příklad 1.

(1)

a) T_0^4 ... $f(x) = \sin 2x$ ... $f(x_0) = 0$

$f'(x) = 2\cos 2x$ ... $f'(x_0) = 2$

$f''(x) = -4\sin 2x$ ... $f''(x_0) = 0$

$f'''(x) = -8\cos 2x$ ... $f'''(x_0) = -8$

$f^{IV}(x) = 16\sin 2x$ ... $f^{IV}(x_0) = 0$

$$T_0^4 = 0 + 2x + 0\cdot x^2 + (-8)\frac{1}{6}x^3 + 0\cdot x^4 = \underline{\underline{2x - \frac{4}{3}x^3}}$$ ✓

b) T_1^4 ... $f(x) = e^{x^2}$ ... $\underline{f(x_0) = e}$

$f'(x) = 2xe^{x^2}$ ... $\underline{f'(x_0) = 2e}$

$f''(x) = 2e^{x^2} + 2x\cdot 2xe^{x^2} = 2e^{x^2} + 4x^2e^{x^2}$ ... $\underline{f''(x_0) = 6e}$

$f'''(x) = 4xe^{x^2} + 8xe^{x^2} + 4x^2\cdot 2xe^{x^2} = 12xe^{x^2} + 8x^3e^{x^2}$ ... $\underline{f'''(x_0) = 20e}$

$f^{IV}(x) = 12e^{x^2} + 12x\cdot 2xe^{x^2} + 24x^2e^{x^2} + 8x^3\cdot 2x\cdot e^{x^2} =$

$12e^{x^2} + 24x^2e^{x^2} + 24x^2e^{x^2} + 16x^4\cdot e^{x^2} =$

$12e^{x^2} + 48x^2\cdot e^{x^2} + 16x^4e^{x^2}$ ... $\underline{f^{IV}(x) = 76e}$

$$T_1^4 = e + 2e(x-1) + 3e(x-1)^2 + \frac{20}{6}e(x-1)^3 + \frac{76e}{24}(x-1)^4 =$$ ✓

$$e + 2e(x-1) + 3e(x-1)^2 + \frac{10}{3}e(x-1)^3 + \frac{19}{6}e(x-1)^4 =$$

... po roznásobení ...

$$T_1^4 = \underline{\underline{\frac{19}{6}ex^4 - \frac{28}{3}ex^3 + 12ex^2 - \frac{20}{3}ex + \frac{11}{6}e}}$$

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íklad 2.

$T_0^6 \quad \dots \quad f(x) = \cos^2(x) \quad \dots \quad f(x_0) = 1$

$f'(x) = -2\cos x \sin x = -\sin 2x \quad \dots \quad f'(x_0) = 0$

$f''(x) = -2\cos 2x \quad \dots \quad f''(x_0) = -2$

$f'''(x) = 4\sin 2x \quad \dots \quad f'''(x_0) = 0$

$f^{IV}(x) = 8\cos 2x \quad \dots \quad f^{IV}(x_0) = 8$

$f^{V}(x) = -16\sin 2x \quad \dots \quad f^{V}(x_0) = 0$

$f^{VI}(x) = -32\cos 2x \quad \dots \quad f^{VI}(x_0) = -32$

$f^{VII}(x) = 64\sin 2x \quad \dots \quad f^{VII}(x_0) = 0$

$$T_0^6 = 1 + (-2)\frac{1}{2}x^2 + 8\frac{1}{24}x^4 + (-32)\frac{1}{720}x^6 =$$

$$\underline{\underline{1 - x^2 + \frac{1}{3}x^4 - \underbrace{\frac{32}{720}}_{-\frac{2}{45}}x^6}}$$

R na intervalu $\langle 0; \frac{\pi}{4} \rangle$ ... max. hodnota = 64 pro $\frac{\pi}{4}$

$$R = \frac{64 \cdot \pi^7}{7! \cdot 4^7} \approx \underline{\underline{0,00234}} \checkmark$$

klad 3.

a) $f(x) = \dfrac{5(x-2)}{(x-1)^2}$ ... $\underline{\underline{D(f) = \mathbb{R} - \{1\}}}$ ✓

$$f'(x) = \frac{5(x-1)^2 - 5(x-2)\cdot 2(x-1)}{(x-1)^4} = \frac{\cancel{(x-1)}\left[5(x-1) - 5(x-2)\cdot 2\right]}{(x-1)^4} =$$

$$\frac{5x-5-10x+20}{(x-1)^3} = \frac{-5x+15}{(x-1)^3} = 5\,\underline{\frac{3-x}{(x-1)^3}}$$

	$(-\infty;1)$	$(1;3)$	$(3;\infty)$
$3-x$	+	+	−
$(x-1)^3$	−	+	+
	⊖	⊕	⊖

$\Rightarrow$ klesající ... $(-\infty;1),(3;\infty)$

rostoucí ... $(1;3\rangle$

$\underline{\underline{\text{lok. maximum pro } x \in \{3\}}}$ ✓

b) $f(x) = \operatorname{arctg}\left(\dfrac{x}{x-1}\right)$ ... $\underline{\underline{D(f) = \mathbb{R} - \{1\}}}$ ✓

$$f'(x) = \frac{1}{1+\frac{x^2}{(x-1)^2}} \cdot \frac{(x-1)-x}{(x-1)^2} = \frac{\cancel{(x-1)^2}}{(x-1)^2+x^2} \cdot \frac{-1}{\cancel{(x-1)^2}} =$$

platí!

$$\underbrace{\frac{-1}{(x-1)^2+x^2}}_{\text{vždy} < 0} = \frac{-1}{x^2-2x+1+x^2} = \frac{-1}{2x^2-2x+1}$$

~~lok. extrém v $\frac{1}{4}$~~

~~$\frac{-1}{\frac{1}{4}} \neq 0$~~

$\Rightarrow$ nemá lok. extrémy

~~↳ řešení v C~~

$\Rightarrow$ nemá lok. extrémy

$$2x^2-2x+1 = 2\left(x^2-x+\frac{1}{2}\right) = \cancel{2\left(x-\frac{1}{2}\right)^2}$$

$$2\left(x^2-x+\frac{1}{4}-\frac{1}{4}+\frac{1}{2}\right) = 2\left[\left(x-\frac{1}{4}\right)^2-\frac{1}{4}+\frac{1}{2}\right] =$$

$$2\left[\left(x-\frac{1}{4}\right)^2+\frac{1}{4}\right] = \underline{2\left(x-\frac{1}{4}\right)^2+\frac{1}{2}}$$ ← úprava na vrcholový tvar

$\Rightarrow$ $\underline{\underline{\text{lok. minimum pro } x \in \left\{\frac{1}{4}\right\}}}$