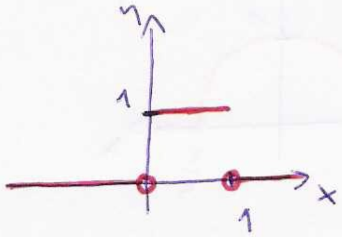
Tomáš Sedmík (255594)

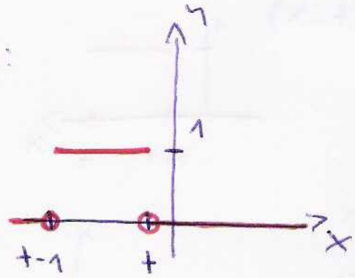
příklad 1.

$$f(x) \text{ i } g(x) = \begin{cases} 1 & \text{pro } x \in \langle 0; 1 \rangle \\ 0 & \text{jinak} \end{cases}$$

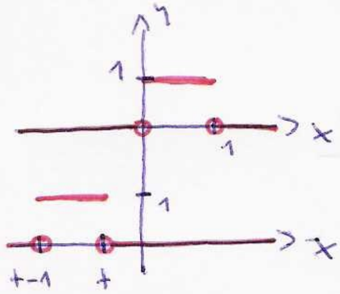
$f(x)$:



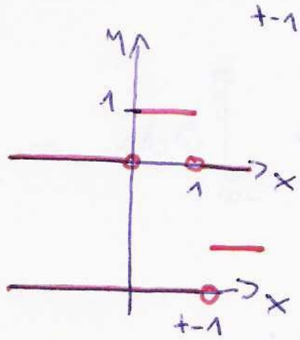
$f(t-x)$:



ⓐ



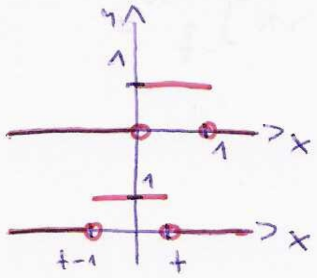
pro $t \in (-\infty; 0)$



pro $t \in (2; \infty)$

$$\int_{-\infty}^{\infty} 0 \, dx = \underline{0}$$

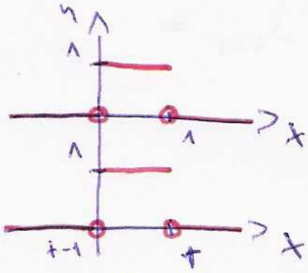
ⓑ



$$\int_{-\infty}^{\infty} f(t-x) g(x) \, dx = \int_{0}^{t} 1 \, dx = [x]_0^t = \underline{t}$$

$\Rightarrow$ pro $t \in \langle 0; 1)$

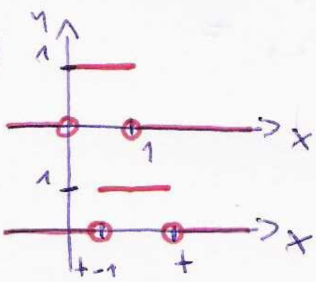
ⓒ



$$\int_{-\infty}^{\infty} f(t-x) g(x) \, dx = \int_{0}^{1} 1 \, dx = [x]_0^1 = \underline{1}$$

$\Rightarrow$ pro $t \in \{1\}$

ⓓ



$$\int_{-\infty}^{\infty} f(t-x) g(x) \, dx = \int_{t-1}^{1} 1 \, dx = [x]_{t-1}^{1} = 1 - t + 1 = \underline{2 - t}$$

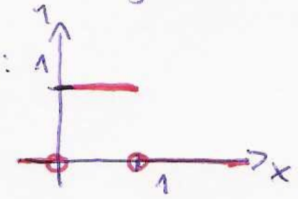
$\Rightarrow$ pro $t \in (1; 2\rangle$

$$f * g = \begin{cases} x & \text{pro } x \in \langle 0; 1) \\ 1 & \text{pro } x \in \{1\} \\ 2-x & \text{pro } x \in (1; 2\rangle \\ 0 & \text{jinak} \end{cases}$$

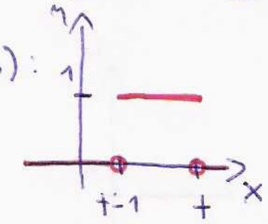
příklad 2.

$$f(x) = \begin{cases} 1 & \text{pro } x \in \langle 0;1\rangle \\ 0 & \text{jinak} \end{cases} \qquad g(x) = \begin{cases} \cos x & \text{pro } x \in \langle -\frac{\pi}{2}; \frac{\pi}{2}\rangle \\ 0 & \text{jinak} \end{cases}$$

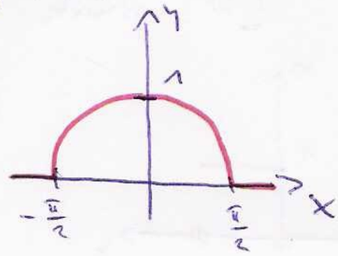
$f(x)$:



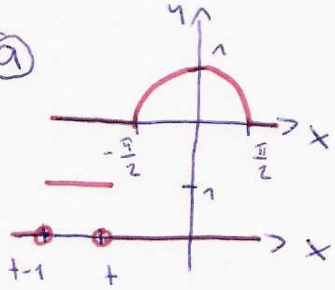
$f(t-x)$:



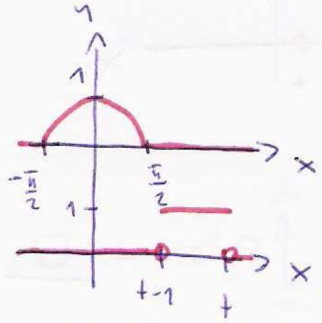
$g(x)$:



(a)



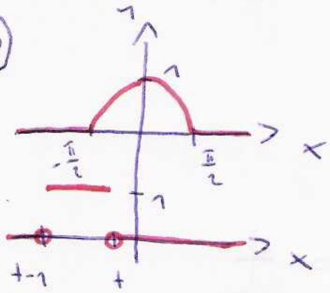
pro $t \in (-\infty; -\frac{\pi}{2})$



pro $t \in (\frac{\pi}{2}+1; \infty)$

$$\int_{-\infty}^{\infty} 0\,dx = \underline{0}$$

(b)

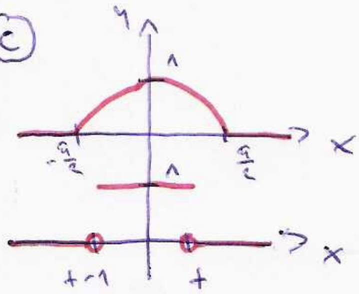


$$\int_{-\infty}^{\infty} f(t-x)g(x) = \int_{-\frac{\pi}{2}}^{t} \cos x\,dx = [\sin x]_{-\frac{\pi}{2}}^{t} =$$

$$\underline{\sin t + \cancel{\sin(t-1)}\ 1}$$

$\Rightarrow$ pro $t \in \langle -\frac{\pi}{2}; -\frac{\pi}{2}+1\rangle$

(c)

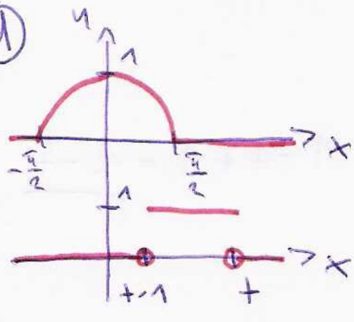


$$\int_{-\infty}^{\infty} f(t-x)g(x) = \int_{t-1}^{t} \cos x\,dx = [\sin x]_{t-1}^{t} =$$

$$\underline{\sin t - \sin(t-1)}$$

$\Rightarrow$ pro $t \in \langle -\frac{\pi}{2}+1; \frac{\pi}{2}\rangle$

(d)



$$\int_{-\infty}^{\infty} f(t-x)g(x) = \int_{t-1}^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_{t-1}^{\frac{\pi}{2}} =$$

$$\underline{1 - \sin(t-1)}$$

$\Rightarrow$ pro $t \in \langle \frac{\pi}{2}; \frac{\pi}{2}+1\rangle$

$$f * g = \begin{cases} \sin x + 1 & \text{pro } x \in \langle -\frac{\pi}{2}; -\frac{\pi}{2}+1\rangle \\ \sin x - \sin(x-1) & \text{pro } x \in \langle -\frac{\pi}{2}+1; \frac{\pi}{2}\rangle \\ 1 - \sin(x-1) & \text{pro } x \in \langle \frac{\pi}{2}; \frac{\pi}{2}+1\rangle \\ 0 & \text{jinde} \end{cases}$$

příklad 3.

$y = x^2$ pro $x \in \langle -\pi ; \pi \rangle$ ... sudá funkce

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x^2 \cos(0 \cdot x)\, dx = \frac{2}{\pi} \int_0^{\pi} x^2 dx = \frac{2}{\pi} \left[\frac{1}{3} x^3 \right]_0^{\pi} =$$

$$\frac{2}{\pi} \left[\frac{1}{3} \pi^3 - 0 \right] = \underline{\frac{2}{3} \pi^2}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} x^2 \cos(nx)\, dx = \ldots$$

$$\int x^2 \cos(nx)\, dx = \left| \begin{matrix} u = x^2 & u' = 2x \\ v' = \cos(nx) & v = \frac{1}{n} \sin(nx) \end{matrix} \right| = \frac{1}{n} x^2 \sin(nx)$$

$$- \frac{2}{n} \int x \sin(nx)\, dx = \left| \begin{matrix} u = x & u' = 1 \\ v' = \sin(nx) & v = \frac{1}{n} [-\cos(nx)] \end{matrix} \right| =$$

$$\frac{1}{n} x^2 \sin(nx) + \frac{2x}{n^2} \cos(nx) + \frac{1}{n} \int \cos(nx)\, dx =$$

$$\frac{2}{\pi} \int_0^{\pi} x^2 \cos(nx)\, dx = \frac{2}{\pi} \left[\frac{x^2}{n} \sin(nx) + \frac{2x}{n^2} \cos(nx) + \frac{1}{n^2} \sin(nx) \right]_0^{\pi} =$$

$$\frac{2}{\pi} \left[0 + \frac{2\pi}{n^2} (-1)^n + 0 - (0 + 0 + 0) \right] = \frac{2}{\cancel{\pi}} \cdot \frac{2\cancel{\pi}}{n^2} (-1)^n = \underline{\frac{4}{n^2} (-1)^n}$$

$$\underline{\underline{F(x) = \frac{2}{3} \pi^2 + \sum_{n=1}^{\infty} \frac{4}{n^2} (-1)^n \cdot \cos(nx)}}$$