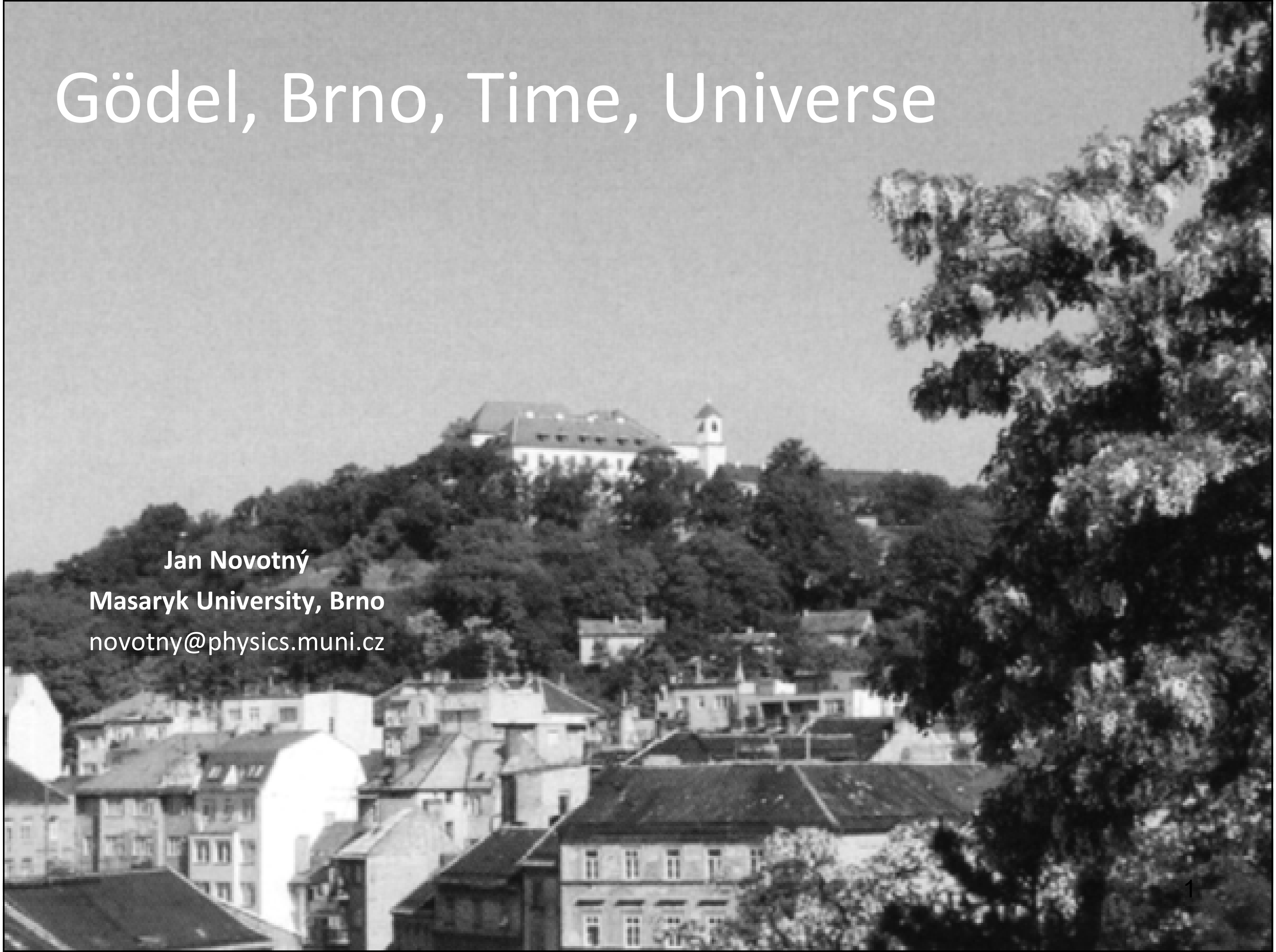
Gödel, Brno, Time, Universe



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I. The city of Brno is associated with famous scientists:

Karel Absolon, archaeologist, paleontologist
(1877 – 1960)

Kurt Gödel, mathematician
(1906 – 1978)

Ernst Mach, physicist and philosopher
(1838 – 1916)

Viktor Kaplan, engineer, inventor
(1876 –1934)

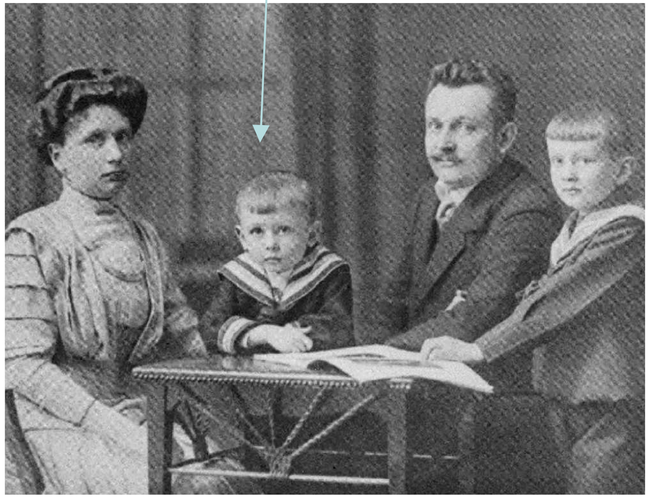
Gregor Mendel, founder of genetics
(1822 – 1884)

George Placzek, physicist
(1905 – 1955)

Pavel Tichý, logician and philosopher
(1936 –1994)

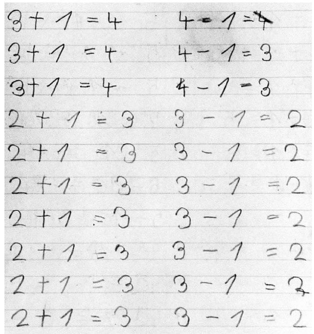
I. Gödel in Brno

Godel*s birthplace, 5 Pekařská



The Godel family, ca. 1910: Marianne, Kurt, father Rudolf, son Rudolf

The Godel villa, Pellicova 8a, near Špilberk forttres



3+1=4	4−1=4
3+1=4	4−1=3
3+1=4	4−1=3
2+1=3	3−1=2
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2+1=3	3−1=2
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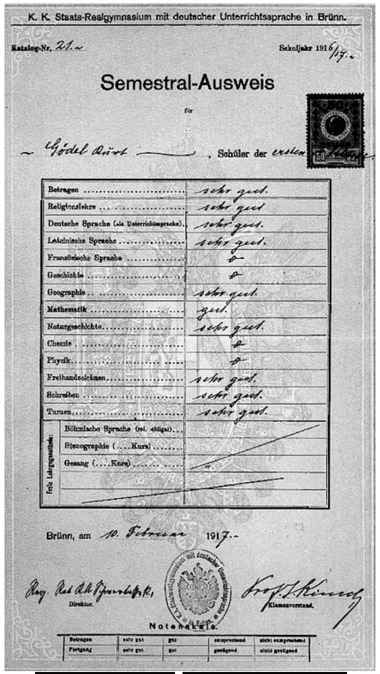
2+1=3	3−1=2
2+1=3	3−1=2

A page from his first arithmetic workbook



K. K. Staats-Realgymnasium mit deutscher Unterrichtssprache in Brünn.

Katalog-Nr. 21.– Schuljahr 1916/17.–

Semestral-Ausweis

für

– Gödel Kurt ——, Schüler der ersten Klasse

Betragen	sehr gut.
Religionslehre	sehr gut
Deutsche Sprache (als Unterrichtssprache)	sehr gut.
Lateinische Sprache	sehr gut.
Französische Sprache	0
Geschichte	0
Geographie	sehr gut.
Mathematik	gut.
Naturgeschichte	sehr gut.
Chemie	0
Physik	0
Freihandzeichnen	sehr gut.
Schreiben	sehr gut.
Turnen	sehr gut.
Freie Lehrgegenstände: Böhmische Sprache (rel. obligat)	
Stenographie (....Kurs)	
Gesang (....Kurs)	

Brünn, am 10. Februar 1917.–

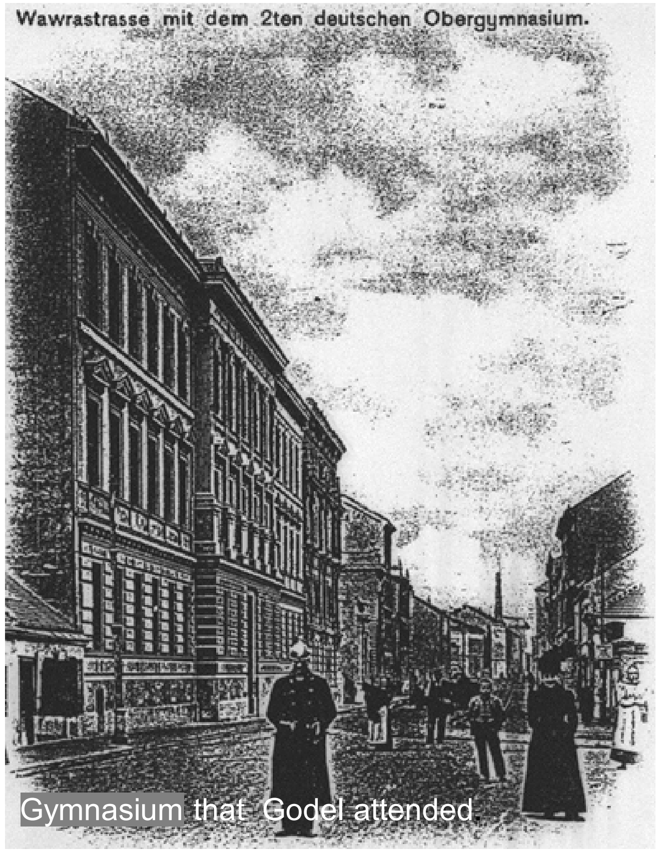
Direktor. Klassenvorstand.

Notenskala.

Betragen	sehr gut	gut	entsprechend	nicht entsprechend
Fortgang	sehr gut	gut	genügend	nicht genügend

Semester report 1917 and his photo 1922

The Evangelische Volks und Bürgerschule

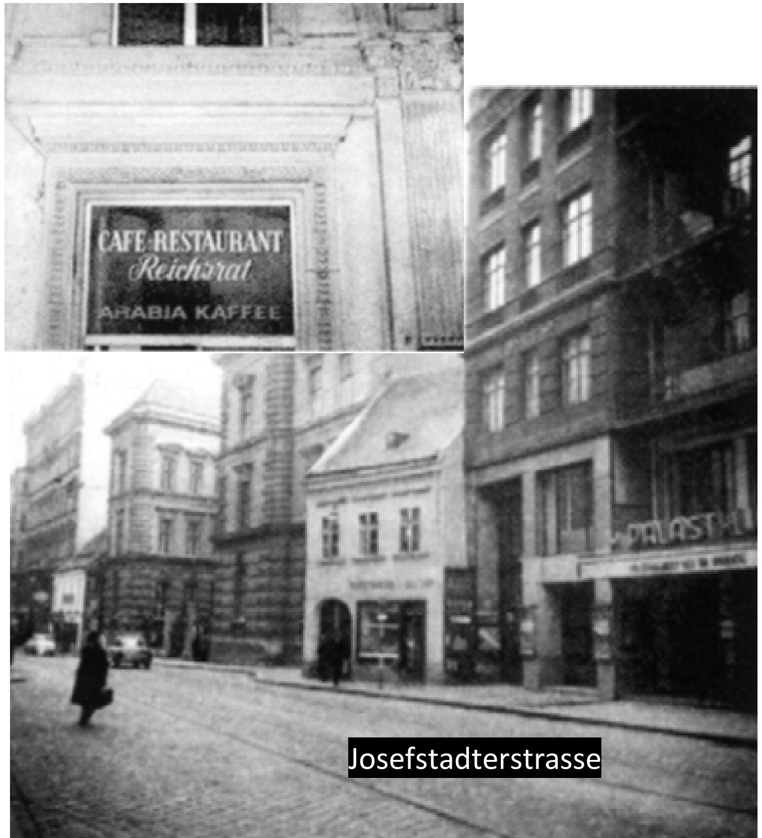


Gymnasium that Godel attended

I. Gödel's life

University of Vienna

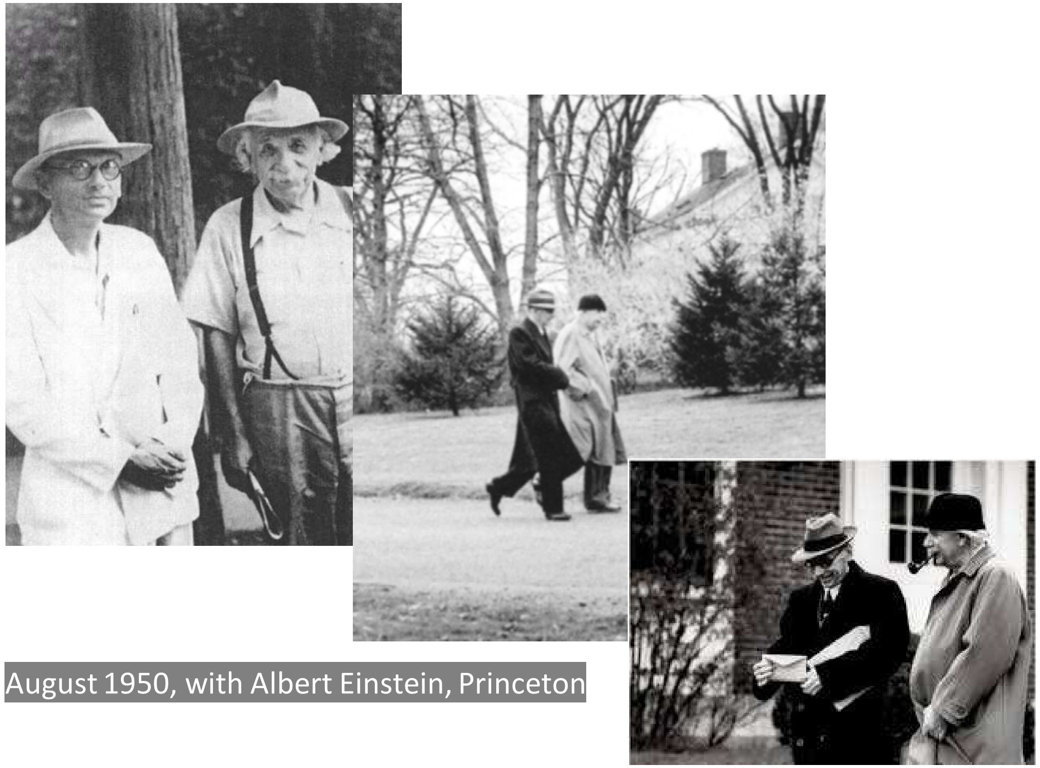
Institutsgebaude der Universitat Wien, Strudlhofgasse



Josefstadterstrasse

Wedding portrait of Kurt and Adele, Vienna, 1938.

Partcipants at Einsteinś 70th birthday celebration (1949)
(Wigner, Weyl, Godel, Rabi, Einstein, Ladenburg, Oppenheimer)



August 1950, with Albert Einstein, Princeton

14.3.1951, receiving the Einstein Prize, Princeton

The Godel home, 145 Linden Lane, Princeton, 1950

II. Overview of mathematical merits

Completness

Every valid logical expression is provable. Equivalently, every logical expression is either satisfiable or refutable.

Incompletness Theorem I

In any consistent theory containing arithmetic there is a sentence which is neither provable nor refutable.

Incompletness Theorem II

Consistency of such theory is not provable.

Consistency

The Continuum Hypothesis and the Axiom of Choice are consistent with standard axioms of set theory.

II. Philosophical Roots

What exactly is time? If no one asks me, I know what it is.
If I wish to explain it to him who asks, I do not know.
Saint Augustine, Confessions

Gödel initial intention to study theoretical physics:

1929-1947 great mathematical works, then a gap

1946-1949 full concentration on philosophy of time,
five manuscripts and final publication

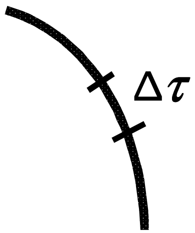
1949-1952 concrete results in physics, public lecture,
two publications

Rare example when initially purely philosophical interest lead to concrete and fundamental results in exact science

III. Time in General Relativity

$$c^2 d\boldsymbol{\tau}^2 = g_{ik}\, dx^i dx^k$$

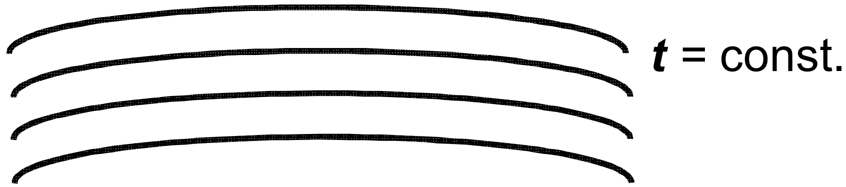
proper Time



Lenght of world line
(absolute)

$$x^i = \left(c\boldsymbol{t}, x^\alpha\right)$$

coordinate Time



Slices of simultaneity
(relative)

$$R_{ik} - \tfrac{1}{2} R\, g_{ik} - \lambda g_{ik} = \kappa T_{ik}$$

geometry | dark energy? | matter

What is missing here in comaprison with our time perception?

I. Arrow (past - future direction)

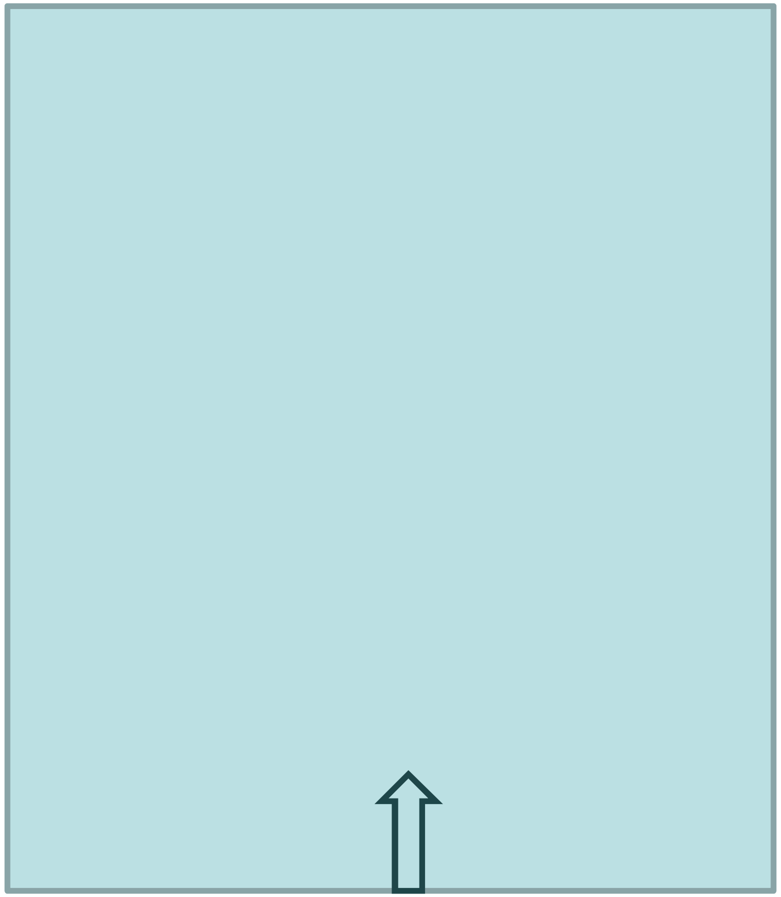
II. Curtain (flow of time)

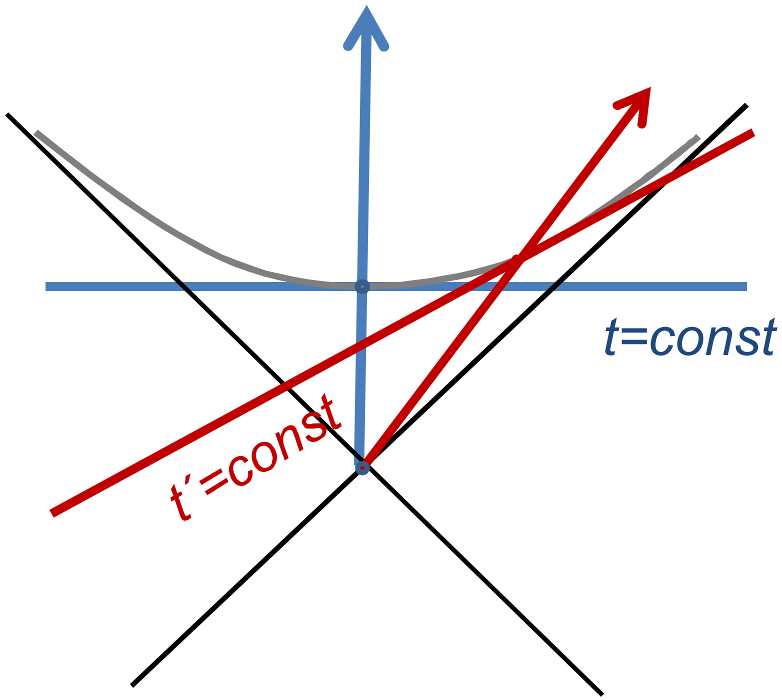
III. Time Arrow and Time Curtain

Newtonian physics – curtain is moved by arrow

Statistical physics – physical motivations for arrow (entropy), is the physical arrow omnipresent?

Special Relativity – doubts about objective existence of curtain





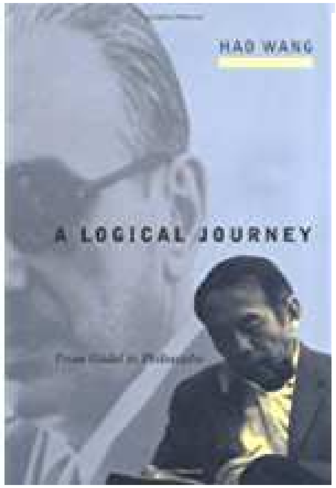
IV. Gödel's Philosophical Views

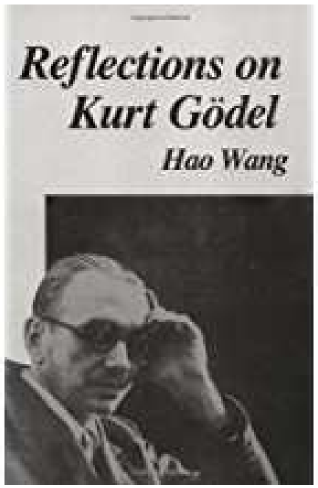
Some observations about the relationship between theory of relativity and Kantian Philosophy (1946-49)

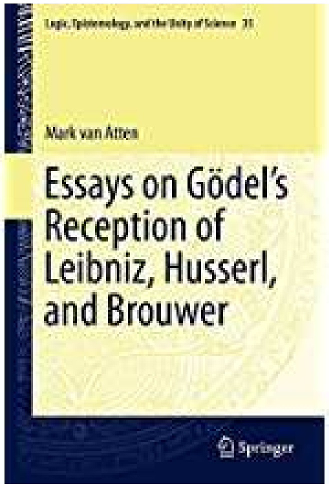
5 manuscripts – 2 in Gödel's Collected Works III

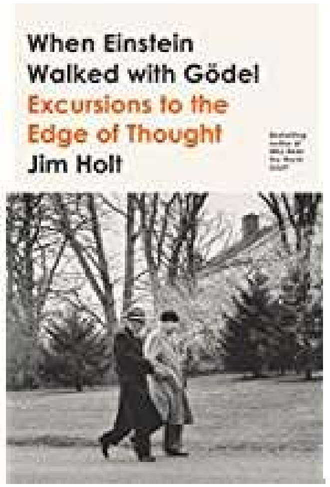
Resulting work:

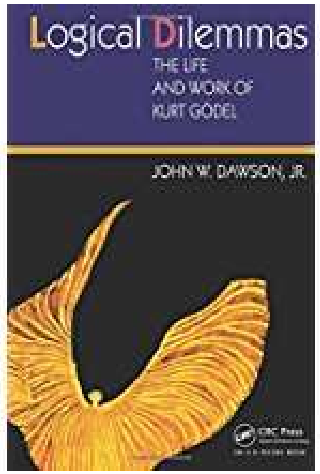
A remark on the relationship between relativity theory and idealistic philosophy, Albert Einstein: Philosopher-Scientist (Library of Living Philosophers), P. Schilpp (ed.), La Salle, IL: Open Court, 1949, pp. 555–562. Reprinted in Gödel 1990, pp. 202–207











IV. Quotations from Gödel

I wish to say rightly in the beginning that I am not an adherent of Kantian philosophy in general. I only try to show that a surprising similiarity exists in some respect between relativity theory and the Kantian doctrine about time and space and that contradictions between them are by far not so fundamental as we frequently understood.

(Kurt Gödel: Collected Works: Volume III: Unpublished Essays and Lectures)

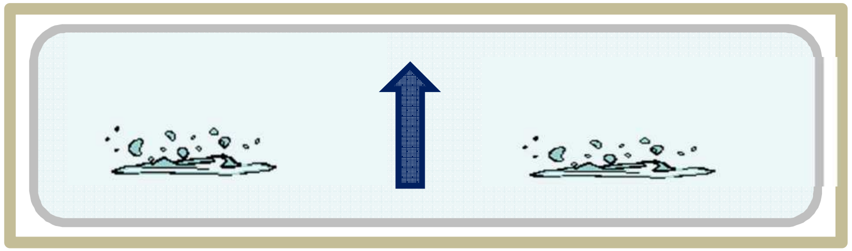
In short, it seems that one obtains an unequivocal proof for the view of those philosophers who, like Parmenides, Kant, and the modern idealists, deny the objectivity of change and consider change as an illusion or an appearance due to our special mode of perception.

(Kurt Gödel: Collected Works: Volume II: Publications 1938-1974)

IV. Philosophical Dilemma

Presentism – only the present is real

Eddington: The great thing about time is that it goes on. Something must be added to the geometrical conceptions comprised in Minkowski´s world it becomes a complete picture of the world as we know it (The nature of physical world 1928)

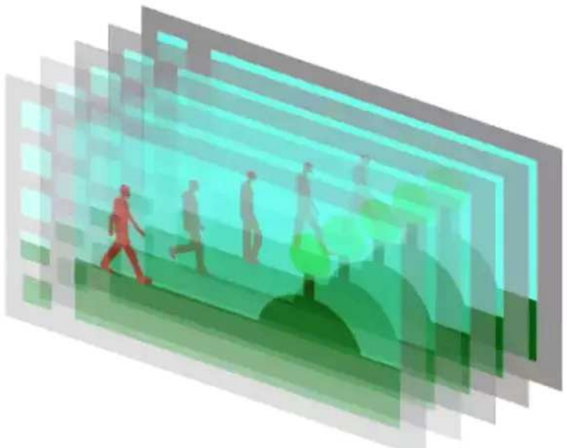


Herakleitos, Bergson, Popper …

Eternalism – all points in time are equally real

Parmenides, Leibniz, Spinoza …

Einstein: For us beliwing physicists, the distinction between past, present and future is only an illusion, even if a stubborn one (private letter 1955)



V. Rotation in cosmology

Special relativity supports eternalism (many curtains means nonobjectivity of flow of of time). At first sight GR goes further in this direction – space-like curtains are arbitrary.

But, according to Jeans, standard relativistic cosmology allows us to return to Newtonian view.

Jeans (*Man and the Universe, Lecture 1935*):

... there is no reason to abandon the intuitive idea of an absolute time lapsing objectively.

On the other side Gamow (*Rotating Universe?, Nature 1946*):

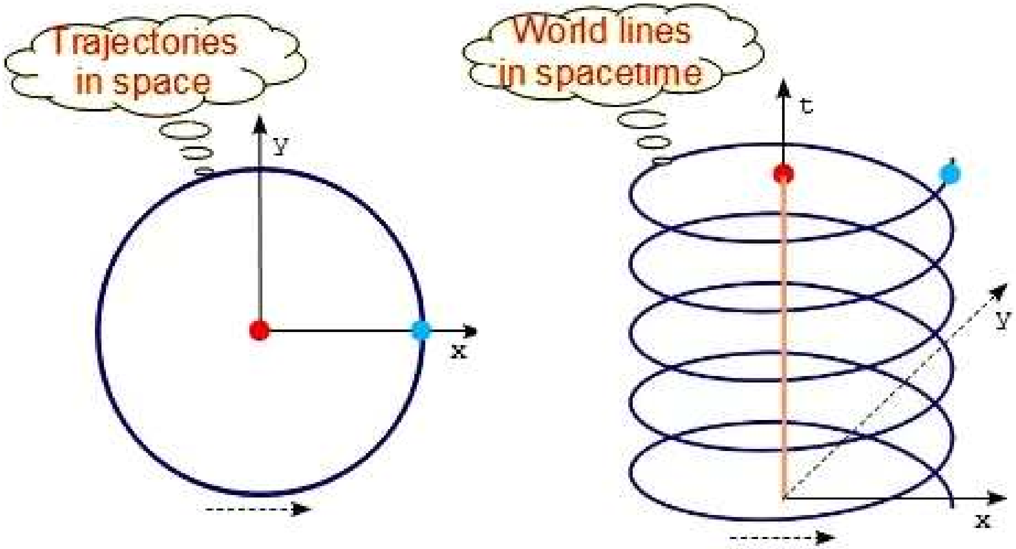
One of the mysterious result of the astronomical studies of the universe lies in the fact that planets, stars and galaxies are formed in the state of more or less rapid rotation…

We can ask ourselves if it is not possible to assume that all matter in the visible universe is in a state of general rotation.

Such a rotating universe can be probably represented by the group of anisotropic solutions of the fundamental equations of cosmology.

V. Rotating Reference Frames

No possibility of cutting space- time by simultaneity surfaces orthogonal to world lines of privileged observers − no geometrical suport for objective flow of time



V. Godel main results ... *he did more than he intended*

$$ds^2 = g^{ik} dx^i dx^k$$

$$ds^2 = a^2 \left[\left(dT + e^X dY \right)^2 - dX^2 - \frac{e^{2X}}{2} dY^2 - dZ^2 \right]$$

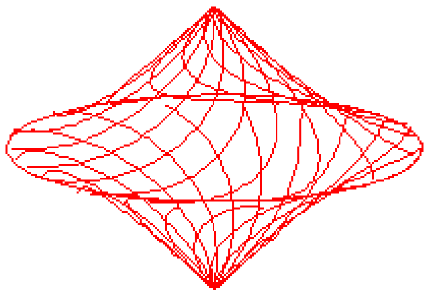
$$\frac{1}{a^2} = 8\pi G\rho \qquad \lambda = -4\pi G\rho$$

coordinates T, X, Y, Z cosmological constant λ density of matter ρ

$$ds^2 = 4a\left[dt^2 - dr^2 - dy^2 + \left(sh^4 r - sh^2 r \right) d\varphi^2 - sh^2 r \, d\varphi dt \right]$$

coordinates t, r, y, φ

φ is timelike for: $sh^4 r > sh^2 r$



V. Basic Properties of Gödel Universe

Gödel universe is rare example of regular (singularity free), geodetically completed and topologically trivial (homeomorphic to R^4) manifold.

Symmetries:

This solution, or rather the 4D space *S* which it defines, has the further properties:

(1) S is homogeneous, i.e., for any two points *P*, *Q* of *S* there exists a transformation of *S* into itself which carries *P* into *Q*. In terms of physics this means that the solution is stationary and spatially homogeneous.

(2) There exists a one-parametric group of transformations of *S* into itself which carries each world line of matter into itself, so that any two world lines of matter are equidistant.

(3) *S* has rotational symmetry, i.e., for each point *P* of *S* there exists a one-parametric group of transformations of *S* into itself which carries *P* into itself.

(4) The totality of time-like and null vectors can be divided into + and - vectors in such a way that: (a) if ξ is a + vector, - ξ is a - vector, (b) a limit of + (or -) vectors, if ǂ 0, is again a + (or -) vector. That is, a positive direction of time can consistently be introduced in the whole solution.

Stability with respect to small perturbances
Barrow, J.D. ,Tsagas, C.G. (2004). Dynamics and stability of the Gödel universe, Class. Quantum Grav., 21,

V. Time Loop

Difference between cyclic time and no-time

Rindler: ...while this dependence of the objectivity of time on the cosmic mass distribution may well be a problem for the philosophers, it presents neither physical nor logical problems for physicists, who have long learned to live and work with this state of affairs.

Gödel, Einstein, Mach, Gamow, and Lanczos: Gödel's remarkable excursion into cosmology, American Journal of Physics 77, 498 (2009)

Who was the first?

Rindler: ... Gödel's universe was the cleanest example, certainly the one that caught the widest attention, and possibly the first where time loops were explicitly recognized.

The first Time loop, but not noticed by author is in:

Lanczos. C., Über eine stationare kosmologie im sinne der einsteinschen gravitationstheorie (In German). Z.Phys. 21 (1924)

V. Lanczos and Wheeler

As the electron can have neither beginning nor end anywhere, it would, after one period of time, have to return to the same point in space from which it started.

... the electron could also continue its existence at another point in space, where it would appear to be a second electron. Thus we are led to the idea that, maybe, the world-lines of all the electrons are only segments of one original world-line and that the individual electrons are only different temporal stages of one single primordial object.

This would explain in a natural manner why all elementary building blocks of matter (at least as far as the negative electrons are concerned) are in principle the same, thereby enriching our conviction about the unity of the universe by fascinating aspects.

Feynman Nobel Lecture:
I received a telephon call one day at prof.Wheeler in which he sad: Feynman,
I know why allelectrons have the same charge and the same mass.
Why? Because they are all the same electron.

V. Gödel and Mach

Gödel: .. the world may be said to rotate as a whole (like a rigid body) ...

Of course, it is also possible and even more suggestive to think of this world as a rigid body at rest and of the compass of inertia as rotating everywhere relative to this body.

Evidently this state of affairs shows that the inertial field is to a large extent independent of the state of motion of matter. This contradicts Mach's principle but it does not contradict relativity theory.

Hawking:
In other words, matter has inertia only relative to other matter in the universe. The principle is generally taken to imply that the local inertial frame defined by gyroscopes should be non-rotating with respect to the frame defined by distant galaxies.

... most physicists nowadays would not accept Mach's principle, because they feel that it makes an untenable distinction between the geometry of space-time, which represents the gravitational and inertial field, and other forms of fields and matter.

Machian programme

Inertial forces →

gravity

other forces

Einstein way to GR

Inertial forces

← gravity

other forces

What exactly means „Mach principle" in General Relativity?

Still open question

VI. Gödel on Rotating Universes

Lecture on rotating universes (1949),

Gödel Celected Works, Volume III: Unpublished Essays and Lectures

Rotating universes in general relativity theory (1952)

Gödel Celected Works, Volume II : Publications 1938–1974 .

$$a_{klm} = v_k \left(\frac{\partial v_l}{\partial x_m} - \frac{\partial v_m}{\partial x_l} \right) + v_l \left(\frac{\partial v_m}{\partial x_k} - \frac{\partial v_k}{\partial x_m} \right) + v_m \left(\frac{\partial v_k}{\partial x_l} - \frac{\partial v_l}{\partial x_k} \right)$$

v^i - vector field tangent to worldlines of privileged observers

$$\omega^i = \frac{\varepsilon^{iklm}}{12\left(-g\right)^{\frac{1}{2}}} a_{klm}$$ angular velocity

Imagine therefore a locally parallel set of such constant-density surfaces, going from greater to lesser density, and the worldline of a fundamental observer cutting across them obliquely. The observer’s proper surface of simultaneity is orthogonal to his worldline, and therefore cuts across the constant-density surfaces. At any given instant, therefore, the observer “sees” more galaxies in one half of the sky where his simultaneity dips into the greater density than in the other half. (cit. Rindler)

Principal Consequences of Gödel Work

Inspiration for time philosophy - Barbour, Penrose, Smolin ...

New ideas on Causality - Carter, Geroch, Hawking, Penrose ...

New cosmological models - mixmaster, rotating Newtonian models...

Role of symmetry groups - Bianchi universes ...

Deeper analysis of frames and world line congruences – expansion, vorticity, shear

Figure 1

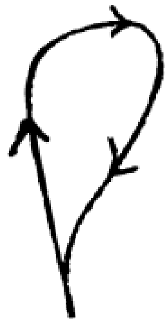


Figure 2

Einstein on Gödel

The distincion „earlier-later“ is abandoned for world points which lie far apart in a cosmological sense and more paradoxes regarding the direction of the causal connection arise. It would be interesting to weigh whether these are not to be excluded on physical grounds.

Every ending is really just a new beginning

Thank You

Source Literature:

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Collected-Works-Volume-III-Unpublished-essays-and-lectures
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Dawson, J. (1997) Logical Dilemmas: the life and work of Kurt Gödel,
Wang, Hao (1987) Reflections on Kurt Gödel, MIT Press, Cambridge
Yourgrau, P. (1999) Gödel meets Einstein. Time Travel in the Godel Universes
Rindler, W.(2009): ..Gödel's remarkable excursion into cosmology, AJP, Vol77, 6