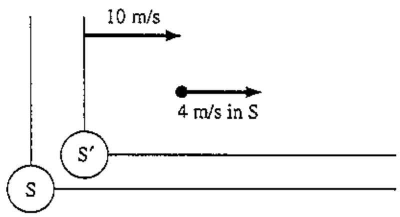
36 Relativity

36.1 Relativity: What's It All About?

36.2 Galilean Relativity

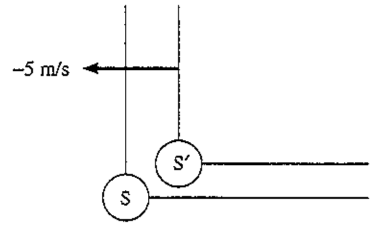
1. In which reference frame, S or S′, does the ball move faster?



The ball moves faster in S′.

$u' = u - v = 4\ \frac{m}{s} - 10\ \frac{m}{s} = -6\ \frac{m}{s}$

2. Frame S′ moves relative to frame S as shown.



a. A ball is at rest in frame S′. What are the speed and direction of the ball in frame S?

$u = u' + v$

$u = 0 + (-5\ m/s) = -5\ m/s$ left

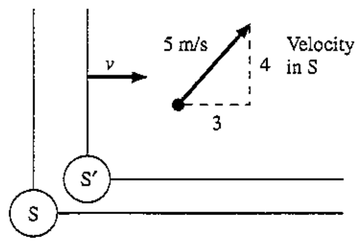
b. A ball is at rest in frame S. What are the speed and direction of the ball in frame S′?

$u = u' + v$

$0\ m/s = u' + (-5\ m/s)$

$u' = +5\ m/s$ right

3. Frame S′ moves parallel to the x-axis of frame S.



a. Is there a value of v for which the ball is at rest in S′? If so, what is v? If not, why not? No.

Because $u_y = u_y' + v_y$ and $v_y = 0$.

Then $u_y = u_y' = 4\ m/s$. There will still be a y-component of u'.

b. Is there a value of v for which the ball has a minimum speed in S′? If so, what is v? If not, why not? Yes.

To minimize u', we can only minimize its x-component.

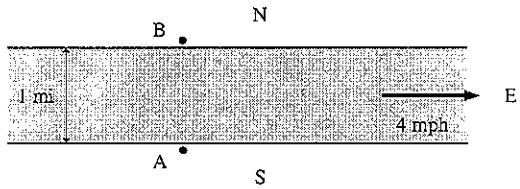
$u_x = u_x' + v$

$3\ m/s = u_x' + v$

Choose $v = 3\ m/s$ and then $u_x' = 0$.

u_y' will still be equal to 4 m/s.

4. Anjay can swim at a steady speed of 2 mph. He needs to cross a river that flows west to east at 4 mph. Anjay jumps in at point A and swims due north (i.e., his head always points due north) until reaching the opposite shore. Where does Anjay land?



$$u_y = \frac{dy}{dt}$$

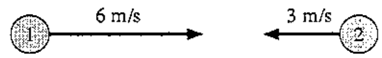
$$2 \text{ mph} = \frac{1 \text{ mi}}{t} \qquad t = 0.5 \text{ h}$$

$$x = vt$$

$$x = (4 \text{ mph})(0.5 \text{ h})$$

$x = 2$ mi downstream from B

5. a. What are the speed and direction of each ball in a reference frame that moves with ball 1?



$$u_{1i} = 6 \text{ m/s} \qquad u_{2i} = -3 \text{ m/s} \qquad v = 6 \text{ m/s}$$

$$u_{1i}' = u_{1i} - v = 6 \text{ m/s} - 6 \text{ m/s} = 0 \text{ m/s}$$

$u_{2i}' = u_{2i} - v = -3 \text{ m/s} - 6 \text{ m/s} = -9 \text{ m/s}$ to the left

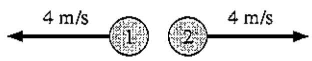
b. What are the speed and direction of each ball in a reference frame that moves with ball 2?

$$u_{1i} = 6 \text{ m/s} \qquad u_{2i} = -3 \text{ m/s} \qquad v = -3 \text{ m/s}$$

$u_{1i}' = u_{1i} - v = 6 \text{ m/s} - (-3 \text{ m/s}) = 9 \text{ m/s}$ to the right

$$u_{2i}' = u_{2i} - v = -3 \text{ m/s} - (-3 \text{ m/s}) = 0 \text{ m/s}$$

6. What are the speed and direction of each ball in a reference frame that moves to the right at 2 m/s?



$$u_{1i} = -4 \text{ m/s} \qquad u_{2i} = 4 \text{ m/s} \qquad v = 2 \text{ m/s}$$

$u_{1i}' = u_{1i} - v = -4 \text{ m/s} - 2 \text{ m/s} = -6 \text{ m/s}$ to the left

$u_{2i}' = u_{2i} - v = 4 \text{ m/s} - 2 \text{ m/s} = 2 \text{ m/s}$ to the right

36.3 Einstein's Principle of Relativity

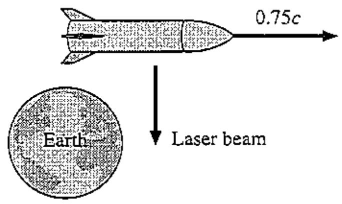
7. A lighthouse beacon alerts ships to the danger of a rocky coastline.
   a. According to the lighthouse keeper, with what speed does the light leave the lighthouse?

   At speed c.

   b. A boat is approaching the coastline at speed $0.5c$. According to the captain, with what speed is the light from the beacon approaching her boat?

   At speed c.

8. As a rocket passes the earth at $0.75c$, it fires a laser perpendicular to its direction of travel.



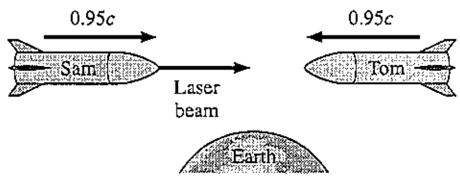
   a. What is the speed of the laser beam relative to the rocket?

   Speed c.

   b. What is the speed of the laser beam relative to the earth?

   Speed c.

9. Teenagers Sam and Tom are playing chicken in their rockets. As seen from the earth, each is traveling at $0.95c$ as he approaches the other. Sam fires a laser beam toward Tom.



   a. What is the speed of the laser beam relative to Sam?

   Speed c.

   b. What is the speed of the laser beam relative to Tom?

   Speed c.

36.4 Events and Measurements

10. It is a bitter cold day at the South Pole, so cold that the speed of sound is only 300 m/s. The speed of light, as always, is 300 m/μs. A firecracker explodes 600 m away from you.

a. How long after the explosion until you see the flash of light? 2 μs

b. How long after the explosion until you see hear the sound? 2 s

c. Suppose you see the flash at $t = 2.000002$ s. At what time was the explosion? 2 s

d. What are the spacetime coordinates for the event "firecracker explodes"? Assume that you are at the origin and that the explosion takes place at a position on the positive x-axis.

(600 m, 0 m, 0 m, 0 s)

11. Firecracker 1 is 300 m from you. Firecracker 2 is 600 m from you in the same direction. You see both explode at the same time. Define event 1 to be "firecracker 1 explodes" and event 2 to be "firecracker 2 explodes." Does event 1 occur before, after, or at the same time as event 2? Explain. Event 1 occurs after event 2.

The flash of light from event 2 has to travel twice as far as the flash of light from event 1 and will take twice as long to travel that longer distance.

12. Firecrackers 1 and 2 are 600 m apart. You are standing exactly halfway between them. Your lab partner is 300 m on the other side of firecracker 1. You see two flashes of light, from the two explosions, at exactly the same instant of time. Define event 1 to be "firecracker 1 explodes" and event 2 to be "firecracker 2 explodes." According to your lab partner, based on measurements he or she makes, does event 1 occur before, after, or at the same time as event 2? Explain. To your lab partner, event 1 occurs before event 2.

You see the two events at the same time and the flash from each travels the same distance to you so the events occur at the same time. Your lab partner sees event 1 first because the flash from 1 has less distance to travel to get to your lab partner.

13. Your clocks and calendars are synchronized with the clocks and calendars in a star system exactly 10 light years from earth that is at rest relative to the earth. You receive a TV transmission from the star system that shows a date and time display. The date it shows is June 17, 2050.

When you glance over at your own wall calendar, the date it shows is June 17, 2060.

14. Two trees are 600 m apart. You are standing exactly halfway between them and your lab partner is at the base of tree 1. Lightning strikes both trees.

a. Your lab partner, based on measurements he or she makes, determines that the two lightning strikes were simultaneous. What did you see? Did you see the lightning hit tree 1 first, hit tree 2 first, or hit them both at the same instant of time? Explain.

You see lightning hit both trees at the same instant because the strikes are simultaneous and the light has the same distance to travel to get to you.

b. Lightning strikes again. This time your lab partner sees both flashes of light at the same instant of time. What did you see? Did you see the lightning hit tree 1 first, hit tree 2 first, or hit them both at the same instant of time? Explain.

You see lightning strike tree 2 first.

To arrive at your lab partner at the same time, the flash of light would have to leave tree 2 first. The two flashes have the same distance to travel to get to you.

c. In the scenario of part b, were the lightning strikes simultaneous? Explain.

No. The lightning actually strikes tree 2 first.

15. You are at the origin of a coordinate system containing clocks, but you're not sure if the clocks have been synchronized. The clocks have reflective faces, allowing you to read them by shining light on them. You flash a bright light at the origin at the instant your clock reads $t = 2.000000$ s.

a. At what time will you see the reflection of the light from a clock at $x = 3000$ m?

$$\Delta t_{trip} = \frac{x}{v} = \frac{6000 \text{ m}}{300 \text{ m}/\mu s} = 20 \mu s$$

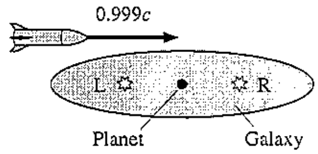
$$t = 2.000020 \text{ s}$$

b. When you see the clock at $x = 3000$ m, it reads 2.000020 s. Is the clock synchronized with your clock at the origin? Explain.

No. Light travels from the origin to the clock 3000 m away and illuminates the time shown on the clock at that instant.

36.5 The Relativity of Simultaneity

16. Two supernovas, labeled L and R, occur on opposite sides of a galaxy, at equal distances from the center. The supernovas are seen at the same instant on a planet at rest in the center of the galaxy. A spaceship is entering the galaxy from the left at a speed of 0.999c relative to the galaxy.



a. According to astronomers on the planet, were the two explosions simultaneous? Explain why.

Yes. They were seen at the same instant and the light traveled the same distance from L and from R to reach the planet.

b. Which supernova, L or R, does the spaceship crew *see* first? L

c. Did the supernova that was *seen* first necessarily *happen* first in the rocket's frame? Explain.

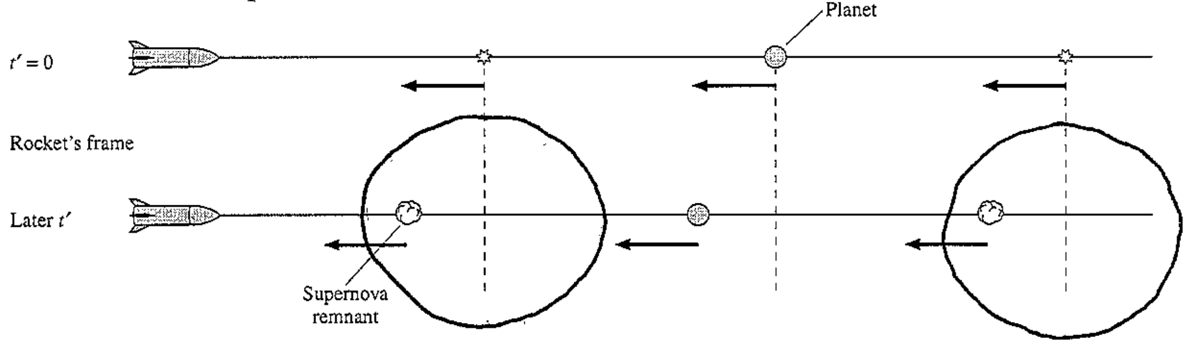
No. Because the rocket is closer to the left supernova the light from the left supernova has a shorter distance to travel to reach the rocket.

d. Is "two light flashes reach the planet at the same instant" an event? To help you decide, could you arrange for something to happen only if two light flashes from opposite directions arrive at the same time? Explain. Yes. You could have a detector on each side of a box and a light on top of the box that would flash only if both detectors detect light at the same time.

If you answered Yes to part d, then the crew on the spaceship will also determine, from their measurements, that the light flashes reach the planet at the same instant. (Experimenters in different reference frames may disagree about when and where an event occurs, but they all agree that it *does* occur.)

e. The figure below shows the supernovas in the spaceship's reference frame with the *assumption* that the supernovas are simultaneous. The second half of the figure is a short time after the explosions. Draw two circular wave fronts to show the light from each supernova. Neither wave front has yet reached the planet. Be sure to consider:

- The points on which the wave fronts are centered.
- The wave speeds of each wave front.



f. According to your diagram, are the two wave fronts going to reach the planet at the same instant of time? Why or why not? No. Because the planet is moving toward the wavefront on the left and away from the wavefront on the right.

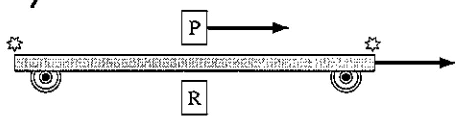
g. Does your answer to part f conflict with your answer to part d? Yes.

If so, what different assumption could you make about the supernovas in the rocket's frame that would bring your wave-front diagram into agreement with your answer to part d?

If the supernovas are not simultaneous in the rocket's frame then the light from each can reach the planet at the same time.

h. So according to the spaceship crew, are the two supernovas simultaneous? If not, which happens first? No. The right one must happen first because the light from the right one must travel a further distance to the planet to reach it at the same time as the light from the left supernova.

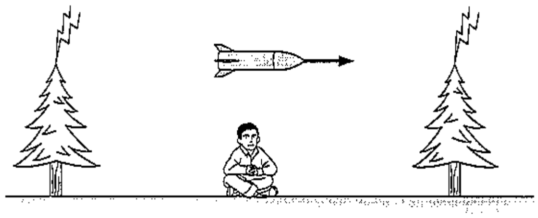
17. Peggy is standing at the center of her railroad car as it passes Ryan on the ground. Firecrackers attached to the ends of the car explode. A short time later, the flashes from the two explosions arrive at Peggy at the same time.



a. Were the explosions simultaneous in Peggy's reference frame? If not, which exploded first? Explain. Yes they are simultaneous. In Peggy's reference frame she and the firecrackers are at rest relative to one another.

b. Were the explosions simultaneous in Ryan's reference frame? If not, which exploded first? Explain. No. The left one had to occur first because it had a further distance for its light flash to travel to reach Peggy, since Peggy was moving toward the right and away from the left.

18. A rocket is traveling from left to right. At the instant it is halfway between two trees, lightning simultaneously (in the rocket's frame) hits both trees.



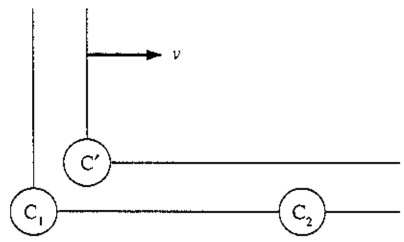
a. Do the light flashes reach the rocket pilot simultaneously? If not, which reaches him first? Explain.

No. During the time the flashes of light are traveling, the rocket is moving to the right. So the flash from the right lightning strike has less distance to travel to get to the rocket and therefore reaches the rocket pilot first.

b. A student was sitting on the ground halfway between the trees as the rocket passed overhead. According to the student, were the lightning strikes simultaneous? If not, which tree was hit first? Explain.

No. Two events occuring simultaneously in one reference frame S are not simultaneous in any reference frame moving relative to S.
The student sees the tree on the left hit first. He is moving to the left relative to the frame of reference of the rocket where the strikes were simultaneous. So he moves toward the wave front on the left and away from the one on the right.

36.6 Time Dilation



19. Clocks C_1 and C_2 in frame S are synchronized. Clock C′ moves at speed v relative to frame S. Clocks C′ and C_1 read exactly the same as C′ goes past. As C′ passes C_2, is the time shown on C′ earlier than, later than, or the same as the time shown on C_2? Explain. Earlier than. In the S′ frame the 2 events are measured by the same clock so that is the proper time $\Delta\tau$. In frame S, $\Delta t > \Delta\tau$.

20. Your friend flies from Los Angeles to New York. She carries an accurate stopwatch with her to measure the flight time. You and your assistants on the ground also measure the flight time.

    a. Identify the two events associated with this measurement.

    Event 1 is the time she leaves Los Angeles.
    Event 2 is the time she arrives in New York.

    b. Who, if anyone, measures the proper time? Your friend.

    c. Who, if anyone, measures the shorter flight time? Your friend.

    d. Who, if anyone, measures the longer flight time? Your assistants.

21. You're passing a car on the highway. You want to know how much time is required to completely pass the car, from no overlap between the cars to no overlap between the cars. Call your car A and the car you are passing B.

    a. Specify two events that can be given spacetime coordinates. In describing the events, refer to cars A and B and to their front bumpers and rear bumpers.

    Event 1 occurs when the front bumper of car A and the rear bumper of car B are at the same point. Event 2 is when the rear bumper of car A and the front bumper of car B are at the same point.

    b. In either reference frame, is there *one* clock that is present at both events? No.

    c. Who, if anyone, measures the proper time between the events? No one.

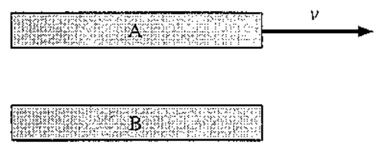
36.7 Length Contraction

22. Your friend flies from Los Angeles to New York. He determines the distance using the tried-and-true $d = vt$. You and your assistants on the ground also measure the distance, using meter sticks and surveying equipment.

   a. Who, if anyone, measures the proper length? You do.

   b. Who, if anyone, measures the shorter distance? Your friend.

23. Experimenters in B's reference frame measure $L_A = L_B$. Do experimenters in A's reference frame agree that A and B are the same length? If not, which do they find to be longer? Explain. Yes they find A and B to be the same length. Each sees the others as being length contracted the same amount because each is moving relative to the other the same amount.



24. As a meter stick flies past you, you simultaneously measure the positions of both ends and determine that $L < 1$ m.

   a. To an experimenter in frame S′, the meter stick's frame, did you make your two measurements simultaneously? If not, which end did you measure first? Explain.

   Hint: Review the reasoning about simultaneity that you used in Exercises 16–18.

   No. You measured the left end first.

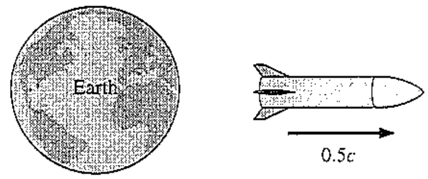
   b. Can experimenters in frame S′ give an explanation for why your measurement is < 1 m?

   Yes. Experimenters in S′ are at rest relative to the meter stick so they are measuring the proper length and the proper time.

25. A 100-m-long train is heading for an 80-m-long tunnel. If the train moves sufficiently fast, is it possible, according to experimenters on the ground, for the entire train to be inside the tunnel at one instant of time? Explain. Yes. There are 2 events. Event 1 is the front of the train is at the entrance to the tunnel and event 2 is that the front of the train is at the tunnel exit. An experimenter on the ground will measure the rear of the train to be at the tunnel entrance at the same time as event 2 due to length contraction.

36.8 The Lorentz Transformations

26. A rocket travels at speed $0.5c$ relative to the earth.



a. The rocket shoots a bullet in the forward direction at speed $0.5c$ relative to the rocket. Is the bullet's speed relative to the earth less than, greater than, or equal to c?

Less than c. $v = 0.5c \quad u' = 0.5c$

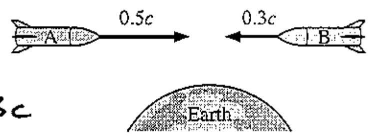
$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{+0.5c + 0.5c}{1 + (0.5c)^2/c^2} = 0.8c$$

b. The rocket shoots a second bullet in the backward direction at speed $0.5c$ relative to the rocket. In the earth's frame, is the bullet moving right, moving left, or at rest?

At rest. $v = 0.5c \quad u' = -0.5c$

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{-0.5c + 0.5c}{1 + (-0.5c)(0.5c)/c^2} = 0$$

27. The rocket speeds are shown relative to the earth. Is the speed of A relative to B greater than, less than, or equal to $0.8c$?



Less than 0.8c. $v = 0.5c \quad u = -0.3c$

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{-0.3c - 0.5c}{1 - (-0.3c)(0.5c)/c^2} = 0.7c$$

36.9 Relativistic Momentum

28. Particle A has half the mass and twice the speed of particle B. Is p_A less than, greater than, or equal to p_B? Explain. Equal. $M_A = \frac{1}{2} M_B \quad V_A = 2V_B$

$$p_A = M_A V_A = \left(\tfrac{1}{2} m_B\right)(2V_B) = m_B V_B = p_B$$

29. Particle A has one-third the mass of particle B. The two particles have equal momenta. Is u_A less than, greater than, or equal to $3u_B$? Explain. Equal.

$M_A = \frac{1}{3} M_B \quad p_A = p_B$

$$M_A u_A = M_B u_B$$
$$\left(\tfrac{1}{3} m_B\right) u_A = m_B u_B$$
$$u_A = 3u_B$$

30. Event A occurs at spacetime coordinates (300 m, 2 μs).

a. Event B occurs at spacetime coordinates (1200 m, 6 μs). Could A possibly be the cause of B? Explain. Yes. $\Delta X = X_B - X_A = 1200m - 300m = 900\ m$

$\Delta t = t_B - t_A = 6\ \mu s - 2\ \mu s = 4\ \mu s \qquad V = \frac{\Delta X}{\Delta t} = \frac{900\ m}{4\ \mu s} = 225\ m/\mu s < c$

b. Event C occurs at spacetime coordinates (2400 m, 8 μs). Could A possibly be the cause of C? Explain. No. $\Delta X = 2400\ m - 300\ m = 2100\ m$

$\Delta t = 8\ \mu s - 2\ \mu s = 6\ \mu s \qquad V = \frac{2100\ m}{6\ \mu s} = 350\ m/\mu s > c$

31. Event B occurs at $t_B = 10.0\ \mu$s. An earlier event A, at $t_A = 5.0\ \mu$s, is the cause of B. What is the maximum possible distance that A can be from B?

$\Delta X = c\,\Delta t = (300\ m/\mu s)(10.0\ \mu s - 5.0\ \mu s) = 1500\ m$

36.10 Relativistic Energy

32. Can a particle of mass m have total energy less than mc^2? Explain.

No. The total energy is equal to the rest energy mc^2 when the kinetic energy $K = 0$ and there is no motion. Whenever there is motion K will be greater than zero and $E > mc^2$.

33. Consider these 4 particles:

Particle	Rest energy	Total energy
1	A	A
2	B	$2B$
3	$2C$	$4C$
4	$3D$	$5D$

Rank in order, from largest to smallest, the particles' speeds u_1 to u_4.

Order: $u_2 = u_3 > u_4 > u_1$

Explanation:

$E^2 - E_0^2 = (pc)^2 = \left(\frac{u}{c}\right)^2 E^2$

$A^2 - A^2 = 0 \qquad u_A = 0$

$(2B)^2 - (B)^2 = \left(\frac{u_B}{c}\right)^2 (2B)^2$

$u_B = \sqrt{\frac{3}{4}}\ c$

$(4C)^2 - (2C)^2 = \left(\frac{u_C}{c}\right)^2 (4C)^2$

$u_C = \sqrt{\frac{3}{4}}\ c$

$(5D)^2 - (3D)^2 = \left(\frac{u_D}{c}\right)^2 (5D)^2$

$u_D = \frac{4}{5}\ c$