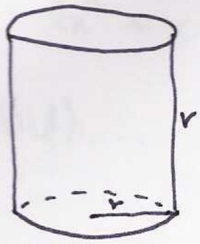
Sada 8

příklad 1.

1



$V = 64\,cm^3$

$V = \pi r^2 v$

$64 = \pi r^2 v$

$v = \dfrac{64}{\pi r^2}$

1) otevřená

$$P_o = \pi r^2 + 2\pi r v = \pi r^2 + 2\pi r \frac{64}{\pi r^2} = \pi r^2 + 128\frac{1}{r}$$

$$P_o' = 2\pi r - 128\frac{1}{r^2} = \frac{2\pi r^3 - 128}{r^2}$$ – r^2 vždy kladné

$$\Rightarrow 2\pi r^3 - 128 = 0$$

$$r^3 = \frac{128}{2\pi}$$

$$\underline{\underline{r = \frac{4}{\sqrt[3]{\pi}}}}$$

$$v = \frac{64}{\pi\left(\frac{4}{\sqrt[3]{\pi}}\right)^2} = \frac{64}{\frac{16\pi}{\sqrt[3]{\pi^2}}} = \underline{\underline{\frac{4\sqrt[3]{\pi^2}}{\pi}}}$$

$\frac{4}{\sqrt[3]{\pi}}$ $\Rightarrow$ lok. minimum

Její rozměry budou $r \doteq 2{,}73\,cm$; $v \doteq 2{,}73\,cm$.

2) zavřená

$$P_z = 2\pi r^2 + 2\pi r v = 2\pi r^2 + 2\pi r \frac{64}{\pi r^2} = 2\pi r^2 + 128\frac{1}{r}$$

$$P_z' = 4\pi r - 128\frac{1}{r^2} = \frac{4\pi r^3 - 128}{r^2}$$ – r^2 vždy kladné

$$\Rightarrow 4\pi r^3 - 128 = 0$$

$$r^3 = \frac{128}{4\pi}$$

$$\underline{\underline{r = \sqrt[3]{\frac{32}{\pi}}}}$$

$$v = \frac{64}{\pi\sqrt[3]{\frac{32}{\pi}}^{2}} = \underline{\underline{\frac{64}{\pi\sqrt[3]{\frac{1024}{\pi^2}}}}}$$

Její rozměry budou $r \doteq 2{,}17\,cm$; $v \doteq 4{,}34\,cm$.

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.2.

$$= \frac{x^2 - x + 2}{2x^2 + x - 1}$$

1) $D(f) \ldots \quad 2x^2 + x - 1 = 0$

$\hookrightarrow D = 1 + 8 = 9$

$x_{1,2} = \frac{-1 \pm 3}{4}$

$x_1 = -1$

$x_2 = \frac{1}{2}$

$\boxed{D(f) = R - \{-1; \frac{1}{2}\}}$

$$\frac{(-x)^2 - (-x) + 2}{2(-x)^2 + (-x) - 1} = \frac{x^2 + x + 2}{2x^2 - x - 1}$$

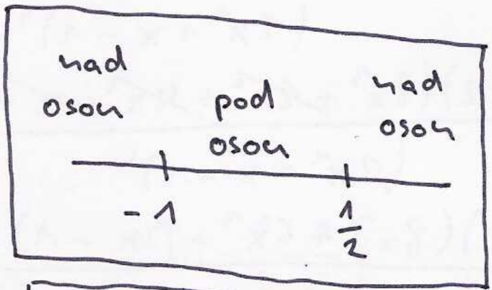
$\Rightarrow$ **NE** sudá / lichá

2) $x^2 - x + 2 = 0$

$\hookrightarrow D = 1 - 8 = -7$

$\Rightarrow$ nemá průsečík s osou x

... vždy kladný



$\boxed{P(x) \ldots \text{není} \quad P(y) = [0; -2]}$

3) $$y' = \frac{(2x-1)(2x^2+x-1) - (x^2-x+2)(4x+1)}{(2x^2+x-1)^2} =$$

$$\frac{\cancel{4x^3} + \cancel{2x^2} - \cancel{2x} - \cancel{2x^2} - \cancel{x} + 1 - (\cancel{4x^3} + x^2 - 4x^2 - x + 8x + 2)}{(2x^2+x-1)^2} =$$

$$\frac{-\cancel{3}x + 1 - \cancel{x^2} + \cancel{4}x^2 + \cancel{x} - \cancel{8}x - 2}{(2x^2+x-1)^2} = \frac{3x^2 - 10x - 1}{\underbrace{(2x^2+x-1)^2}_{\text{vždy kladné}}}$$

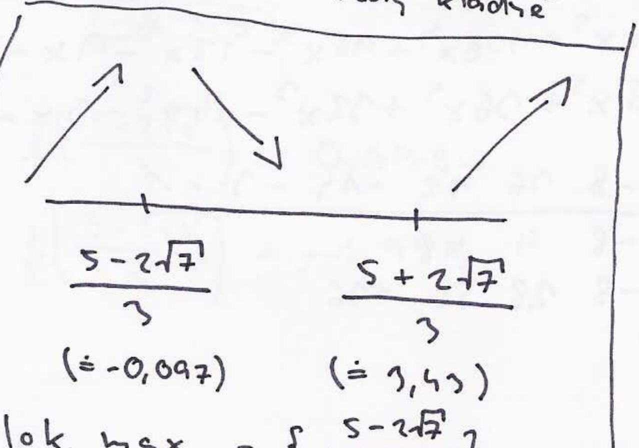
$\Rightarrow 3x^2 - 10x - 1 = 0$

$\hookrightarrow D = 100 + 12 = 112$

$x_{1,2} = \frac{10 \pm \sqrt{112}}{6}$

$x_{1,2} = \frac{10 \pm 4\sqrt{7}}{6}$

$x_{1,2} = \frac{5 \pm 2\sqrt{7}}{3}$



$\Rightarrow$ lok. max. $= \left\{\frac{5-2\sqrt{7}}{3}\right\}$

lok. min. $= \left\{\frac{5+2\sqrt{7}}{3}\right\}$

4) $f''(x) = \dfrac{(6x-10)(2x^2+x-1)^2 - (3x^2-10x-1)\cdot 2(2x^2+x-1)(4x}{(2x^2+x-1)^4}$

$$\frac{(\cancel{12x^3} + \cancel{6x^2} - \cancel{6x} - \cancel{20x^2} - \cancel{10x} + 1)(2x^2+x-1) - \dots}{(2x^2+x-1)^4}$$

$$\frac{(12x^3 - 14x^2 - 16x + 1)(2x^2+x-1) - \dots}{(2x^2+x-1)^4}$$

$$\frac{\cancel{24x^5} + \cancel{12x^4} - \cancel{12x^3} - \cancel{28x^4} - \cancel{14x^3} + \cancel{14x^2} - \cancel{32x^3} - \cancel{16x^2} + 16x + \cancel{2x^2} + x - 1}{(2x^2+x-1)^4}$$

$$\frac{24x^5 - 16x^4 - 58x^3 + 17x - 1 - \dots}{(2x^2+x-1)^4}$$

$$\frac{\dots - (6x^2 - 20x - 2)(8x^3 + \cancel{2x^2} + \cancel{4x^2} + \cancel{x} - \cancel{4x} - 1)}{(2x^2+x-1)^4}$$

$$\frac{\dots - (6x^2 - 20x - 2)(8x^3 + 6x^2 - 3x - 1)}{(2x^2+x-1)^4}$$

$$\frac{\dots - (\cancel{48x^5} + \cancel{36x^4} - \cancel{18x^3} - \cancel{6x^2} - \cancel{160x^4} - \cancel{120x^3} + \cancel{60x^2} + 20x \dots}{(2x^2+x-1)^4}$$

$$\frac{-\cancel{16x^3} - \cancel{12x^2} + 6x + 2}{} = \frac{\dots - (48x^5 - 124x^4 - 154x^3 + 42x^2 + 26x + 2}{(2x^2+x-1)^4}$$

$$\frac{\cancel{24x^5} - \cancel{16x^4} - \cancel{58x^3} + \cancel{17x} - 1 - \cancel{48x^5} + \cancel{124x^4} + \cancel{154x^3} - \cancel{42x^2} - \cancel{26x} - 2}{(2x^2+x-1)^4}$$

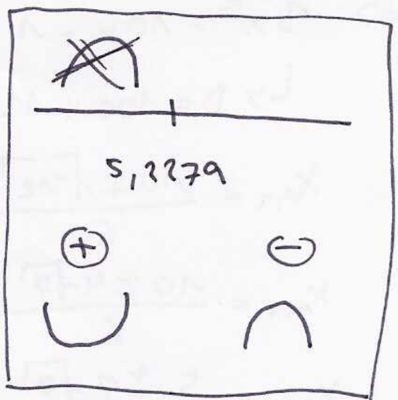
$$\frac{-24x^5 + 108x^4 + 96x^3 - 42x^2 - 9x - 3}{(2x^2+x-1)^4}$$

$$y'' = \frac{-12x^3 + 60x^2 + 12x + 12}{(2x^2+x-1)^3}$$

$\Rightarrow -24x^5 + 108x^4 + 96x^3 - 42x^2 - 9x - 3 = 0$

$-8x^5 + 36x^4 + 32x^3 - 14x^2 - 3x - 1 = 0$

	−8	36	32	−14	−3	−1
−4	−8	4	48	...		
−1	−8	28	50	+36		



$$\lim_{x\to -1^-} \frac{x^2-x+2}{2x^2+x-1} = \lim_{x\to -1^-} \frac{1}{(x+1)} \cdot \lim_{x\to -1^-} \frac{x^2-x+2}{(x-\frac{1}{2})} = +\infty \cdot 4 = +\infty$$

$$\lim_{x\to -1^+} \frac{x^2-x+2}{2x^2+x-1} = \lim_{x\to -1^+} \frac{1}{(x+1)} \cdot \lim_{x\to -1^-} \frac{x^2-x+2}{(x-\frac{1}{2})} = -\infty \cdot 4 = -\infty$$

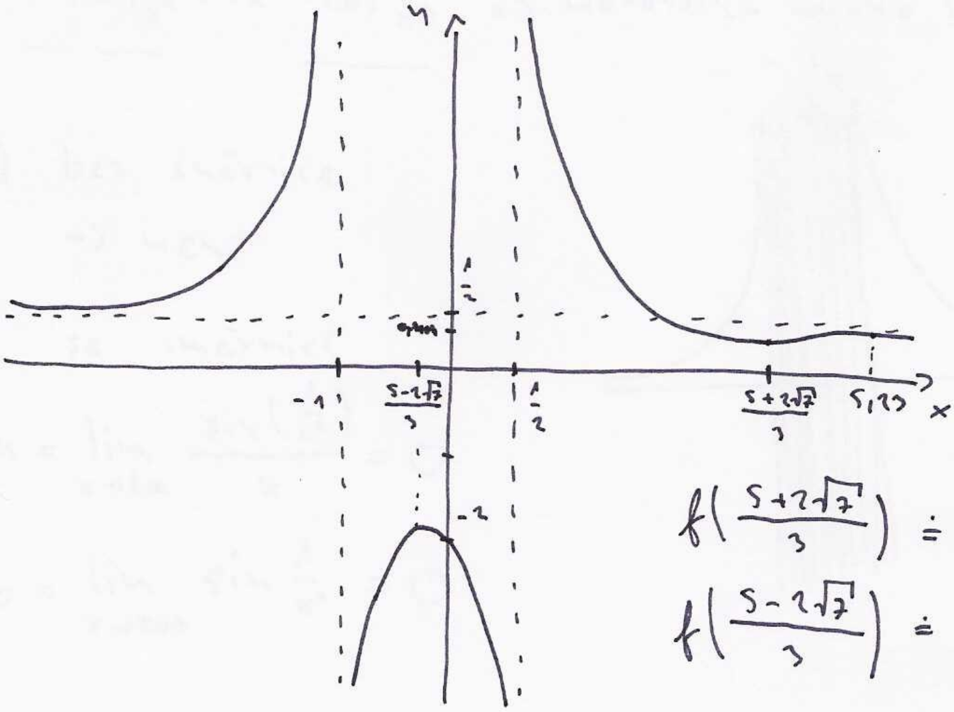
$$\boxed{\Rightarrow \text{asymptoty v bodech } x = \{-1; \tfrac{1}{2}\}}$$

$$a = \lim_{x\to\pm\infty} \frac{\frac{x^2-x+2}{2x^2+x-1}}{x} = \lim_{x\to\pm\infty} \frac{x^2-x+2}{2x^3+x^2-1} = \lim_{x\to\pm\infty} \frac{\cancel{x^3}\left(\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x^3}\right)}{\cancel{x^3}\left(2+\frac{1}{x}-\frac{1}{x^3}\right)}$$

$$= \lim_{x\to\pm\infty} \frac{0}{2} = \underline{0}$$

$$b = \lim_{x\to\pm\infty} \frac{x^2-x+2}{2x^2+x-1} = \lim_{x\to\pm\infty} \frac{\cancel{x^2}\left(1-\frac{1}{x}+\frac{2}{x^2}\right)}{\cancel{x^2}\left(2+\frac{1}{x}-\frac{1}{x^2}\right)} = \lim_{x\to\pm\infty} \frac{1}{2} = \underline{\frac{1}{2}}$$

$$\boxed{\Rightarrow \text{asymptota se směrnicí: } y = \tfrac{1}{2}}$$



$$f\left(\frac{5+2\sqrt{7}}{3}\right) \doteq 0{,}3981$$

$$f\left(\frac{5-2\sqrt{7}}{3}\right) \doteq -1{,}953$$

V.

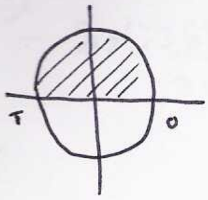
..d 3.

$f(x) = \sin\left(\frac{1}{x^2}\right)$

$\hookrightarrow D(f) = \mathbb{R} - \{0\}$

$\sin\frac{1}{(-x)^2} = \sin\frac{1}{x^2}$

$\Rightarrow$ sudá



2) $\lim_{x\to 0} \frac{1}{x^2} = \infty \Rightarrow \sin\frac{1}{x^2}$ ... velmi mnoho průsečíků s osou x

3) $y' = \cos\left(\frac{1}{x^2}\right)\cdot(-2)\frac{1}{x^3} = 0 \qquad \frac{-2\cos\frac{1}{x^2}}{x^3}$

$\cos\frac{1}{x^2} = 0 \Rightarrow$ nekonečně mnoho lok. max. a lok. min.

4) $y'' = \frac{2\sin\left(\frac{1}{x^2}\right)(-2)\frac{1}{x^3} + 3x^2\cdot 2\cdot\cos\frac{1}{x^2}}{x^6} = -4\sin\frac{1}{x^2} + 6x^5\cdot\cos\frac{1}{x^2}$

$-4\sin\frac{1}{x^2} + 6x^5\cdot\cos\frac{1}{x^2} \Rightarrow$ nekonečně mnoho inflexních bodů

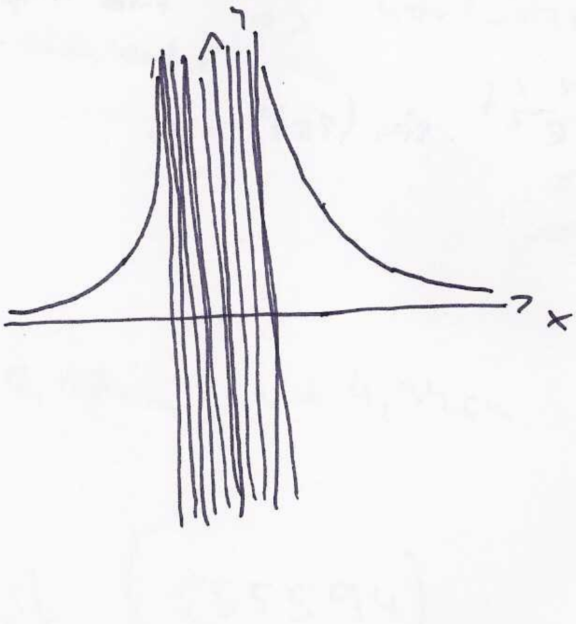
5) bez směrnice

$\Rightarrow$ není

se směrnicí

$a = \lim_{x\to\pm\infty} \frac{\sin\left(\frac{1}{x^2}\right)}{x} = 0$

$b = \lim_{x\to\pm\infty} \sin\frac{1}{x^2} = 0$



b) $f(t) = e^{-\frac{1}{2}t} \sin(2\pi t)$

1) $\boxed{D(t) = \mathbb{R}}$ $\quad - e^{\frac{1}{2}t} \sin(2\pi t) \Rightarrow$ $\boxed{\text{NE sudá lichá}}$

2) $\sin(2\pi t)$ ... periodický nekonečně mnoho průsečíků

e^x ... vždy nad osou x

3) $y' = -\frac{1}{2} e^{-\frac{1}{2}t} \sin(2\pi t) + e^{-\frac{1}{2}t} (\cos 2\pi t) \cdot 2\pi =$

$\underset{0}{\underset{\#}{e^{-\frac{1}{2}t}}} \underbrace{\left(2\pi \cos(2\pi t) - \frac{1}{2} \sin(2\pi t)\right)}_{=0}$

$\Rightarrow$ nekonečně mnoho extrémů

4) ...

5) as. bez směrnice ... nená

se směrnicí

$a = \lim_{t \to \infty} e^{-\frac{1}{2}t} \sin(2\pi t) = \lim_{t \to \infty} \frac{\sin(2\pi t)}{t e^{\frac{1}{2}t}} = 0$

$b = \lim_{t \to \infty} e^{-\frac{1}{2}t} \cdot \sin(2\pi t) = 0$

$\boxed{y = 0}$