events is zero, their separation is said to be *lightlike*.

Two events, 1 and 2, are measured to occur 10 c-ns apart in frame A.

A. Suppose events 1 and 2 are separated by a lightlike interval. Determine the time duration (in ns) between the events in frame A.

B. Suppose instead that in frame A, event 1 occurs 15 ns before event 2.

    1. Is there a reference frame in which the events occur at the same place? If so, describe the motion of that frame relative to frame A. If not, explain why not.

    2. Is there a reference frame in which the events occur at the same time? If so, describe the motion of that frame relative to frame A. If not, explain why not.

    3. Is there a reference frame in which event 1 occurs after event 2? If so, describe the motion of such a frame relative to frame A. If not, explain why not.

C. Suppose instead that in frame A, event 1 occurs 6 ns of time before event 2. Determine the *minimum* spatial separation (in c-ns) between the events in any frame.

Homework: Synchronization and causality

III. Lorentz Transformations

In what follows you will derive the Lorentz transformations using the invariance of the spacetime interval and your knowledge of kinematics. Credit for this problem will be based only on clear and complete reasoning (the correct answers are easily obtained from many textbooks).

A spark jumps between the front end of a train and the track, and another spark jumps between the rear end of the train and the track. In Alan's frame, the sparks jump at the same instant. Beth stands at rest relative to the train. When each spark jumps, it emits a flash of light. Define the following events:

  Event 0:　A spark jumps between the back of the train and the　　　track
  Event 1:　A spark jumps between the front of the train and the track
  Event 2:　The wavefront from the spark that jumped between the front of the train and the track reaches the back of the train

A. Sketch event diagrams showing the location of the train in each of the following frames at the instants of events 0, 1, and 2. Draw a separate picture for each different instant. Indicate the location of each event on the appropriate diagram. (　*Hint:* In Alan's frame, events 0 and 1 are simultaneous. In Beth's frame, events 0 and 2 occur at the same location.)

Alan's frame　　　　　　　　　　　　　*Beth's frame*

Homework: Synchronization and causality

Define $v = \delta x_{02}^{(A)} / \delta t_{02}^{(A)}$

          $u = \delta t_{01}^{(B)} / \delta x_{01}^{(B)}$

B. Give a name for the quantity v in this context. Explain.

Does the quantity u have a name in this context? Explain.

C. *Time dilation*

1. Write $\delta t_{02}^{(B)}$ in terms of $\delta t_{02}^{(A)}$ and $\delta x_{02}^{(A)}$. Explain your reasoning.

2. Use the above expression to write the ratio $\delta t_{02}^{(B)} / \delta t_{02}^{(A)}$ in terms of v. Show your work.

D. *Length Contraction*

1. Write $\delta x_{01}^{(A)}$ in terms of $\delta x_{01}^{(B)}$ and $\delta t_{01}^{(B)}$. Explain your reasoning.

2. Use the above expression to write the ratio $\delta x_{01}^{(A)} / \delta x_{01}^{(B)}$ in terms of u. Show your work.

Homework: Synchronization and causality

E. Define $L = \delta x_{01}^{(B)}$

  $T = \delta t_{02}^{(B)}$

  1. Give a name for the quantity L in this context. Explain.

  2. Give a name for the quantity T in this context. Explain.

  3. Write the following quantities in terms of L, T, u, and v only. Show your work and explain your reasoning.

   a. $\delta x_{12}^{(B)}$

   b. $\delta t_{12}^{(A)}$

   c. $\delta t_{12}^{(B)}$

   d. $\delta x_{12}^{(A)}$

  4. Show that $u = -v c^2$. (*Hint:* What quantity relates the four quantities in part 3?)

F. *Lorentz transformations*

The Lorentz transformations are completely general; they relate event coordinates in two arbitrary frames, which we will refer to as the "primed" and "unprimed" frames, by convention.

Define $\boldsymbol{g} = 1/(\sqrt{1 - v^2/c^2})$ if you haven't already.

Assume the transformations are linear, that is, assume they have the form

$$\delta t' = A\delta t + B\delta x$$
$$\delta x' = C\delta t + D\delta x$$

In parts 1-4 below, let the primed frame be the rest frame of Beth and the unprimed frame be the rest frame of Alan.

1. Considering events 0 and 1, write B in terms of v and $\boldsymbol{g}$ Show your work.

2. Write D in terms of v and $\boldsymbol{g}$. Show your work, stating carefully which events you are considering.

3. Write A in terms of v and $\boldsymbol{g}$. Show your work, stating carefully which events you are considering.

4. Write C in terms of v and $\boldsymbol{g}$. Show your work, stating carefully which events you are considering.

5. Collect your results from parts 1-4 to relate the spatial separation and the time duration between any two events in the primed frame to the spatial separation and the time duration between the same two events in the unprimed frame.

VITA

Rachel Ellen Scherr was born in San Francisco, California, in 1971. She completed a B.A. in Physics from Reed College in Portland, Oregon in 1993. She was then awarded a Thomas J. Watson Fellowship and spent a year abroad investigating extracurricular science education; along with her husband, Dale Hailey, she directed the construction and presentation of science exhibits at an international school in Botswana. In 1996, she was awarded an M.S. in Physics from the University of Washington. Ms. Scherr is presently teaching at The Evergreen State College in Olympia, Washington.