Příklad 1. ①

a) $\lim_{n\to\infty} \frac{e^{n^2}}{n!}$ $\qquad a_{n+1} = q \cdot a_n \rightsquigarrow a_n = \frac{e^{n^2}}{n!}$

$\Rightarrow \frac{e^{(n+1)^2}}{(n+1)\,n!} = q \cdot \frac{e^{n^2}}{n!} \sim q = \frac{e^{(n+1)^2}}{(n+1)n!} \cdot \frac{n!}{e^{n^2}} \sim$

$q = \frac{e^{n^2} \cdot e^{2n} \cdot e}{e^{n^2}} \sim q = e^{2n} + 1$

$\Rightarrow a_{n+1} = \frac{e^{2n+1}}{n+1} \cdot a_n$ $\qquad \lim_{n\to\infty} \frac{e^{n^2}}{n!} = \infty$ ✓

b) $\lim_{x\to\infty} \frac{\arccos \sin x}{x} = \left\| \frac{0}{0} \right\| = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = \lim_{x\to\infty} \frac{1}{\sqrt{1-x^2}} = 1$ ✓

c) $\lim_{x\to 0} \left[x - \ln(x)\sqrt{x}\right] = \|\infty - \infty\| = \lim_{x\to\infty} \frac{x - \ln(x)\sqrt{x}}{1} \cdot \frac{x + \ln(x)\sqrt{x}}{x + \ln(x)\sqrt{x}} =$

$\lim_{x\to\infty} \frac{x^2 + \ln^2(x)\cdot x}{x + \ln(x)\sqrt{x}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1 + \frac{\ln^2 x}{x}}{\frac{1}{x} + \frac{\ln x}{x\sqrt{x}}} = \infty$ ✓

pom. $\lim_{x\to\infty} \frac{\ln^2 x}{x} = \left\| \frac{\infty}{\infty} \right\| = \lim_{x\to\infty} 2\ln x \cdot \frac{1}{x} = 0$ ✓

$\lim_{x\to\infty} \frac{\ln x}{\cdot x^{\frac{3}{2}}} = \left\| \frac{\infty}{\infty} \right\| = \lim \frac{\frac{1}{x}}{\frac{3}{2}\sqrt{x}} = \frac{1}{x} \cdot \frac{2}{3} \cdot \frac{1}{\sqrt{x}} = 0$ ✓

~~pozn.? $\lim_{x\to\infty} (x - \ln(x)\sqrt{x}) = \lim_{x\to\infty} \left[x\left(1 - \frac{1}{\sqrt{x}}\ln(x)\right)\right] = \left\|\infty\left(1 - \frac{1}{\infty}\ln(\infty)\right)\right\|$~~

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Příklad 2.

a) $\operatorname{arctg}(\cos x)$

↳ D(f) ... $\operatorname{arctg} x \Rightarrow \mathbb{R}$, $\cos x \Rightarrow \mathbb{R}$ } $D(f) = \mathbb{R}$ ✓

↳ $y' = \frac{1}{1+\cos^2 x}(-\sin x) = \frac{-\sin x}{1+1-\sin^2 x} = -\frac{\sin x}{2-\sin^2 x}$ ✓

b) $\arccos(e^{x^2})$

$e^{x^2} \leq 1 \quad x^2 \leq 0$

$e^{x^2} \leq e^0 \Rightarrow x = 0$

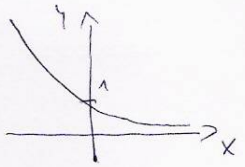
↳ D(f) ... $\arccos x \Rightarrow \langle -1; 1 \rangle$

$e^{x^2} \geq -1$

$e^{x^2} \leq 1$ } $e^{0^2} = 1$; jinak vždy větší $\Rightarrow D(f) = \{0\}$ ✓

↳ $y' = -\frac{1}{\sqrt{1-(e^{x^2})^2}} \cdot e^{x^2} \cdot 2x = -\frac{e^{x^2} \cdot 2x}{\sqrt{1-(e^{x^2})^2}}$ ✓

c) $\arcsin(e^{-x})$



↳ D(f) ... $\arcsin x \Rightarrow \langle -1; 1 \rangle$

$e^{-x} \leq 1$ ... $\langle 0; \infty)$

$e^{x} \geq -1$ ... vždy } $D(f) = \langle 0; \infty)$ ✓

↳ $y' = \frac{1}{\sqrt{1-(e^{-x})^2}} \cdot e^{-x} \cdot (-1) = \frac{-e^{-x}}{\sqrt{1-e^{-2x}}}$ ✓

klady.

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} \overset{?}{=} \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

- víme

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(\frac{a_{n+1}}{a_n} \right) \checkmark$$

$\Rightarrow$ dané limity se ~~nerovnají~~ rovnají ✓