

príklad 1.

$$P(1) = 1; P(2) = -1; P(4) = 1; P(5) = 1$$

$$\hookrightarrow \underline{ax^3 + bx^2 + cx + d = P(x)}$$

$$a + b + c + d = 1$$

$$8a + 4b + 2c + d = -1$$

$$64a + 16b + 4c + d = 1$$

$$\underline{125a + 25b + 5c + d = 1}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 & -1 \\ 64 & 16 & 4 & 1 & 1 \\ 125 & 25 & 5 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & -1 \\ 1 & 4 & 16 & 64 & 1 \\ 1 & 5 & 25 & 125 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 & -2 \\ 0 & 3 & 15 & 63 & 0 \\ 0 & 4 & 24 & 124 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 & -2 \\ 0 & 0 & 6 & 42 & 6 \\ 0 & 0 & 12 & 96 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 & -2 \\ 0 & 0 & 1 & 7 & 1 \\ 0 & 0 & 3 & 24 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 & -2 \\ 0 & 0 & 1 & 7 & 1 \\ 0 & 0 & 0 & 3 & 1 \end{pmatrix}$$

$\begin{matrix} \nearrow & \nearrow & \nearrow & \nearrow \\ d & c & b & a \end{matrix}$

$$\Rightarrow \underline{a = -\frac{1}{3}}$$

$$b = 1 + \frac{7}{3} = \underline{\frac{10}{3}}$$

$$c = -2 + \frac{7}{3} - \frac{20}{3} = \underline{-\frac{29}{3}}$$

$$d = 1 + \frac{29}{3} + \frac{1}{3} - \frac{10}{3} = \underline{\frac{23}{3}}$$

$$\Rightarrow \underline{\underline{P(x) = -\frac{1}{3}x^3 + \frac{10}{3}x^2 - \frac{29}{3}x + \frac{23}{3}}}$$

příklad 2.

$$P(1+i)=1; P(1)=1; P(2)=-i$$

$$\hookrightarrow \underline{ax^2 + bx + c = P(x)}$$

$$2ia + (1+i)b + c = 1$$

$$a + b + c = 1$$

$$\underline{4a + 2b + c = -i}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 2 & 1 & -i \\ 2i & (1+i) & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ \boxed{1} & 2 & 4 & -i \\ \boxed{1} & (1+i) & 2i & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -1-i \\ 0 & \boxed{i} & 2i-1 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -1-i \\ 0 & 0 & -1-i & i-1 \end{array} \right)$$

$\nearrow \quad \nearrow \quad \nearrow$
 $c \quad b \quad a$

$$\Rightarrow a = \frac{i-1}{-1-i} = \frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i-i-1}{2} = \underline{\underline{-i}}$$

$$b = -1-i + 3i = \underline{\underline{2i-1}}$$

$$c = 1+1-2i+i = \underline{\underline{2-i}}$$

$$\Rightarrow \underline{\underline{-ix^2 + (2i-1)x + 2-i = P(x)}}$$

příklad 3.

$$P(1)=0; P'(1)=1; P(2)=2; P'(2)=2$$

$$\hookrightarrow P(x) = ax^3 + bx^2 + cx + d$$

$$P'(x) = 3ax^2 + 2bx + c$$

$$0 = a + b + c + d$$

$$1 = 3a + 2b + c$$

$$2 = 8a + 4b + 2c + d$$

$$2 = 12a + 4b + c$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 3 & 2 & 1 & 0 & 1 \\ 8 & 4 & 2 & 1 & 2 \\ 12 & 4 & 1 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & 1 \\ 0 & -4 & -6 & -7 & 2 \\ 0 & -8 & -11 & -12 & 2 \end{pmatrix} \sim$$
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & 1 \\ 0 & 0 & 2 & 5 & -2 \\ 0 & 0 & 5 & 12 & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & -3 & 1 \\ 0 & 0 & 2 & 5 & -2 \\ 0 & 0 & 0 & -\frac{1}{2} & -1 \end{pmatrix}$$

$$\Rightarrow \underline{d = 2}$$

$$2c + 10 = -2 \Rightarrow \underline{c = -6}$$

$$b + (-12) + 6 = -1 \Rightarrow \underline{b = 5}$$

$$a = -5 + 6 - 2 \Rightarrow \underline{a = -1}$$

$$\Rightarrow \underline{\underline{P(x) = -x^3 + 5x^2 - 6x + 2}}$$